Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.3-Tangent/102-4.3.1.3-d-sin-^m-a+b-tan-^n

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [91]. This is test number [102].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (91)	0.00 (0)
Mathematica	98.90 (90)	1.10 (1)
Maple	91.21 (83)	8.79 (8)
Fricas	91.21 (83)	8.79 (8)
Mupad	91.21 (83)	8.79 (8)
Giac	90.11 (82)	9.89 (9)
Maxima	86.81 (79)	13.19 (12)
Sympy	8.79 (8)	91.21 (83)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

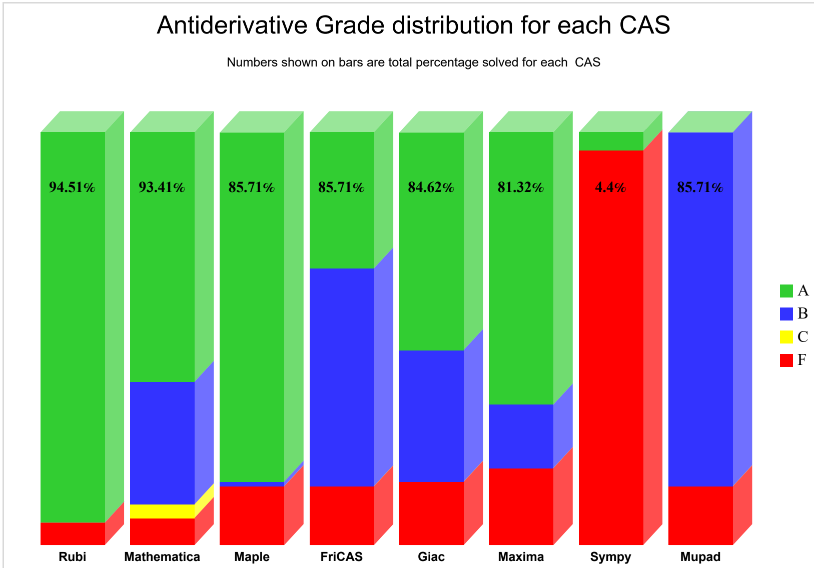
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

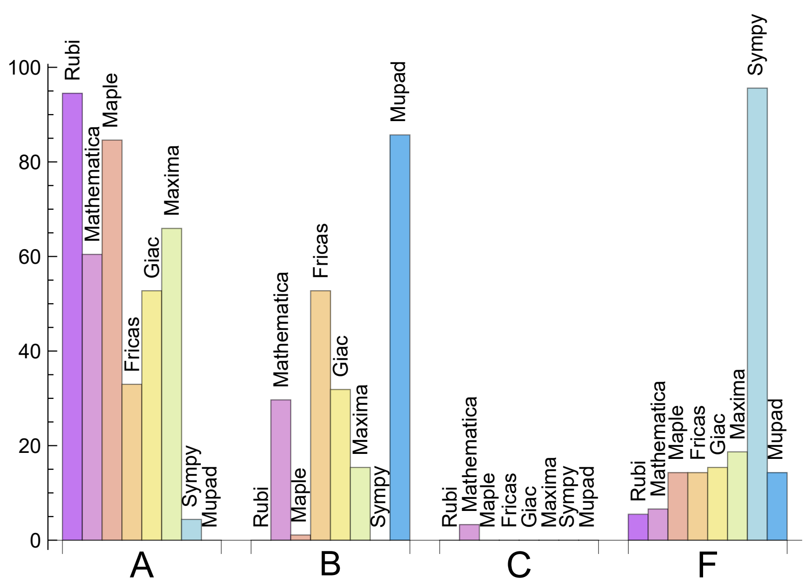
System	% A grade	% B grade	% C grade	% F grade
Rubi	94.505	0.000	0.000	5.495
Maple	84.615	1.099	0.000	14.286
Maxima	65.934	15.385	0.000	18.681
Mathematica	60.440	29.670	3.297	6.593
Giac	52.747	31.868	0.000	15.385
Fricas	32.967	52.747	0.000	14.286
Sympy	4.396	0.000	0.000	95.604
Mupad	0.000	85.714	0.000	14.286

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	1	100.00	0.00	0.00
Fricas	8	100.00	0.00	0.00
Maple	8	100.00	0.00	0.00
Mupad	8	0.00	100.00	0.00
Giac	9	88.89	11.11	0.00
Maxima	12	66.67	0.00	33.33
Sympy	83	84.34	9.64	6.02

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.29
Maxima	0.45
Rubi	0.57
Giac	2.90
Mathematica	3.51
Mupad	5.25
Maple	8.49
Sympy	12.27

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	26.75	0.92	20.00	0.95
Maple	131.22	1.05	106.00	1.00
Rubi	144.00	1.04	109.00	1.00
Maxima	186.62	1.31	120.00	1.10
Mupad	235.73	1.69	146.00	1.25
Mathematica	281.68	1.96	162.00	1.64
Fricas	286.87	2.09	176.00	1.97
Giac	2165.06	15.33	191.00	1.63

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

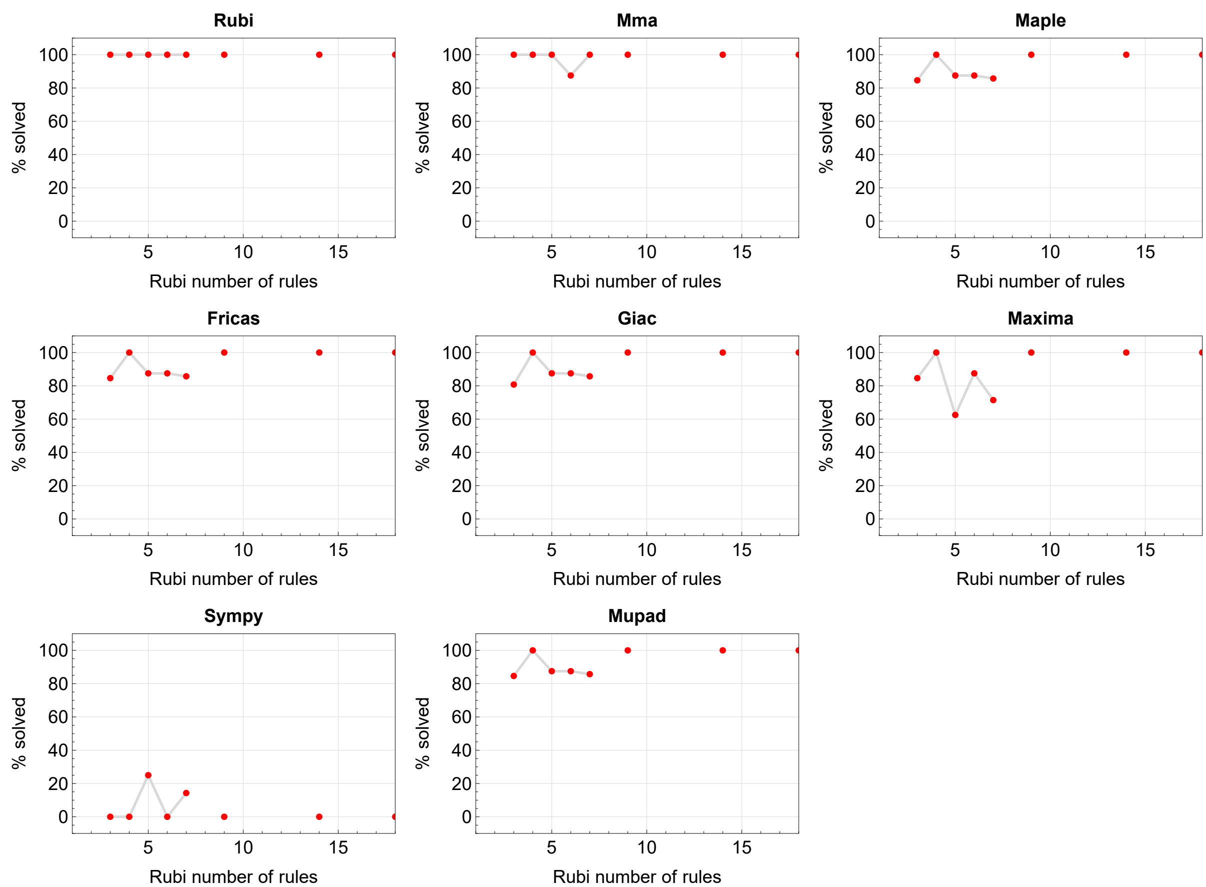


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

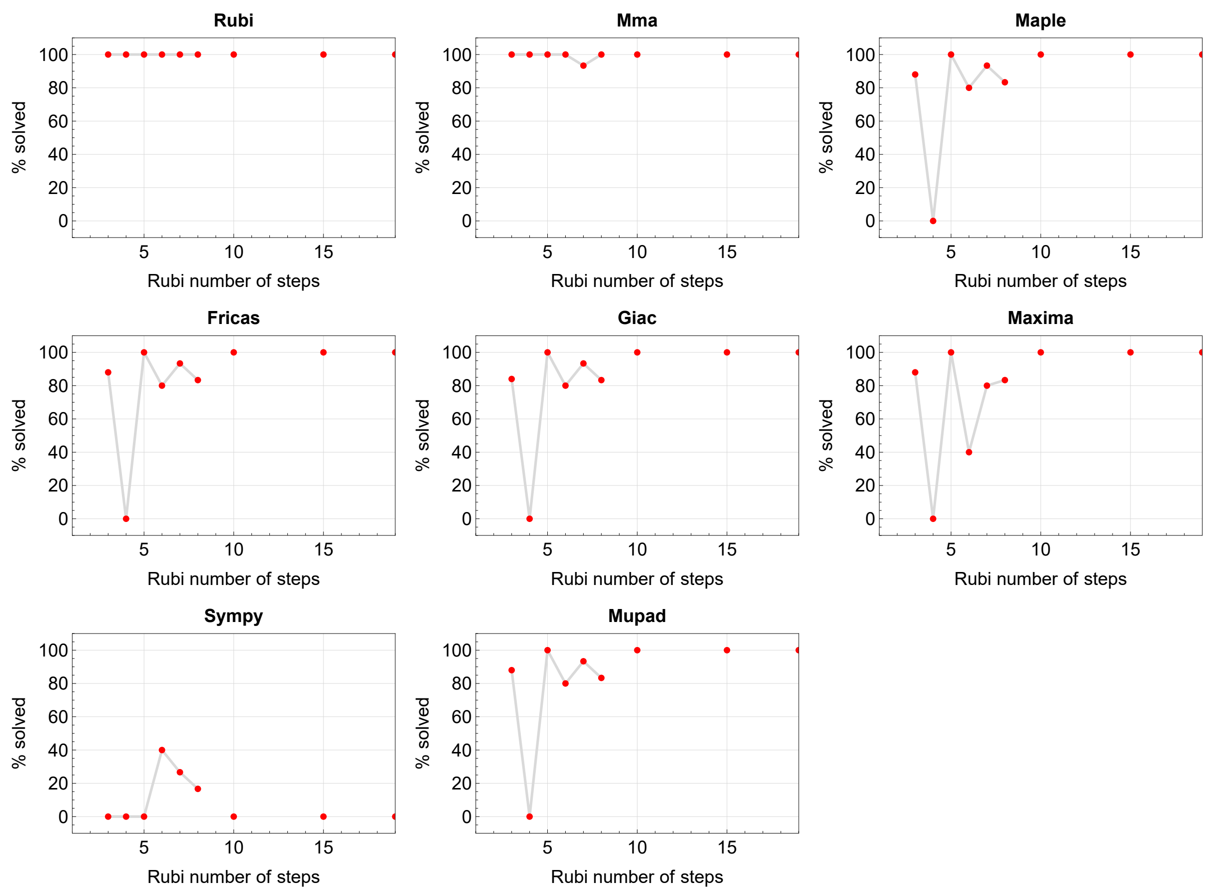


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

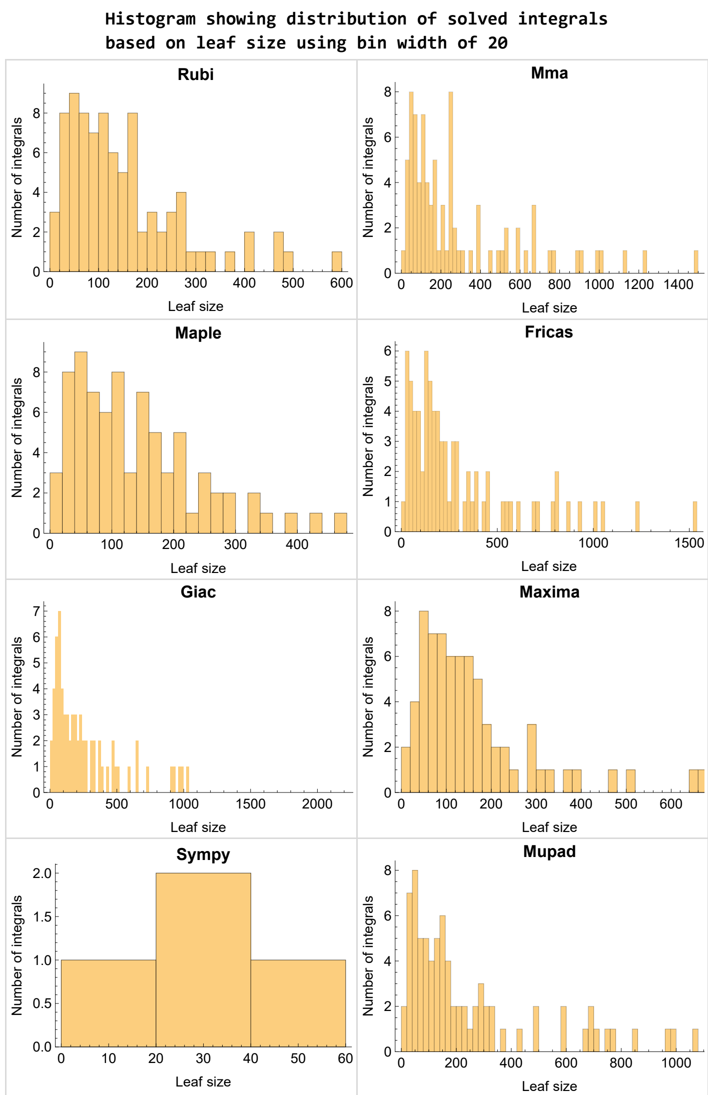


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

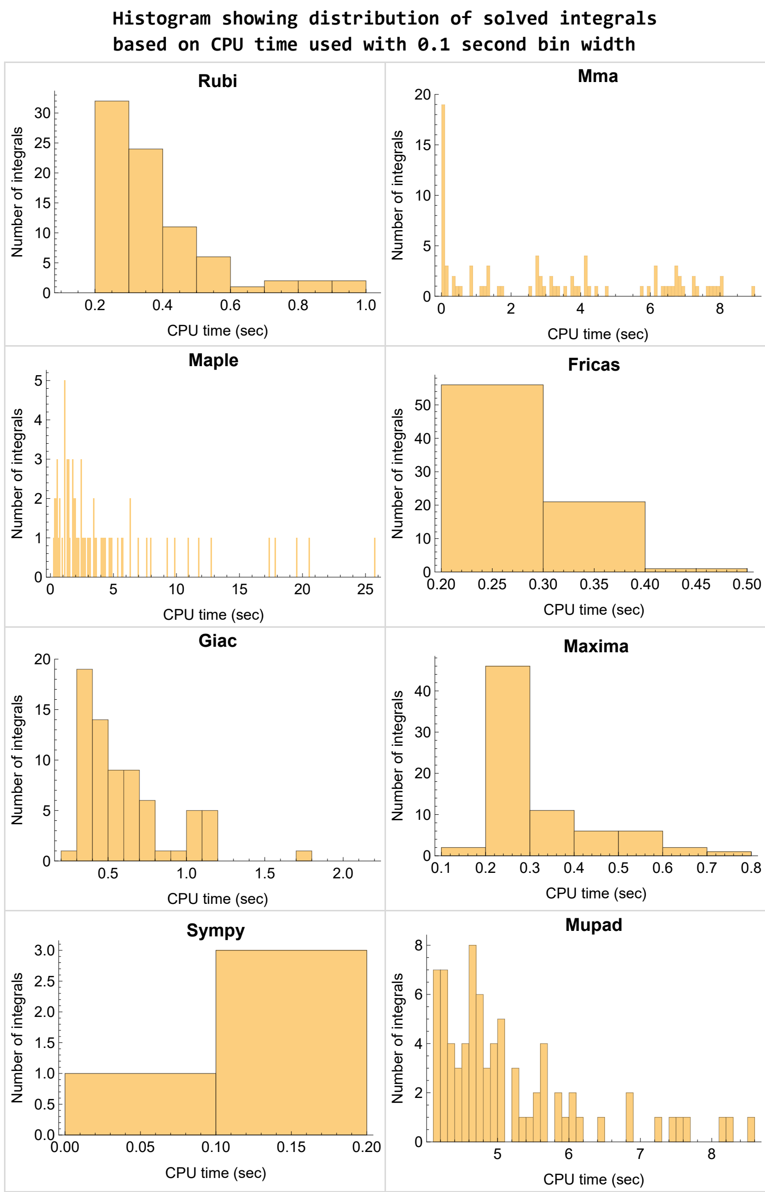


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

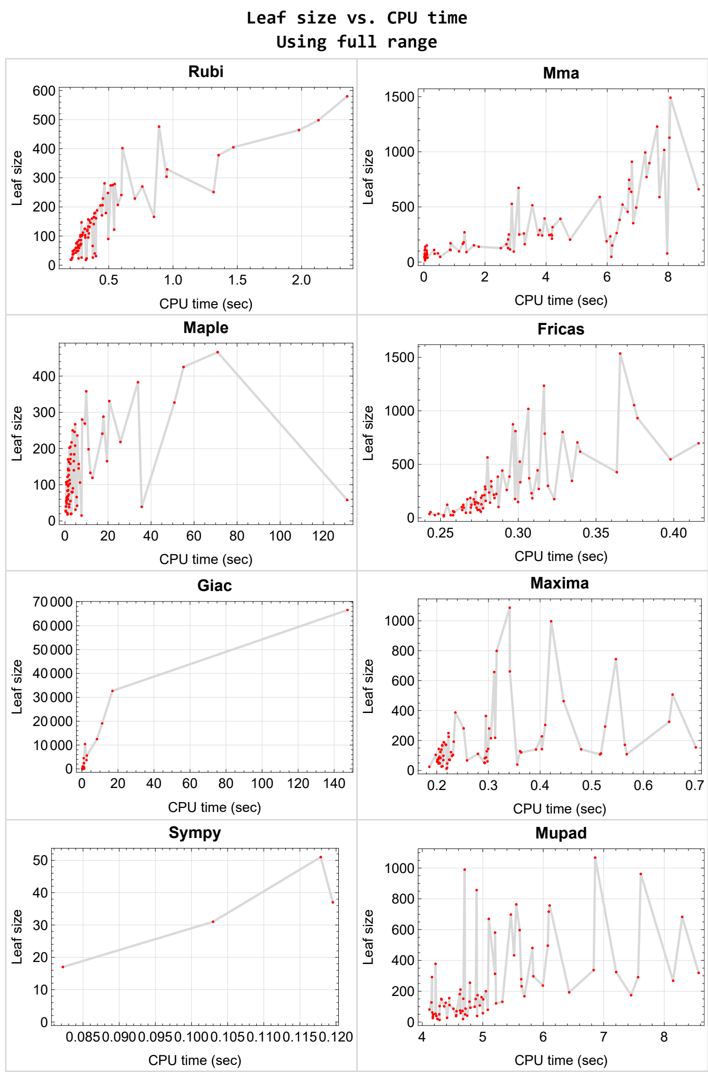


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{83, 88, 89, 90, 91}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {48}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

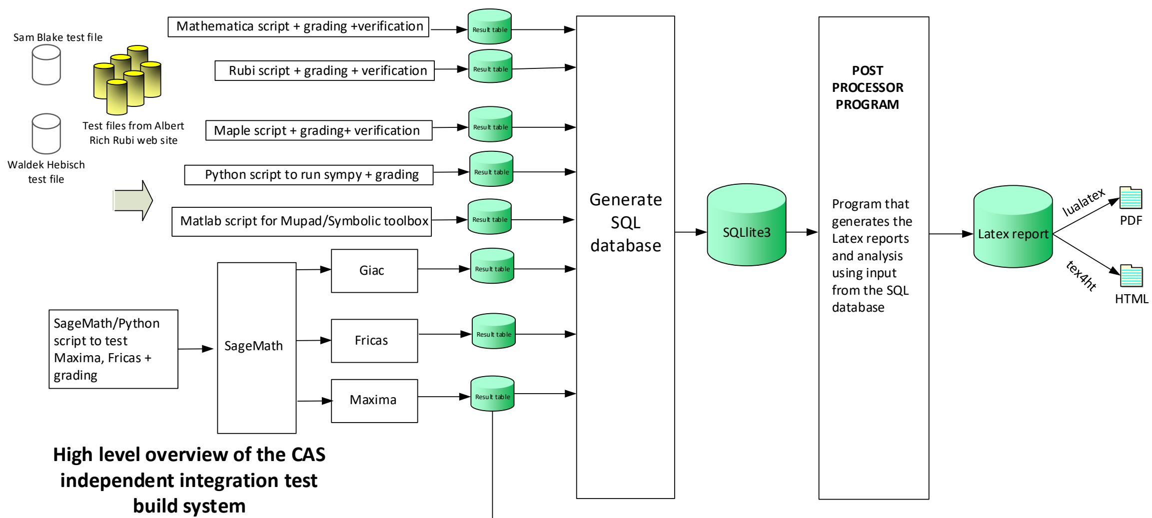
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 19, 21, 23, 25, 29, 31, 33, 36, 38, 42, 45, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 67, 68, 69, 72, 73, 74, 79, 80, 81, 85, 86, 87 }

B grade { 7, 9, 22, 24, 26, 27, 28, 30, 32, 34, 35, 37, 39, 40, 41, 43, 44, 46, 48, 61, 66, 70, 71, 75, 76, 77, 84 }

C grade { 18, 20, 78 }

F normal fail { 82 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

B grade { 7 }

C grade { }

F normal fail { 79, 80, 81, 82, 84, 85, 86, 87 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 6, 11, 12, 13, 14, 15, 22, 23, 24, 25, 32, 33, 34, 35, 41, 42, 43, 44, 45, 50, 51, 52, 53, 54, 57, 59 }

B grade { 5, 7, 8, 9, 10, 16, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 40, 46, 47, 48, 49, 55, 56, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

C grade { }

F normal fail { 79, 80, 81, 82, 84, 85, 86, 87 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 70, 71, 72, 75, 76, 77 }

B grade { 5, 7, 9, 51, 53, 61, 62, 63, 67, 68, 69, 73, 74, 78 }

C grade { }

F normal fail { 79, 80, 81, 82, 84, 85, 86, 87 }

F(-1) timedout fail { }

F(-2) exception fail { 1, 2, 3, 4 }

2.1.6 Giac

A grade { 1, 3, 5, 6, 8, 10, 16, 17, 19, 20, 21, 26, 27, 28, 29, 30, 31, 35, 36, 38, 39, 40, 44, 45, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 70, 71, 72, 75, 76, 77, 78 }

B grade { 2, 4, 7, 9, 11, 12, 13, 14, 15, 18, 22, 23, 24, 25, 32, 33, 34, 37, 42, 43, 52, 54, 61, 62, 67, 68, 69, 73, 74 }

C grade { }

F normal fail { 79, 80, 81, 82, 84, 85, 86, 87 }

F(-1) timedout fail { 41 }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

C grade { }

F normal fail { }

F(-1) timedout fail { 79, 80, 81, 82, 84, 85, 86, 87 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4 }

B grade { }

C grade { }

F normal fail { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87 }

F(-1) timedout fail { 49, 50, 51, 52, 61, 62, 63, 88 }

F(-2) exception fail { 67, 68, 69, 73, 74 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	80	67	39	0	39	51	53	49
N.S.	1	1.03	0.86	0.50	0.00	0.50	0.65	0.68	0.63
time (sec)	N/A	0.248	0.105	35.648	0.000	0.249	0.118	0.326	4.707

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	31	51	31	0	26	37	71	59
N.S.	1	1.07	1.76	1.07	0.00	0.90	1.28	2.45	2.03
time (sec)	N/A	0.398	0.050	4.868	0.000	0.246	0.120	0.328	4.634

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	52	39	19	0	25	31	41	35
N.S.	1	1.04	0.78	0.38	0.00	0.50	0.62	0.82	0.70
time (sec)	N/A	0.234	0.110	2.675	0.000	0.252	0.103	0.296	4.175

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	23	33	18	0	14	17	33	39
N.S.	1	1.21	1.74	0.95	0.00	0.74	0.89	1.74	2.05
time (sec)	N/A	0.334	0.030	1.169	0.000	0.252	0.082	0.311	4.906

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	18	31	21	28	25	0	20	20
N.S.	1	1.12	1.94	1.31	1.75	1.56	0.00	1.25	1.25
time (sec)	N/A	0.336	0.035	1.186	0.211	0.258	0.000	0.307	4.250

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	19	15	20	17	25	0	18	19
N.S.	1	1.06	0.83	1.11	0.94	1.39	0.00	1.00	1.06
time (sec)	N/A	0.213	0.039	2.969	0.220	0.257	0.000	0.325	4.675

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	26	75	42	59	73	0	46	41
N.S.	1	1.08	3.12	1.75	2.46	3.04	0.00	1.92	1.71
time (sec)	N/A	0.384	0.042	5.619	0.200	0.264	0.000	0.329	4.742

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	29	15	12	36	0	12	13
N.S.	1	1.00	1.53	0.79	0.63	1.89	0.00	0.63	0.68
time (sec)	N/A	0.215	0.042	7.676	0.219	0.243	0.000	0.311	4.286

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	139	58	83	123	0	62	57
N.S.	1	1.00	3.48	1.45	2.08	3.08	0.00	1.55	1.42
time (sec)	N/A	0.417	0.044	131.157	0.207	0.254	0.000	0.330	5.001

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	41	28	24	54	0	24	27
N.S.	1	1.00	1.11	0.76	0.65	1.46	0.00	0.65	0.73
time (sec)	N/A	0.228	0.041	0.274	0.186	0.244	0.000	0.302	4.410

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	103	80	91	97	0	10412	121
N.S.	1	1.00	1.02	0.79	0.90	0.96	0.00	103.09	1.20
time (sec)	N/A	0.290	0.092	3.127	0.205	0.270	0.000	1.730	5.219

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	95	82	73	87	74	0	976	155
N.S.	1	1.14	0.99	0.88	1.05	0.89	0.00	11.76	1.87
time (sec)	N/A	0.352	0.095	1.734	0.296	0.275	0.000	0.535	4.446

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	71	60	70	74	0	4486	87
N.S.	1	1.00	1.03	0.87	1.01	1.07	0.00	65.01	1.26
time (sec)	N/A	0.274	0.028	1.351	0.225	0.274	0.000	1.005	4.715

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	59	56	52	52	47	0	373	50
N.S.	1	1.20	1.14	1.06	1.06	0.96	0.00	7.61	1.02
time (sec)	N/A	0.273	0.052	0.586	0.292	0.266	0.000	0.376	4.281

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	48	40	46	49	0	1020	53
N.S.	1	1.00	1.30	1.08	1.24	1.32	0.00	27.57	1.43
time (sec)	N/A	0.225	0.021	0.385	0.220	0.269	0.000	0.424	4.218

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	52	40	46	58	0	49	86
N.S.	1	1.00	2.00	1.54	1.77	2.23	0.00	1.88	3.31
time (sec)	N/A	0.217	0.021	0.454	0.203	0.276	0.000	0.346	4.515

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	23	38	26	25	62	0	35	25
N.S.	1	0.92	1.52	1.04	1.00	2.48	0.00	1.40	1.00
time (sec)	N/A	0.258	0.029	0.426	0.209	0.258	0.000	0.356	4.174

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	107	68	83	142	0	118	149
N.S.	1	1.00	1.78	1.13	1.38	2.37	0.00	1.97	2.48
time (sec)	N/A	0.268	0.030	1.352	0.204	0.282	0.000	0.366	4.883

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	49	78	46	50	122	0	62	49
N.S.	1	0.86	1.37	0.81	0.88	2.14	0.00	1.09	0.86
time (sec)	N/A	0.271	0.032	1.965	0.205	0.265	0.000	0.367	4.180

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	151	90	123	213	0	177	211
N.S.	1	1.00	1.54	0.92	1.26	2.17	0.00	1.81	2.15
time (sec)	N/A	0.305	0.096	3.683	0.207	0.278	0.000	0.392	4.631

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	73	116	66	72	174	0	84	70
N.S.	1	0.84	1.33	0.76	0.83	2.00	0.00	0.97	0.80
time (sec)	N/A	0.291	0.040	5.391	0.202	0.267	0.000	0.390	4.566

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	171	250	140	128	137	0	5604	127
N.S.	1	1.51	2.21	1.24	1.13	1.21	0.00	49.59	1.12
time (sec)	N/A	0.468	3.122	1.872	0.297	0.274	0.000	2.742	4.151

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	152	113	104	126	0	32694	174
N.S.	1	1.00	1.25	0.93	0.85	1.03	0.00	267.98	1.43
time (sec)	N/A	0.365	1.639	2.102	0.198	0.272	0.000	16.946	7.452

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	101	162	109	82	101	0	981	75
N.S.	1	1.33	2.13	1.43	1.08	1.33	0.00	12.91	0.99
time (sec)	N/A	0.297	2.701	1.171	0.293	0.264	0.000	0.619	4.624

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	111	83	67	90	0	2405	93
N.S.	1	1.00	1.63	1.22	0.99	1.32	0.00	35.37	1.37
time (sec)	N/A	0.290	0.855	1.121	0.259	0.277	0.000	1.076	4.795

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	97	56	60	102	0	74	125
N.S.	1	1.00	2.26	1.30	1.40	2.37	0.00	1.72	2.91
time (sec)	N/A	0.250	1.152	0.592	0.298	0.287	0.000	0.447	4.387

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	40	91	38	39	96	0	51	44
N.S.	1	0.95	2.17	0.90	0.93	2.29	0.00	1.21	1.05
time (sec)	N/A	0.238	1.396	1.908	0.356	0.265	0.000	0.458	4.562

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	250	101	122	230	0	172	292
N.S.	1	1.00	2.63	1.06	1.28	2.42	0.00	1.81	3.07
time (sec)	N/A	0.325	2.783	1.808	0.228	0.309	0.000	0.508	4.160

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	73	127	80	69	174	0	91	72
N.S.	1	0.92	1.61	1.01	0.87	2.20	0.00	1.15	0.91
time (sec)	N/A	0.271	2.754	3.493	0.213	0.271	0.000	0.491	4.665

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	994	145	187	333	0	269	378
N.S.	1	1.00	6.02	0.88	1.13	2.02	0.00	1.63	2.29
time (sec)	N/A	0.404	7.247	6.330	0.214	0.301	0.000	0.505	4.219

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	112	114	119	104	240	0	131	107
N.S.	1	0.92	0.93	0.98	0.85	1.97	0.00	1.07	0.88
time (sec)	N/A	0.309	2.820	12.763	0.232	0.273	0.000	0.502	4.951

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	771	184	173	188	0	66584	291
N.S.	1	1.00	3.76	0.90	0.84	0.92	0.00	324.80	1.42
time (sec)	N/A	0.457	7.288	2.718	0.217	0.276	0.000	147.410	7.567

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	125	203	163	113	149	0	2370	151
N.S.	1	1.21	1.97	1.58	1.10	1.45	0.00	23.01	1.47
time (sec)	N/A	0.323	4.787	1.759	0.518	0.272	0.000	1.178	4.679

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	637	144	128	144	0	12476	193
N.S.	1	1.00	4.79	1.08	0.96	1.08	0.00	93.80	1.45
time (sec)	N/A	0.352	6.788	1.467	0.361	0.273	0.000	8.347	6.429

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	241	107	111	148	0	144	278
N.S.	1	1.00	2.80	1.24	1.29	1.72	0.00	1.67	3.23
time (sec)	N/A	0.300	3.873	0.551	0.280	0.300	0.000	0.679	5.636

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	59	126	55	56	127	0	70	62
N.S.	1	0.92	1.97	0.86	0.88	1.98	0.00	1.09	0.97
time (sec)	N/A	0.247	2.520	2.518	0.202	0.276	0.000	0.708	4.165

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	897	140	171	299	0	304	581
N.S.	1	1.00	6.36	0.99	1.21	2.12	0.00	2.16	4.12
time (sec)	N/A	0.364	7.383	4.121	0.218	0.319	0.000	0.747	5.204

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	104	212	106	98	237	0	133	103
N.S.	1	0.92	1.88	0.94	0.87	2.10	0.00	1.18	0.91
time (sec)	N/A	0.293	4.193	6.966	0.213	0.281	0.000	0.695	4.287

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	1229	198	250	427	0	373	698
N.S.	1	1.00	5.37	0.86	1.09	1.86	0.00	1.63	3.05
time (sec)	N/A	0.480	7.635	10.945	0.223	0.363	0.000	0.758	5.465

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	154	515	165	142	343	0	189	146
N.S.	1	0.92	3.08	0.99	0.85	2.05	0.00	1.13	0.87
time (sec)	N/A	0.352	3.550	19.516	0.403	0.283	0.000	0.767	4.318

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	1017	267	218	224	0	0	319
N.S.	1	1.00	3.70	0.97	0.79	0.81	0.00	0.00	1.16
time (sec)	N/A	0.549	7.867	4.709	0.313	0.286	0.000	0.000	8.568

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	157	263	250	154	186	0	3651	161
N.S.	1	1.13	1.89	1.80	1.11	1.34	0.00	26.27	1.16
time (sec)	N/A	0.350	6.311	3.530	0.701	0.269	0.000	2.567	4.979

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	383	217	166	176	0	19074	268
N.S.	1	1.00	2.13	1.21	0.92	0.98	0.00	105.97	1.49
time (sec)	N/A	0.418	6.406	3.035	0.211	0.298	0.000	11.223	8.146

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	352	170	139	175	0	193	496
N.S.	1	1.00	2.98	1.44	1.18	1.48	0.00	1.64	4.20
time (sec)	N/A	0.346	6.852	1.374	0.209	0.323	0.000	1.035	6.075

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	75	162	79	72	159	0	86	81
N.S.	1	0.90	1.95	0.95	0.87	1.92	0.00	1.04	0.98
time (sec)	N/A	0.262	3.302	3.400	0.224	0.279	0.000	1.119	4.119

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	1128	157	188	346	0	300	670
N.S.	1	1.00	7.01	0.98	1.17	2.15	0.00	1.86	4.16
time (sec)	N/A	0.398	8.039	6.386	0.213	0.335	0.000	1.065	5.099

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	125	188	133	120	267	0	161	132
N.S.	1	0.91	1.37	0.97	0.88	1.95	0.00	1.18	0.96
time (sec)	N/A	0.322	5.981	11.767	0.364	0.279	0.000	1.159	4.780

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	1491	241	304	547	0	479	857
N.S.	1	1.00	5.44	0.88	1.11	2.00	0.00	1.75	3.13
time (sec)	N/A	0.529	8.071	17.346	0.410	0.398	0.000	1.093	4.898

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	178	233	218	171	386	0	235	181
N.S.	1	0.92	1.20	1.12	0.88	1.99	0.00	1.21	0.93
time (sec)	N/A	0.381	6.102	25.775	0.564	0.294	0.000	1.175	4.620

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	660	327	387	697	0	647	990
N.S.	1	1.00	1.64	0.81	0.96	1.73	0.00	1.61	2.46
time (sec)	N/A	0.649	8.994	50.856	0.236	0.416	0.000	1.153	4.700

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	251	289	358	658	370	0	464	683
N.S.	1	0.92	1.05	1.31	2.40	1.35	0.00	1.69	2.49
time (sec)	N/A	1.309	3.796	9.862	0.311	0.307	0.000	0.452	8.297

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	241	249	208	280	216	0	334	313
N.S.	1	1.53	1.58	1.32	1.77	1.37	0.00	2.11	1.98
time (sec)	N/A	0.586	4.171	4.691	0.301	0.284	0.000	0.404	5.202

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	166	139	202	364	261	0	241	324
N.S.	1	0.99	0.83	1.20	2.17	1.55	0.00	1.43	1.93
time (sec)	N/A	0.902	1.798	2.096	0.295	0.293	0.000	0.465	7.206

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	146	170	122	144	122	0	184	147
N.S.	1	1.55	1.81	1.30	1.53	1.30	0.00	1.96	1.56
time (sec)	N/A	0.366	0.873	1.153	0.300	0.270	0.000	0.389	5.008

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	79	101	141	185	0	118	110
N.S.	1	1.00	0.88	1.12	1.57	2.06	0.00	1.31	1.22
time (sec)	N/A	0.518	0.460	0.698	0.479	0.285	0.000	0.430	4.450

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	75	63	107	183	0	94	174
N.S.	1	1.00	1.14	0.95	1.62	2.77	0.00	1.42	2.64
time (sec)	N/A	0.398	0.352	0.750	0.516	0.309	0.000	0.432	4.913

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	49	47	48	47	95	0	60	39
N.S.	1	0.98	0.94	0.96	0.94	1.90	0.00	1.20	0.78
time (sec)	N/A	0.260	0.534	0.787	0.206	0.273	0.000	0.422	4.231

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	179	140	215	270	0	209	764
N.S.	1	1.00	1.47	1.15	1.76	2.21	0.00	1.71	6.26
time (sec)	N/A	0.564	1.306	1.507	0.305	0.313	0.000	0.497	5.554

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	103	95	96	97	208	0	144	102
N.S.	1	0.95	0.88	0.89	0.90	1.93	0.00	1.33	0.94
time (sec)	N/A	0.313	2.944	1.425	0.229	0.277	0.000	0.399	4.362

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	160	150	165	168	385	0	251	167
N.S.	1	0.95	0.89	0.98	0.99	2.28	0.00	1.49	0.99
time (sec)	N/A	0.375	6.166	4.075	0.210	0.287	0.000	0.437	5.689

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	405	664	383	799	619	0	735	757
N.S.	1	1.36	2.24	1.29	2.69	2.08	0.00	2.47	2.55
time (sec)	N/A	1.522	6.724	33.908	0.316	0.340	0.000	0.545	6.106

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	304	392	269	507	444	0	513	481
N.S.	1	1.40	1.81	1.24	2.34	2.05	0.00	2.36	2.22
time (sec)	N/A	0.984	4.469	9.209	0.657	0.312	0.000	0.534	5.821

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	207	246	171	293	292	0	263	255
N.S.	1	1.40	1.66	1.16	1.98	1.97	0.00	1.78	1.72
time (sec)	N/A	0.586	3.747	2.489	0.526	0.279	0.000	0.510	4.788

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	67	109	67	74	293	0	74	79
N.S.	1	0.93	1.51	0.93	1.03	4.07	0.00	1.03	1.10
time (sec)	N/A	0.270	0.873	0.964	0.203	0.283	0.000	0.454	5.084

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	131	244	127	144	442	0	203	150
N.S.	1	0.94	1.74	0.91	1.03	3.16	0.00	1.45	1.07
time (sec)	N/A	0.348	4.103	1.725	0.204	0.290	0.000	0.505	4.314

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	206	589	205	225	787	0	332	237
N.S.	1	0.94	2.69	0.94	1.03	3.59	0.00	1.52	1.08
time (sec)	N/A	0.446	7.713	2.484	0.223	0.317	0.000	0.494	5.993

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	498	746	466	1088	932	0	923	1068
N.S.	1	1.30	1.95	1.22	2.85	2.44	0.00	2.42	2.80
time (sec)	N/A	2.191	6.717	70.944	0.341	0.377	0.000	0.686	6.858

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	378	521	331	744	705	0	588	717
N.S.	1	1.33	1.83	1.16	2.61	2.47	0.00	2.06	2.52
time (sec)	N/A	1.402	6.503	20.582	0.547	0.338	0.000	0.682	6.090

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	270	316	236	463	526	0	482	433
N.S.	1	1.31	1.53	1.15	2.25	2.55	0.00	2.34	2.10
time (sec)	N/A	0.796	4.212	5.726	0.446	0.301	0.000	0.636	5.518

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	86	241	85	108	565	0	113	99
N.S.	1	0.91	2.54	0.89	1.14	5.95	0.00	1.19	1.04
time (sec)	N/A	0.281	4.194	1.467	0.568	0.280	0.000	0.568	4.867

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	165	456	158	192	811	0	237	200
N.S.	1	0.93	2.56	0.89	1.08	4.56	0.00	1.33	1.12
time (sec)	N/A	0.383	6.671	2.467	0.233	0.298	0.000	0.600	5.051

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	248	494	246	281	1018	0	382	297
N.S.	1	0.94	1.86	0.93	1.06	3.84	0.00	1.44	1.12
time (sec)	N/A	0.482	6.951	4.388	0.252	0.306	0.000	0.687	5.836

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	464	590	425	997	1053	0	902	962
N.S.	1	1.27	1.61	1.16	2.72	2.88	0.00	2.46	2.63
time (sec)	N/A	2.000	5.757	55.089	0.421	0.374	0.000	0.908	7.612

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	329	395	288	662	802	0	642	597
N.S.	1	1.25	1.50	1.09	2.51	3.04	0.00	2.43	2.26
time (sec)	N/A	0.982	3.952	17.837	0.342	0.329	0.000	0.828	5.610

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	103	259	103	140	874	0	129	131
N.S.	1	0.89	2.23	0.89	1.21	7.53	0.00	1.11	1.13
time (sec)	N/A	0.294	3.276	2.236	0.392	0.297	0.000	0.674	5.324

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	188	528	184	228	1235	0	222	232
N.S.	1	0.92	2.58	0.90	1.11	6.02	0.00	1.08	1.13
time (sec)	N/A	0.407	2.879	4.263	0.403	0.317	0.000	0.702	5.642

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	279	673	280	325	1536	0	428	337
N.S.	1	0.93	2.24	0.93	1.08	5.12	0.00	1.43	1.12
time (sec)	N/A	0.542	3.101	7.964	0.650	0.366	0.000	0.761	6.834

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	41	26	50	55	0	44	38
N.S.	1	1.00	1.58	1.00	1.92	2.12	0.00	1.69	1.46
time (sec)	N/A	0.301	0.120	0.373	0.293	0.259	0.000	0.364	4.558

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	229	205	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.727	2.756	0.000	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	166	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.488	1.274	0.000	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	109	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.361	0.337	0.000	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	765	580	0	0	0	0	0	0	0
N.S.	1	0.76	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.415	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	20	23	23
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.10
time (sec)	N/A	2.847	6.546	1.297	2.514	0.298	54.374	0.717	8.005

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	435	476	910	0	0	0	0	0	0
N.S.	1	1.09	2.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.898	6.810	0.000	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	276	281	270	0	0	0	0	0	0
N.S.	1	1.02	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.480	1.329	0.000	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	6.136	0.000	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	147	78	0	0	0	0	0	0
N.S.	1	1.05	0.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.299	7.965	0.000	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	32	0	23	23
N.S.	1	1.00	1.10	1.00	1.10	1.52	0.00	1.10	1.10
time (sec)	N/A	2.367	4.963	3.144	4.326	0.243	0.000	1.344	7.380

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	21	21	19	21	21
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.00	1.11	1.11
time (sec)	N/A	1.311	2.767	0.568	1.577	0.236	4.717	1.209	5.479

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	21	21	19	21	23
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.00	1.11	1.21
time (sec)	N/A	0.823	4.425	1.531	1.487	0.238	3.695	0.857	4.562

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	20	23	23
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.10
time (sec)	N/A	2.114	23.126	1.567	2.959	0.246	34.947	1.238	6.322

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [51] had the largest ratio of [.857142999999999988]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.03	13	0.385
2	A	7	7	1.07	13	0.538
3	A	6	5	1.04	13	0.385
4	A	7	7	1.21	11	0.636
5	A	7	7	1.12	11	0.636
6	A	5	4	1.06	13	0.308
7	A	7	7	1.08	13	0.538
8	A	6	5	1.00	13	0.385
9	A	7	7	1.00	13	0.538
10	A	6	5	1.00	13	0.385
11	A	3	3	1.00	19	0.158
12	A	8	7	1.14	19	0.368
13	A	3	3	1.00	19	0.158
14	A	8	7	1.20	19	0.368
15	A	3	3	1.00	17	0.176
16	A	3	3	1.00	17	0.176
17	A	5	4	0.92	19	0.211
18	A	3	3	1.00	19	0.158
19	A	5	4	0.86	19	0.211
20	A	3	3	1.00	19	0.158
21	A	5	4	0.84	19	0.211
22	A	7	6	1.51	21	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	3	3	1.00	21	0.143
24	A	8	7	1.33	21	0.333
25	A	3	3	1.00	19	0.158
26	A	3	3	1.00	19	0.158
27	A	5	4	0.95	21	0.190
28	A	3	3	1.00	21	0.143
29	A	5	4	0.92	21	0.190
30	A	3	3	1.00	21	0.143
31	A	5	4	0.92	21	0.190
32	A	3	3	1.00	21	0.143
33	A	8	7	1.21	21	0.333
34	A	3	3	1.00	19	0.158
35	A	3	3	1.00	19	0.158
36	A	5	4	0.92	21	0.190
37	A	3	3	1.00	21	0.143
38	A	5	4	0.92	21	0.190
39	A	3	3	1.00	21	0.143
40	A	5	4	0.92	21	0.190
41	A	3	3	1.00	21	0.143
42	A	8	7	1.13	21	0.333
43	A	3	3	1.00	19	0.158
44	A	3	3	1.00	19	0.158
45	A	5	4	0.90	21	0.190
46	A	3	3	1.00	21	0.143
47	A	5	4	0.91	21	0.190
48	A	3	3	1.00	21	0.143
49	A	5	4	0.92	21	0.190
50	A	3	3	1.00	21	0.143
51	A	19	18	0.92	21	0.857
52	A	8	7	1.53	21	0.333
53	A	15	14	0.99	21	0.667
54	A	8	7	1.55	21	0.333
55	A	10	9	1.00	19	0.474
56	A	5	5	1.00	19	0.263

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	5	4	0.98	21	0.190
58	A	5	5	1.00	21	0.238
59	A	5	4	0.95	21	0.190
60	A	5	4	0.95	21	0.190
61	A	10	9	1.36	21	0.429
62	A	7	6	1.40	21	0.286
63	A	7	6	1.40	21	0.286
64	A	5	4	0.93	21	0.190
65	A	5	4	0.94	21	0.190
66	A	5	4	0.94	21	0.190
67	A	10	9	1.30	21	0.429
68	A	7	6	1.33	21	0.286
69	A	7	6	1.31	21	0.286
70	A	5	4	0.91	21	0.190
71	A	5	4	0.93	21	0.190
72	A	5	4	0.94	21	0.190
73	A	7	6	1.27	21	0.286
74	A	7	6	1.25	21	0.286
75	A	5	4	0.89	21	0.190
76	A	5	4	0.92	21	0.190
77	A	5	4	0.93	21	0.190
78	A	5	5	1.00	9	0.556
79	A	3	3	1.00	21	0.143
80	A	3	3	1.00	21	0.143
81	A	3	3	1.00	19	0.158
82	A	7	6	0.76	21	0.286
83	N/A	8	0	1.00	21	0.000
84	A	8	7	1.09	21	0.333
85	A	8	7	1.02	21	0.333
86	A	4	3	1.00	21	0.143
87	A	6	5	1.05	21	0.238
88	N/A	8	0	1.00	21	0.000
89	N/A	7	0	1.00	19	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
90	N/A	7	0	1.00	19	0.000
91	N/A	8	0	1.00	21	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{\sin^4(x)}{i+\tan(x)}\,dx$

3.1.1 Optimal result

Integrand size = 13, antiderivative size = 78

$$\int \frac{\sin^4(x)}{i+\tan(x)}\,dx = -\frac{ix}{16} - \frac{1}{32(i-\tan(x))^2} - \frac{i}{8(i-\tan(x))} + \frac{i}{24(i+\tan(x))^3} - \frac{5}{32(i+\tan(x))^2} - \frac{3i}{16(i+\tan(x))}$$

output
```
-1/16*I*x-1/32/(I-tan(x))^2-1/8*I/(I-tan(x))+1/24*I/(I+tan(x))^3-5/32/(I+t
an(x))^2-3/16*I/(I+tan(x))
```

3.1.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

$$\int \frac{\sin^4(x)}{i+\tan(x)}\,dx = \frac{\sec(x)(-56i\cos(x) - 9i\cos(3x) + i\cos(5x) + 24\arctan(\tan(x))(\cos(x) - i\sin(x)) - 32\sin(x) - 27\sin(3}{384(i+\tan(x))}$$

input
```
Integrate[Sin[x]^4/(I + Tan[x]),x]
```

output
```
(Sec[x]*((-56*I)*Cos[x] - (9*I)*Cos[3*x] + I*Cos[5*x] + 24*ArcTan[Tan[x]]*
(Cos[x] - I*Sin[x]) - 32*Sin[x] - 27*Sin[3*x] + 5*Sin[5*x]))/(384*(I + Tan
[x]))
```

3.1.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3999, 516, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(x)}{\tan(x)+i} \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(x)^4}{\tan(x)+i}dx$$

$$\Big\downarrow \ 3999$$

$$\int \frac{\tan^4(x)}{(\tan(x)+i)\left(\tan^2(x)+1\right)^3}d\tan(x)$$

$$\Big\downarrow \ 516$$

$$\int \frac{\tan^4(x)}{(\tan(x)-i)^3(\tan(x)+i)^4}d\tan(x)$$

$$\Big\downarrow \ 99$$

$$\int \left(-\frac{i}{16\left(\tan^2(x)+1\right)}-\frac{i}{8(\tan(x)-i)^2}+\frac{3i}{16(\tan(x)+i)^2}+\frac{1}{16(\tan(x)-i)^3}+\frac{5}{16(\tan(x)+i)^3}-\frac{i}{8(\tan(x)+i}\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{1}{16}i\arctan(\tan(x))-\frac{i}{8(-\tan(x)+i)}-\frac{3i}{16(\tan(x)+i)}-\frac{1}{32(-\tan(x)+i)^2}-\frac{5}{32(\tan(x)+i)^2}+\frac{i}{24(\tan(x)+i)^3}$$

input
```
Int[Sin[x]^4/(I + Tan[x]),x]
```

output
```
(-1/16*I)*ArcTan[Tan[x]] - 1/(32*(I - Tan[x])^2) - (I/8)/(I - Tan[x]) + (I
/24)/(I + Tan[x])^3 - 5/(32*(I + Tan[x])^2) - ((3*I)/16)/(I + Tan[x])
```

3.1.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.1.4 Maple [A] (verified)

Time = 35.65 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.50

method	result
risch	$-\frac{ix}{16}-\frac{e^{6ix}}{192}+\frac{\cos(4x)}{32}+\frac{i\sin(4x)}{64}-\frac{5\cos(2x)}{64}+\frac{i\sin(2x)}{64}$
parallelrisch	$-\frac{7}{480}-\frac{ix}{12}+\ln\left(\frac{1}{(i+\tan(x))^{\frac{1}{48}}}\right)+\ln\left(\left(\sec^2(x)\right)^{\frac{1}{96}}\right)+\frac{i\sin(2x)}{64}-\frac{i\sin(6x)}{192}+\frac{i\sin(4x)}{64}-\frac{\cos(6x)}{192}+$ co
default	$\frac{i}{8\tan(x)-8i}-\frac{1}{32(\tan(x)-i)^2}-\frac{\ln(\tan(x)-i)}{32}+\frac{i}{24(i+\tan(x))^3}-\frac{3i}{16(i+\tan(x))}-\frac{5}{32(i+\tan(x))^2}+\frac{\ln(i+\tan(x))}{32}$

input
```
int(sin(x)^4/(I+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/16*I*x-1/192*exp(6*I*x)+1/32*cos(4*x)+1/64*I*sin(4*x)-5/64*cos(2*x)+1/6
4*I*sin(2*x)
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.50

$$\int \frac{\sin^4(x)}{i+\tan(x)}\,dx = \frac{1}{384}\left(-24i\,xe^{(4i\,x)} - 2\,e^{(10i\,x)} + 9\,e^{(8i\,x)} - 12\,e^{(6i\,x)} - 18\,e^{(2i\,x)} + 3\right)e^{(-4i\,x)}$$

input
```
integrate(sin(x)^4/(I+tan(x)),x, algorithm="fricas")
```

output
```
1/384*(-24*I*x*e^(4*I*x) - 2*e^(10*I*x) + 9*e^(8*I*x) - 12*e^(6*I*x) - 18*
e^(2*I*x) + 3)*e^(-4*I*x)
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{\sin^4(x)}{i+\tan(x)}\,dx = -\frac{ix}{16} - \frac{e^{6ix}}{192} + \frac{3e^{4ix}}{128} - \frac{e^{2ix}}{32} - \frac{3e^{-2ix}}{64} + \frac{e^{-4ix}}{128}$$

input
```
integrate(sin(x)**4/(I+tan(x)),x)
```

output
```
-I*x/16 - exp(6*I*x)/192 + 3*exp(4*I*x)/128 - exp(2*I*x)/32 - 3*exp(-2*I*x
)/64 + exp(-4*I*x)/128
```

3.1.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^4(x)}{i+\tan(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sin(x)^4/(I+tan(x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.68

$$\int \frac{\sin^4(x)}{i+\tan(x)}\,dx = -\frac{3i\tan(x)^4+21\tan(x)^3+13i\tan(x)^2+11\tan(x)+8i}{48(\tan(x)+i)^3(\tan(x)-i)^2} + \frac{1}{32}\log(\tan(x)+i) - \frac{1}{32}\log(\tan(x)-i)$$

input
```
integrate(sin(x)^4/(I+tan(x)),x, algorithm="giac")
```

output
```
-1/48*(3*I*tan(x)^4 + 21*tan(x)^3 + 13*I*tan(x)^2 + 11*tan(x) + 8*I)/((tan
(x) + I)^3*(tan(x) - I)^2) + 1/32*log(tan(x) + I) - 1/32*log(tan(x) - I)
```

3.1.9 Mupad [B] (verification not implemented)

Time = 4.71 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.63

$$\int \frac{\sin^4(x)}{i+\tan(x)}\,dx = -\frac{x\,1\mathrm{i}}{16} + \frac{\frac{\tan(x)^4\,1\mathrm{i}}{16}+\frac{7\tan(x)^3}{16}+\frac{\tan(x)^2\,13\mathrm{i}}{48}+\frac{11\tan(x)}{48}+\frac{1}{6}\mathrm{i}}{(\tan(x)+1\mathrm{i})^3\,(1+\tan(x)\,1\mathrm{i})^2}$$

input
```
int(sin(x)^4/(tan(x) + 1i),x)
```

output
```
((11*tan(x))/48 + (tan(x)^2*13i)/48 + (7*tan(x)^3)/16 + (tan(x)^4*1i)/16 +
 1i/6)/((tan(x) + 1i)^3*(tan(x)*1i + 1)^2) - (x*1i)/16
```

3.2 $\int \frac{\sin^3(x)}{i+\tan(x)} \, dx$

3.2.1 Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{\sin^3(x)}{i+\tan(x)} \, dx = \frac{1}{3} i \cos^3(x) - \frac{1}{5} i \cos^5(x) + \frac{\sin^5(x)}{5}$$

output
```
1/3*I*cos(x)^3-1/5*I*cos(x)^5+1/5*sin(x)^5
```

3.2.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.76

$$\int \frac{\sin^3(x)}{i+\tan(x)} \, dx = \frac{1}{8} i \cos(x) + \frac{1}{48} i \cos(3x) - \frac{1}{80} i \cos(5x) + \frac{\sin(x)}{8} - \frac{1}{16} \sin(3x) + \frac{1}{80} \sin(5x)$$

input
```
Integrate[Sin[x]^3/(I + Tan[x]),x]
```

output
```
(I/8)*Cos[x] + (I/48)*Cos[3*x] - (I/80)*Cos[5*x] + Sin[x]/8 - Sin[3*x]/16
+ Sin[5*x]/80
```

3.2.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3042, 4001, 3042, 3587, 3042, 3586, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{\tan(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^3}{\tan(x)+i}dx$$

$$\Big\downarrow \text{4001}$$

$$\int \frac{\sin^3(x)\cos(x)}{\sin(x)+i\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^3\cos(x)}{\sin(x)+i\cos(x)}dx$$

$$\Big\downarrow \text{3587}$$

$$-i\int \cos(x)(\cos(x)+i\sin(x))\sin^3(x)dx$$

$$\Big\downarrow \text{3042}$$

$$-i\int \cos(x)(\cos(x)+i\sin(x))\sin(x)^3dx$$

$$\Big\downarrow \text{3586}$$

$$-i\int \left(i\cos(x)\sin^4(x)+\cos^2(x)\sin^3(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{1}{5}i\sin^5(x)+\frac{\cos^5(x)}{5}-\frac{\cos^3(x)}{3}\right)$$

input
```
Int[Sin[x]^3/(I + Tan[x]),x]
```

output
```
(-I)*(-1/3*Cos[x]^3 + Cos[x]^5/5 + (I/5)*Sin[x]^5)
```

3.2.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3586
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

rule 3587
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[a^p*b^p   Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
 d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && IL
tQ[p, 0]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

3.2.4 Maple [A] (verified)

Time = 4.87 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{i e^{5 i x}}{80}+\frac{i e^{-i x}}{8}+\frac{i \cos \left(3 x\right)}{48}-\frac{\sin \left(3 x\right)}{16}$
parallelrisch	$\frac{2 i}{15}+\frac{i \cos \left(3 x\right)}{48}+\frac{i \cos \left(x\right)}{8}-\frac{i \cos \left(5 x\right)}{80}+\frac{\sin \left(5 x\right)}{80}-\frac{\sin \left(3 x\right)}{16}+\frac{\sin \left(x\right)}{8}$
default	$\frac{i}{\left(\tan \left(\frac{x}{2}\right)+i\right)^{4}}+\frac{2}{5 \left(\tan \left(\frac{x}{2}\right)+i\right)^{5}}-\frac{2}{3 \left(\tan \left(\frac{x}{2}\right)+i\right)^{3}}-\frac{1}{8 \left(\tan \left(\frac{x}{2}\right)+i\right)}-\frac{i}{4 \left(\tan \left(\frac{x}{2}\right)-i\right)^{2}}+\frac{1}{6 \left(\tan \left(\frac{x}{2}\right)-i\right)^{3}}+\frac{1}{8 \tan \left(\frac{x}{2}\right)-8i}$

input
```
int(sin(x)^3/(I+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/80*I*exp(5*I*x)+1/8*I*exp(-I*x)+1/48*I*cos(3*x)-1/16*sin(3*x)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{\sin^3(x)}{i+\tan(x)}\,dx = \frac{1}{240}\left(-3i\,e^{(8i\,x)}+10i\,e^{(6i\,x)}+30i\,e^{(2i\,x)}-5i\right)e^{(-3i\,x)}$$

input
```
integrate(sin(x)^3/(I+tan(x)),x, algorithm="fricas")
```

output
```
1/240*(-3*I*e^(8*I*x) + 10*I*e^(6*I*x) + 30*I*e^(2*I*x) - 5*I)*e^(-3*I*x)
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{\sin^3(x)}{i+\tan(x)}\,dx = -\frac{ie^{5ix}}{80}+\frac{ie^{3ix}}{24}+\frac{ie^{-ix}}{8}-\frac{ie^{-3ix}}{48}$$

input
```
integrate(sin(x)**3/(I+tan(x)),x)
```

output
```
-I*exp(5*I*x)/80 + I*exp(3*I*x)/24 + I*exp(-I*x)/8 - I*exp(-3*I*x)/48
```

3.2.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^3(x)}{i+\tan(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sin(x)^3/(I+tan(x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.2.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(19) = 38$.

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.45

$$\int \frac{\sin^3(x)}{i+\tan(x)}\,dx = -\frac{-3i\tan\left(\frac{1}{2}x\right)^2 - 12\tan\left(\frac{1}{2}x\right) + 5i}{24\left(-i\tan\left(\frac{1}{2}x\right) - 1\right)^3} - \frac{15\tan\left(\frac{1}{2}x\right)^4 + 60i\tan\left(\frac{1}{2}x\right)^3 - 10\tan\left(\frac{1}{2}x\right)^2 - 20i\tan\left(\frac{1}{2}x\right) + 7}{120\left(\tan\left(\frac{1}{2}x\right) + i\right)^5}$$

input
```
integrate(sin(x)^3/(I+tan(x)),x, algorithm="giac")
```

output
```
-1/24*(-3*I*tan(1/2*x)^2 - 12*tan(1/2*x) + 5*I)/(-I*tan(1/2*x) - 1)^3 - 1/
120*(15*tan(1/2*x)^4 + 60*I*tan(1/2*x)^3 - 10*tan(1/2*x)^2 - 20*I*tan(1/2*
x) + 7)/(tan(1/2*x) + I)^5
```

3.2.9 Mupad [B] (verification not implemented)

Time = 4.63 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.03

$$\int \frac{\sin^3(x)}{i+\tan(x)}\,dx = -\frac{4\left(-\tan\left(\frac{x}{2}\right)^4 15\mathrm{i} + 6\tan\left(\frac{x}{2}\right)^3 + \tan\left(\frac{x}{2}\right)^2 2\mathrm{i} + 2\tan\left(\frac{x}{2}\right) + 1\mathrm{i}\right)}{15\left(-1+\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)^5\left(1+\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)^3}$$

input
```
int(sin(x)^3/(tan(x) + 1i),x)
```

output
```
-(4*(2*tan(x/2) + tan(x/2)^2*2i + 6*tan(x/2)^3 - tan(x/2)^4*15i + 1i))/(15
*(tan(x/2)*1i - 1)^5*(tan(x/2)*1i + 1)^3)
```

3.3 $\int \frac{\sin^2(x)}{i+\tan(x)} \, dx$

3.3.1 Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \frac{\sin^2(x)}{i+\tan(x)} \, dx = -\frac{ix}{8} - \frac{i}{8(i-\tan(x))} - \frac{1}{8(i+\tan(x))^2} - \frac{i}{4(i+\tan(x))}$$

output
```
-1/8*I*x-1/8*I/(I-tan(x))-1/8/(I+tan(x))^2-1/4*I/(I+tan(x))
```

3.3.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{\sin^2(x)}{i+\tan(x)} \, dx = -\frac{i(3+\cos(2x)-3i\sin(2x)+2\arctan(\tan(x))(i+\tan(x)))}{16(i+\tan(x))}$$

input
```
Integrate[Sin[x]^2/(I + Tan[x]),x]
```

output
```
((-1/16*I)*(3 + Cos[2*x] - (3*I)*Sin[2*x] + 2*ArcTan[Tan[x]]*(I + Tan[x]))
)/(I + Tan[x])
```

3.3.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3999, 516, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{\tan(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^2}{\tan(x)+i}dx$$

$$\Big\downarrow \text{3999}$$

$$\int \frac{\tan^2(x)}{(\tan(x)+i)\left(\tan^2(x)+1\right)^2}d\tan(x)$$

$$\Big\downarrow \text{516}$$

$$\int \frac{\tan^2(x)}{(\tan(x)-i)^2(\tan(x)+i)^3}d\tan(x)$$

$$\Big\downarrow \text{99}$$

$$\int \left(-\frac{i}{8\left(\tan^2(x)+1\right)}-\frac{i}{8(\tan(x)-i)^2}+\frac{i}{4(\tan(x)+i)^2}+\frac{1}{4(\tan(x)+i)^3}\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{8}i\arctan(\tan(x))-\frac{i}{8(-\tan(x)+i)}-\frac{i}{4(\tan(x)+i)}-\frac{1}{8(\tan(x)+i)^2}$$

input
```
Int[Sin[x]^2/(I + Tan[x]),x]
```

output
```
(-1/8*I)*ArcTan[Tan[x]] - (I/8)/(I - Tan[x]) - 1/(8*(I + Tan[x])^2) - (I/4
)/(I + Tan[x])
```

3.3.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.3.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.38

method	result
risch	$-\frac{ix}{8}+\frac{{\mathrm e}^{4 i x}}{32}-\frac{\cos\left(2 x\right)}{8}$
parallelrisch	$-\frac{i x}{4}-\frac{7}{96}+\ln\left(\frac{1}{\left(i+\tan\left(x\right)\right)^{\frac{1}{8}}}\right)+\ln\left(\left(\sec^{2}\left(x\right)\right)^{\frac{1}{16}}\right)+\frac{i \sin\left(4 x\right)}{32}+\frac{\cos\left(4 x\right)}{32}-\frac{\cos\left(2 x\right)}{8}$
default	$\frac{i}{8 \tan\left(x\right)-8 i}-\frac{\ln\left(\tan\left(x\right)-i\right)}{16}-\frac{i}{4 \left(i+\tan\left(x\right)\right)}-\frac{1}{8 \left(i+\tan\left(x\right)\right)^{2}}+\frac{\ln\left(i+\tan\left(x\right)\right)}{16}$
norman	$-\frac{1}{4}-\frac{\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{4}-\frac{\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\frac{i x}{8}+i x \tan\left(x\right) \tan\left(\frac{x}{2}\right)-\frac{3 i x \left(\tan^{2}\left(x\right)\right) \left(\tan^{4}\left(\frac{x}{2}\right)\right)}{8}-i x \tan\left(x\right) \left(\tan^{3}\left(\frac{x}{2}\right)\right)+\frac{i x \left(\tan^{2}\left(x\right)\right) \left(\tan^{2}\left(\frac{x}{2}\right)\right)}{4}-$

input
```
int(sin(x)^2/(I+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*I*x+1/32*exp(4*I*x)-1/8*cos(2*x)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(x)}{i+\tan(x)}\,dx = \frac{1}{32}\left(-4i\,xe^{(2i\,x)} + e^{(6i\,x)} - 2\,e^{(4i\,x)} - 2\right)e^{(-2i\,x)}$$

input
```
integrate(sin(x)^2/(I+tan(x)),x, algorithm="fricas")
```

output
```
1/32*(-4*I*x*e^(2*I*x) + e^(6*I*x) - 2*e^(4*I*x) - 2)*e^(-2*I*x)
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.62

$$\int \frac{\sin^2(x)}{i+\tan(x)}\,dx = -\frac{ix}{8} + \frac{e^{4ix}}{32} - \frac{e^{2ix}}{16} - \frac{e^{-2ix}}{16}$$

input
```
integrate(sin(x)**2/(I+tan(x)),x)
```

output
```
-I*x/8 + exp(4*I*x)/32 - exp(2*I*x)/16 - exp(-2*I*x)/16
```

3.3.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(x)}{i+\tan(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sin(x)^2/(I+tan(x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{\sin^2(x)}{i+\tan(x)}\,dx = -\frac{i\tan(x)^2+3\tan(x)+2i}{8(\tan(x)+i)^2(\tan(x)-i)}+\frac{1}{16}\log(\tan(x)+i)-\frac{1}{16}\log(\tan(x)-i)$$

input
```
integrate(sin(x)^2/(I+tan(x)),x, algorithm="giac")
```

output
```
-1/8*(I*tan(x)^2 + 3*tan(x) + 2*I)/((tan(x) + I)^2*(tan(x) - I)) + 1/16*lo
g(tan(x) + I) - 1/16*log(tan(x) - I)
```

3.3.9 Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int \frac{\sin^2(x)}{i+\tan(x)}\,dx = -\frac{x\,1\mathrm{i}}{8}+\frac{\frac{\tan(x)^2}{8}-\frac{\tan(x)\,3\mathrm{i}}{8}+\frac{1}{4}}{(\tan(x)+1\mathrm{i})^2\,(1+\tan(x)\,1\mathrm{i})}$$

input
```
int(sin(x)^2/(tan(x) + 1i),x)
```

output
```
(tan(x)^2/8 - (tan(x)*3i)/8 + 1/4)/((tan(x) + 1i)^2*(tan(x)*1i + 1)) - (x*
1i)/8
```

3.4 $\int \frac{\sin(x)}{i+\tan(x)} \, dx$

3.4.1 Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\sin(x)}{i+\tan(x)} \, dx = \frac{1}{3} i \cos^3(x) + \frac{\sin^3(x)}{3}$$

output
```
1/3*I*cos(x)^3+1/3*sin(x)^3
```

3.4.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{\sin(x)}{i+\tan(x)} \, dx = \frac{1}{4} i \cos(x) + \frac{1}{12} i \cos(3x) + \frac{\sin(x)}{4} - \frac{1}{12} \sin(3x)$$

input
```
Integrate[Sin[x]/(I + Tan[x]),x]
```

output
```
(I/4)*Cos[x] + (I/12)*Cos[3*x] + Sin[x]/4 - Sin[3*x]/12
```

3.4.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3042, 4001, 3042, 3587, 3042, 3586, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\tan(x)+i}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\tan(x)+i}dx$$

$$\Big\downarrow \text{4001}$$

$$\int \frac{\sin(x)\cos(x)}{\sin(x)+i\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)\cos(x)}{\sin(x)+i\cos(x)}dx$$

$$\Big\downarrow \text{3587}$$

$$-i\int \cos(x)(\cos(x)+i\sin(x))\sin(x)dx$$

$$\Big\downarrow \text{3042}$$

$$-i\int \cos(x)(\cos(x)+i\sin(x))\sin(x)dx$$

$$\Big\downarrow \text{3586}$$

$$-i\int \left(\sin(x)\cos^2(x)+i\sin^2(x)\cos(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(-\frac{\cos^3(x)}{3}+\frac{1}{3}i\sin^3(x)\right)$$

input
```
Int[Sin[x]/(I + Tan[x]),x]
```

output
```
(-I)*(-1/3*Cos[x]^3 + (I/3)*Sin[x]^3)
```

3.4.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3586
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

rule 3587
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[a^p*b^p   Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
 d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && IL
tQ[p, 0]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

3.4.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{i e^{3 i x}}{12}+\frac{i e^{-i x}}{4}$	18
parallelrisch	$\frac{2 i}{3}+\frac{i \cos\left(3 x\right)}{12}+\frac{i \cos\left(x\right)}{4}-\frac{\sin\left(3 x\right)}{12}+\frac{\sin\left(x\right)}{4}$	26
default	$\frac{1}{2 \tan\left(\frac{x}{2}\right)-2 i}+\frac{i}{\left(\tan\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{2}{3 \left(\tan\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{1}{2 \left(\tan\left(\frac{x}{2}\right)+i\right)}$	47
norman	$\frac{\frac{i \left(\tan^{2}\left(\frac{x}{2}\right)\right)}{3}+\frac{4 i \left(\tan^{2}\left(x\right)\right)}{3}+\frac{2 \left(\tan^{2}\left(x\right)\right) \tan\left(\frac{x}{2}\right)}{3}-\frac{\left(\tan^{2}\left(\frac{x}{2}\right)\right) \tan\left(x\right)}{3}-\frac{4 i \tan\left(x\right) \tan\left(\frac{x}{2}\right)}{3}+\frac{\tan\left(x\right)}{3}-\frac{2 \tan\left(\frac{x}{2}\right)}{3}+i}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right) \left(\tan^{2}\left(x\right)+1\right)}$	78

input
```
int(sin(x)/(I+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
1/12*I*exp(3*I*x)+1/4*I*exp(-I*x)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{\sin(x)}{i+\tan(x)}\,dx = \frac{1}{12}\left(i\,e^{(4i\,x)}+3i\right)e^{(-i\,x)}$$

input
```
integrate(sin(x)/(I+tan(x)),x, algorithm="fricas")
```

output
```
1/12*(I*e^(4*I*x) + 3*I)*e^(-I*x)
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\sin(x)}{i+\tan(x)}\,dx = \frac{ie^{3ix}}{12}+\frac{ie^{-ix}}{4}$$

input
```
integrate(sin(x)/(I+tan(x)),x)
```

output
```
I*exp(3*I*x)/12 + I*exp(-I*x)/4
```

3.4.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin(x)}{i+\tan(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sin(x)/(I+tan(x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.4.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(13) = 26.

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{\sin(x)}{i+\tan(x)}\,dx = -\frac{i}{2\left(-i\tan\left(\frac{1}{2}x\right)-1\right)} - \frac{3\tan\left(\frac{1}{2}x\right)^2-1}{6\left(\tan\left(\frac{1}{2}x\right)+i\right)^3}$$

input
```
integrate(sin(x)/(I+tan(x)),x, algorithm="giac")
```

output
```
-1/2*I/(-I*tan(1/2*x) - 1) - 1/6*(3*tan(1/2*x)^2 - 1)/(tan(1/2*x) + I)^3
```

3.4.9 Mupad [B] (verification not implemented)

Time = 4.91 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{\sin(x)}{i+\tan(x)}\,dx = -\frac{2\left(3\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)\,2\mathrm{i}-1\right)}{3\,\left(1+\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\,\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)^3}$$

input
```
int(sin(x)/(tan(x) + 1i),x)
```

output
```
-(2*(tan(x/2)*2i + 3*tan(x/2)^2 - 1))/(3*(tan(x/2)*1i + 1)*(tan(x/2) + 1i)
^3)
```

3.5 $\int \frac{\csc(x)}{i+\tan(x)} dx$

3.5.1 Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{\csc(x)}{i+\tan(x)} dx = i\text{arctanh}(\cos(x)) - i\cos(x) + \sin(x)$$

output
```
I*arctanh(cos(x))-I*cos(x)+sin(x)
```

3.5.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \frac{\csc(x)}{i+\tan(x)} dx = -i\cos(x) + i\log\left(\cos\left(\frac{x}{2}\right)\right) - i\log\left(\sin\left(\frac{x}{2}\right)\right) + \sin(x)$$

input
```
Integrate[Csc[x]/(I + Tan[x]),x]
```

output
```
(-I)*Cos[x] + I*Log[Cos[x/2]] - I*Log[Sin[x/2]] + Sin[x]
```

3.5.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3042, 4001, 3042, 3587, 3042, 3586, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{\tan(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(x)(\tan(x)+i)}dx$$

$$\Big\downarrow \text{4001}$$

$$\int \frac{\cot(x)}{\sin(x)+i\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\sin(x)(\sin(x)+i\cos(x))}dx$$

$$\Big\downarrow \text{3587}$$

$$-i\int \cot(x)(\cos(x)+i\sin(x))dx$$

$$\Big\downarrow \text{3042}$$

$$-i\int \frac{\cos(x)(\cos(x)+i\sin(x))}{\sin(x)}dx$$

$$\Big\downarrow \text{3586}$$

$$-i\int (\cot(x)\cos(x)+i\cos(x))dx$$

$$\Big\downarrow \text{2009}$$

$$-i(-\text{arctanh}(\cos(x))+i\sin(x)+\cos(x))$$

input
```
Int[Csc[x]/(I + Tan[x]),x]
```

output
```
(-I)*(-ArcTanh[Cos[x]] + Cos[x] + I*Sin[x])
```

3.5.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3586
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

rule 3587
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[a^p*b^p   Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
 d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && IL
tQ[p, 0]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

3.5.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

method	result	size
default	$\frac{2}{\tan\left(\frac{x}{2}\right)+i}-i\ln\left(\tan\left(\frac{x}{2}\right)\right)$	21
risch	$-i\mathrm{e}^{ix}+i\ln\left(\mathrm{e}^{ix}+1\right)-i\ln\left(\mathrm{e}^{ix}-1\right)$	32

input
```
int(csc(x)/(I+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(tan(1/2*x)+I)-I*ln(tan(1/2*x))
```

3.5.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{\csc(x)}{i+\tan(x)}\,dx = -i\,e^{(i\,x)} + i\,\log\left(e^{(i\,x)}+1\right) - i\,\log\left(e^{(i\,x)}-1\right)$$

input
```
integrate(csc(x)/(I+tan(x)),x, algorithm="fricas")
```

output
```
-I*e^(I*x) + I*log(e^(I*x) + 1) - I*log(e^(I*x) - 1)
```

3.5.6 Sympy [F]

$$\int \frac{\csc(x)}{i+\tan(x)}\,dx = \int \frac{\csc(x)}{\tan(x)+i}\,dx$$

input
```
integrate(csc(x)/(I+tan(x)),x)
```

output
```
Integral(csc(x)/(tan(x) + I), x)
```

3.5.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{\csc(x)}{i+\tan(x)}\,dx = \frac{2}{\frac{\sin(x)}{\cos(x)+1}+i} - i\,\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(csc(x)/(I+tan(x)),x, algorithm="maxima")
```

output
```
2/(sin(x)/(cos(x) + 1) + I) - I*log(sin(x)/(cos(x) + 1))
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\csc(x)}{i+\tan(x)}\,dx = -\frac{2i}{-i\tan\left(\frac{1}{2}x\right)+1} - i\log\left(\tan\left(\frac{1}{2}x\right)\right)$$

input
```
integrate(csc(x)/(I+tan(x)),x, algorithm="giac")
```

output
```
-2*I/(-I*tan(1/2*x) + 1) - I*log(tan(1/2*x))
```

3.5.9 Mupad [B] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\csc(x)}{i+\tan(x)}\,dx = -\ln\left(\tan\left(\frac{x}{2}\right)\right)\,1\mathrm{i} + \frac{2}{\tan\left(\frac{x}{2}\right)+1\mathrm{i}}$$

input
```
int(1/(sin(x)*(tan(x) + 1i)),x)
```

output
```
2/(tan(x/2) + 1i) - log(tan(x/2))*1i
```

3.6 $\int \frac{\csc^2(x)}{i+\tan(x)}\,dx$

3.6.1 Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{\csc^2(x)}{i+\tan(x)}\,dx = ix + i\cot(x) + \log(\cos(x)) + \log(\tan(x))$$

output
```
I*x+I*cot(x)+ln(cos(x))+ln(tan(x))
```

3.6.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\csc^2(x)}{i+\tan(x)}\,dx = ix + i\cot(x) + \log(\sin(x))$$

input
```
Integrate[Csc[x]^2/(I + Tan[x]),x]
```

output
```
I*x + I*Cot[x] + Log[Sin[x]]
```

3.6.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3999, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)}{\tan(x)+i}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sin(x)^2(\tan(x)+i)}dx$$
$$\Big\downarrow \text{ 3999}$$
$$\int \frac{\cot^2(x)}{\tan(x)+i}d\tan(x)$$
$$\Big\downarrow \text{ 54}$$
$$\int \left(\frac{1}{-\tan(x)-i}-i\cot^2(x)+\cot(x)\right)d\tan(x)$$
$$\Big\downarrow \text{ 2009}$$
$$i\cot(x)+\log(\tan(x))-\log(\tan(x)+i)$$

input
```
Int[Csc[x]^2/(I + Tan[x]),x]
```

output
```
I*Cot[x] + Log[Tan[x]] - Log[I + Tan[x]]
```

3.6.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.6.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

method	result	size
default	$\ln\left(\tan\left(x\right)\right)+\frac{i}{\tan\left(x\right)}-\ln\left(i+\tan\left(x\right)\right)$	20
risch	$-\frac{2}{{\mathrm e}^{2 i x}-1}+\ln\left({\mathrm e}^{2 i x}-1\right)$	21

input
```
int(csc(x)^2/(I+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(tan(x))+I/tan(x)-ln(I+tan(x))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{\csc^2(x)}{i+\tan(x)}\,dx = \frac{\left(e^{(2i\,x)}-1\right)\log\left(e^{(2i\,x)}-1\right)-2}{e^{(2i\,x)}-1}$$

input
```
integrate(csc(x)^2/(I+tan(x)),x, algorithm="fricas")
```

output
```
((e^(2*I*x) - 1)*log(e^(2*I*x) - 1) - 2)/(e^(2*I*x) - 1)
```

3.6.6 Sympy [F]

$$\int \frac{\csc^2(x)}{i+\tan(x)}\,dx = \int \frac{\csc^2(x)}{\tan(x)+i}\,dx$$

input
```
integrate(csc(x)**2/(I+tan(x)),x)
```

output
```
Integral(csc(x)**2/(tan(x) + I), x)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\csc^2(x)}{i+\tan(x)}\,dx = \frac{i}{\tan(x)} - \log(\tan(x)+i) + \log(\tan(x))$$

input
```
integrate(csc(x)^2/(I+tan(x)),x, algorithm="maxima")
```

output
```
I/tan(x) - log(tan(x) + I) + log(tan(x))
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(x)}{i+\tan(x)}\,dx = \frac{i}{\tan(x)} - \log(\tan(x)+i) + \log(|\tan(x)|)$$

input
```
integrate(csc(x)^2/(I+tan(x)),x, algorithm="giac")
```

output
```
I/tan(x) - log(tan(x) + I) + log(abs(tan(x)))
```

3.6.9 Mupad [B] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\csc^2(x)}{i+\tan(x)}\,dx = \operatorname{atan}(2\tan(x)+1\mathrm{i})\ 2\mathrm{i}+\frac{1\mathrm{i}}{\tan{(x)}}$$

input
```
int(1/(sin(x)^2*(tan(x) + 1i)),x)
```

output
```
atan(2*tan(x) + 1i)*2i + 1i/tan(x)
```

3.7 $\int \frac{\csc^3(x)}{i+\tan(x)}\,dx$

3.7.1 Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{\csc^3(x)}{i+\tan(x)}\,dx = -\frac{1}{2}i\text{arctanh}(\cos(x)) - \csc(x) + \frac{1}{2}i\cot(x)\csc(x)$$

output
```
-1/2*I*arctanh(cos(x))-csc(x)+1/2*I*cot(x)*csc(x)
```

3.7.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 75 vs. 2(24) = 48.

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.12

$$\int \frac{\csc^3(x)}{i+\tan(x)}\,dx = -\frac{1}{2}\cot\left(\frac{x}{2}\right) + \frac{1}{8}i\csc^2\left(\frac{x}{2}\right) - \frac{1}{2}i\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{1}{2}i\log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{8}i\sec^2\left(\frac{x}{2}\right) - \frac{1}{2}\tan\left(\frac{x}{2}\right)$$

input
```
Integrate[Csc[x]^3/(I + Tan[x]),x]
```

output
```
-1/2*Cot[x/2] + (I/8)*Csc[x/2]^2 - (I/2)*Log[Cos[x/2]] + (I/2)*Log[Sin[x/2
]] - (I/8)*Sec[x/2]^2 - Tan[x/2]/2
```

3.7.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3042, 4001, 3042, 3587, 3042, 3586, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(x)}{\tan(x)+i}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(x)^3(\tan(x)+i)}dx$$

$$\Big\downarrow \; 4001$$

$$\int \frac{\cot(x)\csc^2(x)}{\sin(x)+i\cos(x)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(x)}{\sin(x)^3(\sin(x)+i\cos(x))}dx$$

$$\Big\downarrow \; 3587$$

$$-i\int \cot(x)\csc^2(x)(\cos(x)+i\sin(x))dx$$

$$\Big\downarrow \; 3042$$

$$-i\int \frac{\cos(x)(\cos(x)+i\sin(x))}{\sin(x)^3}dx$$

$$\Big\downarrow \; 3586$$

$$-i\int \left(\csc(x)\cot^2(x)+i\csc(x)\cot(x)\right)dx$$

$$\Big\downarrow \; 2009$$

$$-i\left(\frac{1}{2}\text{arctanh}(\cos(x))-i\csc(x)-\frac{1}{2}\cot(x)\csc(x)\right)$$

input
```
Int[Csc[x]^3/(I + Tan[x]),x]
```

output
```
(-I)*(ArcTanh[Cos[x]]/2 - I*Csc[x] - (Cot[x]*Csc[x])/2)
```

3.7.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3586
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

rule 3587
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[a^p*b^p   Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
 d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && IL
tQ[p, 0]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

3.7.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(18) = 36$.

Time = 5.62 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

method	result	size
default	$-\frac{\tan\left(\frac{x}{2}\right)}{2}-\frac{i\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{8}+\frac{i}{8\tan\left(\frac{x}{2}\right)^{2}}+\frac{i\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2}-\frac{1}{2\tan\left(\frac{x}{2}\right)}$	42
risch	$-\frac{i\left(3\,e^{3ix}-e^{ix}\right)}{\left(e^{2ix}-1\right)^{2}}-\frac{i\ln\left(e^{ix}+1\right)}{2}+\frac{i\ln\left(e^{ix}-1\right)}{2}$	51

input
```
int(csc(x)^3/(I+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*tan(1/2*x)-1/8*I*tan(1/2*x)^2+1/8*I/tan(1/2*x)^2+1/2*I*ln(tan(1/2*x))
-1/2/tan(1/2*x)
```

3.7.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.04

$$\int \frac{\csc^3(x)}{i+\tan(x)}\,dx = \frac{\left(-i\,e^{(4i\,x)}+2i\,e^{(2i\,x)}-i\right)\log\left(e^{(i\,x)}+1\right)+\left(i\,e^{(4i\,x)}-2i\,e^{(2i\,x)}+i\right)\log\left(e^{(i\,x)}-1\right)-6i\,e^{(3i\,x)}+2i\,e^{(i\,x)}}{2\left(e^{(4i\,x)}-2\,e^{(2i\,x)}+1\right)}$$

input
```
integrate(csc(x)^3/(I+tan(x)),x, algorithm="fricas")
```

output
```
1/2*((-I*e^(4*I*x) + 2*I*e^(2*I*x) - I)*log(e^(I*x) + 1) + (I*e^(4*I*x) -
2*I*e^(2*I*x) + I)*log(e^(I*x) - 1) - 6*I*e^(3*I*x) + 2*I*e^(I*x))/(e^(4*I
*x) - 2*e^(2*I*x) + 1)
```

3.7.6 Sympy [F]

$$\int \frac{\csc^3(x)}{i+\tan(x)}\,dx = \int \frac{\csc^3(x)}{\tan(x)+i}\,dx$$

input
```
integrate(csc(x)**3/(I+tan(x)),x)
```

output
```
Integral(csc(x)**3/(tan(x) + I), x)
```

3.7.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(16) = 32.

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

$$\int \frac{\csc^3(x)}{i+\tan(x)}\,dx = -\frac{\left(\frac{4\sin(x)}{\cos(x)+1}-i\right)(\cos(x)+1)^2}{8\sin(x)^2} - \frac{\sin(x)}{2(\cos(x)+1)} - \frac{i\sin(x)^2}{8(\cos(x)+1)^2} + \frac{1}{2}i\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(csc(x)^3/(I+tan(x)),x, algorithm="maxima")
```

output
```
-1/8*(4*sin(x)/(cos(x) + 1) - I)*(cos(x) + 1)^2/sin(x)^2 - 1/2*sin(x)/(cos
(x) + 1) - 1/8*I*sin(x)^2/(cos(x) + 1)^2 + 1/2*I*log(sin(x)/(cos(x) + 1))
```

3.7.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(16) = 32.

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{\csc^3(x)}{i+\tan(x)}\,dx = -\frac{1}{8}i\tan\left(\frac{1}{2}x\right)^2 - \frac{6i\tan\left(\frac{1}{2}x\right)^2+4\tan\left(\frac{1}{2}x\right)-i}{8\tan\left(\frac{1}{2}x\right)^2} + \frac{1}{2}i\log\left(\tan\left(\frac{1}{2}x\right)\right) - \frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

input
```
integrate(csc(x)^3/(I+tan(x)),x, algorithm="giac")
```

output
```
-1/8*I*tan(1/2*x)^2 - 1/8*(6*I*tan(1/2*x)^2 + 4*tan(1/2*x) - I)/tan(1/2*x)
^2 + 1/2*I*log(tan(1/2*x)) - 1/2*tan(1/2*x)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 4.74 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{\csc^3(x)}{i+\tan(x)}\,dx = -\frac{\tan\left(\frac{x}{2}\right)}{2} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)\ 1\mathrm{i}}{2} - \frac{2\tan\left(\frac{x}{2}\right) - \frac{1}{2}\mathrm{i}}{4\tan\left(\frac{x}{2}\right)^2} - \frac{\tan\left(\frac{x}{2}\right)^2\ 1\mathrm{i}}{8}$$

input
```
int(1/(sin(x)^3*(tan(x) + 1i)),x)
```

output
```
(log(tan(x/2))*1i)/2 - tan(x/2)/2 - (2*tan(x/2) - 1i/2)/(4*tan(x/2)^2) - (
tan(x/2)^2*1i)/8
```

3.8 $\int \frac{\csc^4(x)}{i+\tan(x)}\,dx$

3.8.1 Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{\csc^4(x)}{i+\tan(x)}\,dx = -\frac{1}{2}\cot^2(x) + \frac{1}{3}i\cot^3(x)$$

output
```
-1/2*cot(x)^2+1/3*I*cot(x)^3
```

3.8.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{\csc^4(x)}{i+\tan(x)}\,dx = -\frac{1}{3}i\cot(x) - \frac{\csc^2(x)}{2} + \frac{1}{3}i\cot(x)\csc^2(x)$$

input
```
Integrate[Csc[x]^4/(I + Tan[x]),x]
```

output
```
(-1/3*I)*Cot[x] - Csc[x]^2/2 + (I/3)*Cot[x]*Csc[x]^2
```

3.8.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3999, 516, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(x)}{\tan(x)+i} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(x)^4(\tan(x)+i)}dx$$

$$\Big\downarrow \text{3999}$$

$$\int \frac{(\tan^2(x)+1)\cot^4(x)}{\tan(x)+i}d\tan(x)$$

$$\Big\downarrow \text{516}$$

$$\int (\tan(x)-i)\cot^4(x)d\tan(x)$$

$$\Big\downarrow \text{53}$$

$$\int \left(\cot^3(x)-i\cot^4(x)\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cot^2(x)}{2}+\frac{1}{3}i\cot^3(x)$$

input
```
Int[Csc[x]^4/(I + Tan[x]),x]
```

output
```
-1/2*Cot[x]^2 + (I/3)*Cot[x]^3
```

3.8.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.8.4 Maple [A] (verified)

Time = 7.68 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{1}{2\tan(x)^{2}}+\frac{i}{3\tan(x)^{3}}$	15
default	$-\frac{1}{2\tan(x)^{2}}+\frac{i}{3\tan(x)^{3}}$	15
risch	$\frac{4\,\mathrm{e}^{4ix}-2\,\mathrm{e}^{2ix}+\frac{2}{3}}{\left(\mathrm{e}^{2ix}-1\right)^{3}}$	28
parallelrisch	$\frac{(7-4i\cot(x))\cos(2x)+5-4i\cot(x)}{-12+12\cos(2x)}$	31

input
```
int(csc(x)^4/(I+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/tan(x)^2+1/3*I/tan(x)^3
```

3.8.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{\csc^4(x)}{i+\tan(x)}\,dx = \frac{2\left(6\,e^{(4i\,x)}-3\,e^{(2i\,x)}+1\right)}{3\left(e^{(6i\,x)}-3\,e^{(4i\,x)}+3\,e^{(2i\,x)}-1\right)}$$

input
```
integrate(csc(x)^4/(I+tan(x)),x, algorithm="fricas")
```

output
```
2/3*(6*e^(4*I*x) - 3*e^(2*I*x) + 1)/(e^(6*I*x) - 3*e^(4*I*x) + 3*e^(2*I*x)
 - 1)
```

3.8.6 Sympy [F]

$$\int \frac{\csc^4(x)}{i+\tan(x)}\,dx = \int \frac{\csc^4(x)}{\tan(x)+i}\,dx$$

input
```
integrate(csc(x)**4/(I+tan(x)),x)
```

output
```
Integral(csc(x)**4/(tan(x) + I), x)
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{\csc^4(x)}{i+\tan(x)}\,dx = -\frac{i\,(-3i\,\tan(x)-2)}{6\,\tan(x)^3}$$

input
```
integrate(csc(x)^4/(I+tan(x)),x, algorithm="maxima")
```

output
```
-1/6*I*(-3*I*tan(x) - 2)/tan(x)^3
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{\csc^4(x)}{i+\tan(x)}\,dx = -\frac{3\,\tan\left(x\right)-2i}{6\,\tan\left(x\right)^3}$$

input
```
integrate(csc(x)^4/(I+tan(x)),x, algorithm="giac")
```

output
```
-1/6*(3*tan(x) - 2*I)/tan(x)^3
```

3.8.9 Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\csc^4(x)}{i+\tan(x)}\,dx = \frac{\cot(x)^2\,(-3+\cot(x)\;2\mathrm{i})}{6}$$

input
```
int(1/(sin(x)^4*(tan(x) + 1i)),x)
```

output
```
(cot(x)^2*(cot(x)*2i - 3))/6
```

3.9 $\int \frac{\csc^5(x)}{i+\tan(x)}\,dx$

3.9.1 Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\csc^5(x)}{i+\tan(x)}\,dx=-\frac{1}{8}i\operatorname{arctanh}(\cos(x))-\frac{1}{8}i\cot(x)\csc(x)-\frac{\csc^3(x)}{3}+\frac{1}{4}i\cot(x)\csc^3(x)$$

output
```
-1/8*I*arctanh(cos(x))-1/8*I*cot(x)*csc(x)-1/3*csc(x)^3+1/4*I*cot(x)*csc(x
)^3
```

3.9.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 139 vs. 2(40) = 80.

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.48

$$\begin{aligned}\int \frac{\csc^5(x)}{i+\tan(x)}\,dx=&-\frac{1}{12}\cot\left(\frac{x}{2}\right)-\frac{1}{32}i\csc^2\left(\frac{x}{2}\right)-\frac{1}{24}\cot\left(\frac{x}{2}\right)\csc^2\left(\frac{x}{2}\right)+\frac{1}{64}i\csc^4\left(\frac{x}{2}\right)\\&-\frac{1}{8}i\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{1}{8}i\log\left(\sin\left(\frac{x}{2}\right)\right)+\frac{1}{32}i\sec^2\left(\frac{x}{2}\right)\\&-\frac{1}{64}i\sec^4\left(\frac{x}{2}\right)-\frac{1}{12}\tan\left(\frac{x}{2}\right)-\frac{1}{24}\sec^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)\end{aligned}$$

input
```
Integrate[Csc[x]^5/(I + Tan[x]),x]
```

output
```
-1/12*Cot[x/2] - (I/32)*Csc[x/2]^2 - (Cot[x/2]*Csc[x/2]^2)/24 + (I/64)*Csc
[x/2]^4 - (I/8)*Log[Cos[x/2]] + (I/8)*Log[Sin[x/2]] + (I/32)*Sec[x/2]^2 -
(I/64)*Sec[x/2]^4 - Tan[x/2]/12 - (Sec[x/2]^2*Tan[x/2])/24
```

3.9.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3042, 4001, 3042, 3587, 3042, 3586, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(x)}{\tan(x)+i}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(x)^5(\tan(x)+i)}dx$$

$$\Big\downarrow \text{4001}$$

$$\int \frac{\cot(x)\csc^4(x)}{\sin(x)+i\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\sin(x)^5(\sin(x)+i\cos(x))}dx$$

$$\Big\downarrow \text{3587}$$

$$-i\int \cot(x)\csc^4(x)(\cos(x)+i\sin(x))dx$$

$$\Big\downarrow \text{3042}$$

$$-i\int \frac{\cos(x)(\cos(x)+i\sin(x))}{\sin(x)^5}dx$$

$$\Big\downarrow \text{3586}$$

$$-i\int \left(\cot^2(x)\csc^3(x)+i\cot(x)\csc^3(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{1}{8}\text{arctanh}(\cos(x))-\frac{1}{3}i\csc^3(x)-\frac{1}{4}\cot(x)\csc^3(x)+\frac{1}{8}\cot(x)\csc(x)\right)$$

input
```
Int[Csc[x]^5/(I + Tan[x]),x]
```

output
```
(-I)*(ArcTanh[Cos[x]]/8 + (Cot[x]*Csc[x])/8 - (I/3)*Csc[x]^3 - (Cot[x]*Csc
[x]^3)/4)
```

3.9.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3586
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

rule 3587
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[a^p*b^p   Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
 d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && IL
tQ[p, 0]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

3.9.4 Maple [A] (verified)

Time = 131.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{\tan\left(\frac{x}{2}\right)}{8}-\frac{i\left(\tan^4\left(\frac{x}{2}\right)\right)}{64}-\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}+\frac{i}{64\tan\left(\frac{x}{2}\right)^4}-\frac{1}{24\tan\left(\frac{x}{2}\right)^3}+\frac{i\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8}-\frac{1}{8\tan\left(\frac{x}{2}\right)}$	58
risch	$\frac{i\left(3\,{e}^{7ix}+53\,{e}^{5ix}-11\,{e}^{3ix}+3\,{e}^{ix}\right)}{12\left({e}^{2ix}-1\right)^4}+\frac{i\ln\left({e}^{ix}-1\right)}{8}-\frac{i\ln\left({e}^{ix}+1\right)}{8}$	65

input
```
int(csc(x)^5/(I+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*tan(1/2*x)-1/64*I*tan(1/2*x)^4-1/24*tan(1/2*x)^3+1/64*I/tan(1/2*x)^4-
1/24/tan(1/2*x)^3+1/8*I*ln(tan(1/2*x))-1/8/tan(1/2*x)
```

3.9.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.08

$$\int \frac{\csc^5(x)}{i+\tan(x)}\,dx = -\frac{3\left(i\,e^{(8i\,x)}-4i\,e^{(6i\,x)}+6i\,e^{(4i\,x)}-4i\,e^{(2i\,x)}+i\right)\log\left(e^{(i\,x)}+1\right)+3\left(-i\,e^{(8i\,x)}+4i\,e^{(6i\,x)}-6i\,e^{(4i\,x)}+4i\,e^{(2}\right.}{24\left(e^{(8i\,x)}-4\,e^{(6i\,x)}+6\,e^{(4i\,x)}-4\,e^{(2i\,x)}\right.}$$

input
```
integrate(csc(x)^5/(I+tan(x)),x, algorithm="fricas")
```

output
```
-1/24*(3*(I*e^(8*I*x) - 4*I*e^(6*I*x) + 6*I*e^(4*I*x) - 4*I*e^(2*I*x) + I)
*log(e^(I*x) + 1) + 3*(-I*e^(8*I*x) + 4*I*e^(6*I*x) - 6*I*e^(4*I*x) + 4*I*
e^(2*I*x) - I)*log(e^(I*x) - 1) - 6*I*e^(7*I*x) - 106*I*e^(5*I*x) + 22*I*e
^(3*I*x) - 6*I*e^(I*x))/(e^(8*I*x) - 4*e^(6*I*x) + 6*e^(4*I*x) - 4*e^(2*I*
x) + 1)
```

3.9.6 Sympy [F]

$$\int \frac{\csc^5(x)}{i+\tan(x)}\,dx = \int \frac{\csc^5(x)}{\tan(x)+i}\,dx$$

input
```
integrate(csc(x)**5/(I+tan(x)),x)
```

output
```
Integral(csc(x)**5/(tan(x) + I), x)
```

3.9.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(26) = 52$.

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.08

$$\int \frac{\csc^5(x)}{i+\tan(x)}\,dx = -\frac{\left(\frac{8\sin(x)}{\cos(x)+1}+\frac{24\sin(x)^3}{(\cos(x)+1)^3}-3i\right)(\cos(x)+1)^4}{192\sin(x)^4} - \frac{\sin(x)}{8(\cos(x)+1)} - \frac{\sin(x)^3}{24(\cos(x)+1)^3} - \frac{i\sin(x)^4}{64(\cos(x)+1)^4} + \frac{1}{8}i\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(csc(x)^5/(I+tan(x)),x, algorithm="maxima")
```

output
```
-1/192*(8*sin(x)/(cos(x) + 1) + 24*sin(x)^3/(cos(x) + 1)^3 - 3*I)*(cos(x)
+ 1)^4/sin(x)^4 - 1/8*sin(x)/(cos(x) + 1) - 1/24*sin(x)^3/(cos(x) + 1)^3 -
 1/64*I*sin(x)^4/(cos(x) + 1)^4 + 1/8*I*log(sin(x)/(cos(x) + 1))
```

3.9.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(26) = 52$.

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.55

$$\int \frac{\csc^5(x)}{i+\tan(x)}\,dx = -\frac{1}{64}i\tan\left(\frac{1}{2}x\right)^4 - \frac{1}{24}\tan\left(\frac{1}{2}x\right)^3 - \frac{50i\tan\left(\frac{1}{2}x\right)^4+24\tan\left(\frac{1}{2}x\right)^3+8\tan\left(\frac{1}{2}x\right)-3i}{192\tan\left(\frac{1}{2}x\right)^4} + \frac{1}{8}i\log\left(\tan\left(\frac{1}{2}x\right)\right) - \frac{1}{8}\tan\left(\frac{1}{2}x\right)$$

input
```
integrate(csc(x)^5/(I+tan(x)),x, algorithm="giac")
```

output
```
-1/64*I*tan(1/2*x)^4 - 1/24*tan(1/2*x)^3 - 1/192*(50*I*tan(1/2*x)^4 + 24*t
an(1/2*x)^3 + 8*tan(1/2*x) - 3*I)/tan(1/2*x)^4 + 1/8*I*log(tan(1/2*x)) - 1
/8*tan(1/2*x)
```

3.9.9 Mupad [B] (verification not implemented)

Time = 5.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.42

$$\int \frac{\csc^5(x)}{i+\tan(x)}\,dx = -\frac{\tan\left(\frac{x}{2}\right)}{8} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)\ 1\mathrm{i}}{8} - \frac{2\tan\left(\frac{x}{2}\right)^3 + \frac{2\tan\left(\frac{x}{2}\right)}{3} - \frac{1}{4}\mathrm{i}}{16\tan\left(\frac{x}{2}\right)^4} - \frac{\tan\left(\frac{x}{2}\right)^3}{24} - \frac{\tan\left(\frac{x}{2}\right)^4\ 1\mathrm{i}}{64}$$

input
```
int(1/(sin(x)^5*(tan(x) + 1i)),x)
```

output
```
(log(tan(x/2))*1i)/8 - tan(x/2)/8 - ((2*tan(x/2))/3 + 2*tan(x/2)^3 - 1i/4)
/(16*tan(x/2)^4) - tan(x/2)^3/24 - (tan(x/2)^4*1i)/64
```

3.10 $\int \frac{\csc^6(x)}{i+\tan(x)}\,dx$

3.10.1 Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\csc^6(x)}{i+\tan(x)}\,dx=-\frac{1}{2}\cot^2(x)+\frac{1}{3}i\cot^3(x)-\frac{\cot^4(x)}{4}+\frac{1}{5}i\cot^5(x)$$

output
```
-1/2*cot(x)^2+1/3*I*cot(x)^3-1/4*cot(x)^4+1/5*I*cot(x)^5
```

3.10.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{\csc^6(x)}{i+\tan(x)}\,dx=-\frac{2}{15}i\cot(x)-\frac{1}{15}i\cot(x)\csc^2(x)-\frac{\csc^4(x)}{4}+\frac{1}{5}i\cot(x)\csc^4(x)$$

input
```
Integrate[Csc[x]^6/(I + Tan[x]),x]
```

output
```
((-2*I)/15)*Cot[x] - (I/15)*Cot[x]*Csc[x]^2 - Csc[x]^4/4 + (I/5)*Cot[x]*Cs
c[x]^4
```

3.10.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3999, 516, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(x)}{\tan(x)+i}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(x)^6(\tan(x)+i)}dx$$

$$\Big\downarrow \text{3999}$$

$$\int \frac{\left(\tan^2(x)+1\right)^2\cot^6(x)}{\tan(x)+i}d\tan(x)$$

$$\Big\downarrow \text{516}$$

$$\int (\tan(x)-i)^2(\tan(x)+i)\cot^6(x)d\tan(x)$$

$$\Big\downarrow \text{84}$$

$$\int \left(-i\cot^6(x)+\cot^5(x)-i\cot^4(x)+\cot^3(x)\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}i\cot^5(x)-\frac{\cot^4(x)}{4}+\frac{1}{3}i\cot^3(x)-\frac{\cot^2(x)}{2}$$

input
```
Int[Csc[x]^6/(I + Tan[x]),x]
```

output
```
-1/2*Cot[x]^2 + (I/3)*Cot[x]^3 - Cot[x]^4/4 + (I/5)*Cot[x]^5
```

3.10.3.1 Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.10.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$-\frac{1}{4\tan\left(x\right)^{4}}+\frac{i}{5\tan\left(x\right)^{5}}+\frac{i}{3\tan\left(x\right)^{3}}-\frac{1}{2\tan\left(x\right)^{2}}$$

input
```
int(csc(x)^6/(I+tan(x)),x)
```

output
```
-1/4/tan(x)^4+1/5*I/tan(x)^5+1/3*I/tan(x)^3-1/2/tan(x)^2
```

3.10.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \frac{\csc^6(x)}{i+\tan(x)}\,dx = -\frac{4\left(30\,e^{(6i\,x)}-10\,e^{(4i\,x)}+5\,e^{(2i\,x)}-1\right)}{15\left(e^{(10i\,x)}-5\,e^{(8i\,x)}+10\,e^{(6i\,x)}-10\,e^{(4i\,x)}+5\,e^{(2i\,x)}-1\right)}$$

input
```
integrate(csc(x)^6/(I+tan(x)),x, algorithm="fricas")
```

output
```
-4/15*(30*e^(6*I*x) - 10*e^(4*I*x) + 5*e^(2*I*x) - 1)/(e^(10*I*x) - 5*e^(8
*I*x) + 10*e^(6*I*x) - 10*e^(4*I*x) + 5*e^(2*I*x) - 1)
```

3.10.6 Sympy [F]

$$\int \frac{\csc^6(x)}{i+\tan(x)}\,dx = \int \frac{\csc^6(x)}{\tan(x)+i}\,dx$$

input
```
integrate(csc(x)**6/(I+tan(x)),x)
```

output
```
Integral(csc(x)**6/(tan(x) + I), x)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \frac{\csc^6(x)}{i+\tan(x)}\,dx = \frac{i\left(30i\,\tan(x)^3+20\,\tan(x)^2+15i\,\tan(x)+12\right)}{60\,\tan(x)^5}$$

input
```
integrate(csc(x)^6/(I+tan(x)),x, algorithm="maxima")
```

output
```
1/60*I*(30*I*tan(x)^3 + 20*tan(x)^2 + 15*I*tan(x) + 12)/tan(x)^5
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \frac{\csc^6(x)}{i+\tan(x)}\,dx = -\frac{30\,\tan(x)^3 - 20i\,\tan(x)^2 + 15\,\tan(x) - 12i}{60\,\tan(x)^5}$$

input
```
integrate(csc(x)^6/(I+tan(x)),x, algorithm="giac")
```

output
```
-1/60*(30*tan(x)^3 - 20*I*tan(x)^2 + 15*tan(x) - 12*I)/tan(x)^5
```

3.10.9 Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{\csc^6(x)}{i+\tan(x)}\,dx = \frac{\cot(x)^5\,1\mathrm{i}}{5} - \frac{\cot(x)^4}{4} + \frac{\cot(x)^3\,1\mathrm{i}}{3} - \frac{\cot(x)^2}{2}$$

input
```
int(1/(sin(x)^6*(tan(x) + 1i)),x)
```

output
```
(cot(x)^3*1i)/3 - cot(x)^2/2 - cot(x)^4/4 + (cot(x)^5*1i)/5
```

3.11 $\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx$

3.11.1 Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{a\cos(c+dx)}{d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos^5(c+dx)}{5d}-\frac{b\sin(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin^5(c+dx)}{5d}$$

output
```
b*arctanh(sin(d*x+c))/d-a*cos(d*x+c)/d+2/3*a*cos(d*x+c)^3/d-1/5*a*cos(d*x+
c)^5/d-b*sin(d*x+c)/d-1/3*b*sin(d*x+c)^3/d-1/5*b*sin(d*x+c)^5/d
```

3.11.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{5a\cos(c+dx)}{8d}+\frac{5a\cos(3(c+dx))}{48d}-\frac{a\cos(5(c+dx))}{80d}-\frac{b\sin(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin^5(c+dx)}{5d}$$

input
```
Integrate[Sin[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

output
```
(b*ArcTanh[Sin[c + d*x]])/d - (5*a*Cos[c + d*x])/(8*d) + (5*a*Cos[3*(c + d
*x)])/(48*d) - (a*Cos[5*(c + d*x)])/(80*d) - (b*Sin[c + d*x])/d - (b*Sin[c
 + d*x]^3)/(3*d) - (b*Sin[c + d*x]^5)/(5*d)
```

3.11.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)^5(a+b\tan(c+dx))dx$$

$$\Big\downarrow \text{4000}$$

$$\int \left(a\sin^5(c+dx)+b\sin^5(c+dx)\tan(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\cos^5(c+dx)}{5d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}+\frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{b\sin^5(c+dx)}{5d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}$$

input
```
Int[Sin[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

output
```
(b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d + (2*a*Cos[c + d*x]^3)/(3
*d) - (a*Cos[c + d*x]^5)/(5*d) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(
3*d) - (b*Sin[c + d*x]^5)/(5*d)
```

3.11.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.11.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{-\frac{a\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+b\left(-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)\right.}{d}$
default	$\frac{-\frac{a\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+b\left(-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)\right.}{d}$
risch	$\frac{11i e^{i(dx+c)}b}{16d}-\frac{5\,e^{i(dx+c)}a}{16d}-\frac{11i e^{-i(dx+c)}b}{16d}-\frac{5\,e^{-i(dx+c)}a}{16d}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}-\frac{a\cos(5a}{80}$

input
```
int(sin(d*x+c)^5*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/5*a*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+b*(-1/5*sin(d*x
+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx = -\frac{6\,a\cos\left(dx+c\right)^5-20\,a\cos\left(dx+c\right)^3+30\,a\cos\left(dx+c\right)-15\,b\log\left(\sin\left(dx+c\right)+1\right)+15\,b\log\left(-\sin\left(d\right.\right.}{30\,d}$$

input
```
integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/30*(6*a*cos(d*x + c)^5 - 20*a*cos(d*x + c)^3 + 30*a*cos(d*x + c) - 15*b
*log(sin(d*x + c) + 1) + 15*b*log(-sin(d*x + c) + 1) + 2*(3*b*cos(d*x + c)
^4 - 11*b*cos(d*x + c)^2 + 23*b)*sin(d*x + c))/d
```

3.11.6 Sympy [F]

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan\left(c+dx\right))\sin^5\left(c+dx\right)\,dx$$

input
```
integrate(sin(d*x+c)**5*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*sin(c + d*x)**5, x)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx = -\frac{2\left(3\cos\left(dx+c\right)^5-10\cos\left(dx+c\right)^3+15\cos\left(dx+c\right)\right)a+\left(6\sin\left(dx+c\right)^5+10\sin\left(dx+c\right)^3-15\log\right.}{30\,d}$$

input
```
integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/30*(2*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*a + (6*s
in(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1) + 15*log(sin(
d*x + c) - 1) + 30*sin(d*x + c))*b)/d
```

3.11.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10412 vs. $2(93) = 186$.

Time = 1.73 (sec) , antiderivative size = 10412, normalized size of antiderivative = 103.09

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/30*(15*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*
c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 - 15*b*log(2*(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/
(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2
*d*x)^10*tan(1/2*c)^10 + 16*a*tan(1/2*d*x)^10*tan(1/2*c)^10 + 75*b*log(2*(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)
*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2
*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)
)*tan(1/2*d*x)^10*tan(1/2*c)^8 - 75*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^
8 - 60*b*tan(1/2*d*x)^10*tan(1/2*c)^9 + 75*b*log(2*(tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/
2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)
^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1
/2*c)^10 - 75*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*t...
```

3.11.9 Mupad [B] (verification not implemented)

Time = 5.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.20

$$\int \sin^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,a\cos\left(c+d\,x\right)^3}{3\,d} - \frac{a\cos\left(c+d\,x\right)^5}{5\,d} - \frac{a\,\cos\left(c+d\,x\right)}{d} - \frac{23\,b\,\sin\left(c+d\,x\right)}{15\,d} + \frac{11\,b\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{15\,d} - \frac{b\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{5\,d}$$

input
```
int(sin(c + d*x)^5*(a + b*tan(c + d*x)),x)
```

output
```
(2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*a*cos(c + d*x)^3
)/(3*d) - (a*cos(c + d*x)^5)/(5*d) - (a*cos(c + d*x))/d - (23*b*sin(c + d*
x))/(15*d) + (11*b*cos(c + d*x)^2*sin(c + d*x))/(15*d) - (b*cos(c + d*x)^4
*sin(c + d*x))/(5*d)
```

3.12 $\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx$

3.12.1 Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3ax}{8}-\frac{b\log(\cos(c+dx))}{d} - \frac{\cos(c+dx)\sin^3(c+dx)(a+b\tan(c+dx))}{4d} - \frac{\cos(c+dx)\sin(c+dx)(3a+4b\tan(c+dx))}{8d}$$

output
```
3/8*a*x-b*ln(cos(d*x+c))/d-1/4*cos(d*x+c)*sin(d*x+c)^3*(a+b*tan(d*x+c))/d-
1/8*cos(d*x+c)*sin(d*x+c)*(3*a+4*b*tan(d*x+c))/d
```

3.12.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3a(c+dx)}{8d}-\frac{b\left(-\cos^2(c+dx)+\frac{1}{4}\cos^4(c+dx)+\log(\cos(c+dx))\right)}{d} - \frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}$$

input
```
Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

output
```
(3*a*(c + d*x))/(8*d) - (b*(-Cos[c + d*x]^2 + Cos[c + d*x]^4/4 + Log[Cos[c
 + d*x]]))/d - (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

3.12.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 4889, 530, 2345, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)^4(a+b\tan(c+dx))dx$$

$$\Big\downarrow \text{4889}$$

$$\frac{\int \frac{\tan^4(c+dx)(a+b\tan(c+dx))}{(\tan^2(c+dx)+1)^3}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{530}$$

$$\frac{-\frac{1}{4}\int \frac{-4b\tan^3(c+dx)-4a\tan^2(c+dx)+4b\tan(c+dx)+a}{(\tan^2(c+dx)+1)^2}d\tan(c+dx)-\frac{b-a\tan(c+dx)}{4(\tan^2(c+dx)+1)^2}}{d}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{3a+8b\tan(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)+\frac{8b-5a\tan(c+dx)}{2(\tan^2(c+dx)+1)}\right)-\frac{b-a\tan(c+dx)}{4(\tan^2(c+dx)+1)^2}}{d}$$

$$\Big\downarrow \text{452}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3a\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)+8b\int \frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)\right)+\frac{8b-5a\tan(c+dx)}{2(\tan^2(c+dx)+1)}\right)-\frac{b-a\tan(c+dx)}{4(\tan^2(c+dx)+1)^2}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8b\int \frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)+3a\arctan(\tan(c+dx))\right)+\frac{8b-5a\tan(c+dx)}{2(\tan^2(c+dx)+1)}\right)-\frac{b-a\tan(c+dx)}{4(\tan^2(c+dx)+1)^2}}{d}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3a\arctan(\tan(c+dx))+4b\log\left(\tan^2(c+dx)+1\right)\right)+\frac{8b-5a\tan(c+dx)}{2(\tan^2(c+dx)+1)}\right)-\frac{b-a\tan(c+dx)}{4(\tan^2(c+dx)+1)^2}}{d}$$

input
```
Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

output
```
(-1/4*(b - a*Tan[c + d*x])/(1 + Tan[c + d*x]^2)^2 + ((3*a*ArcTan[Tan[c + d
*x]] + 4*b*Log[1 + Tan[c + d*x]^2])/2 + (8*b - 5*a*Tan[c + d*x])/(2*(1 + T
an[c + d*x]^2)))/4)/d
```

3.12.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.12.4 Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^{3}(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(-\frac{\left(\sin^{4}(dx+c)\right)}{4}-\frac{\left(\sin^{2}(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$
default	$\frac{a\left(-\frac{\left(\sin^{3}(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(-\frac{\left(\sin^{4}(dx+c)\right)}{4}-\frac{\left(\sin^{2}(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$
risch	$ibx+\frac{3ax}{8}+\frac{3\,\mathrm{e}^{2i(dx+c)}b}{16d}+\frac{i\mathrm{e}^{2i(dx+c)}a}{8d}+\frac{3\,\mathrm{e}^{-2i(dx+c)}b}{16d}-\frac{i\mathrm{e}^{-2i(dx+c)}a}{8d}+\frac{2ibc}{d}-\frac{b\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}-$ bc

input
```
int(sin(d*x+c)^4*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+b*(-1
/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c))))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx = -\frac{2\,b\cos\left(dx+c\right)^4 - 3\,adx - 8\,b\cos\left(dx+c\right)^2 + 8\,b\log\left(-\cos\left(dx+c\right)\right) - \left(2\,a\cos\left(dx+c\right)^3 - 5\,a\cos\left(dx+\right.\right.}{8\,d}$$

input
```
integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/8*(2*b*cos(d*x + c)^4 - 3*a*d*x - 8*b*cos(d*x + c)^2 + 8*b*log(-cos(d*x
 + c)) - (2*a*cos(d*x + c)^3 - 5*a*cos(d*x + c))*sin(d*x + c))/d
```

3.12.6 Sympy [F]

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan\left(c+dx\right))\sin^4\left(c+dx\right)\,dx$$

input
```
integrate(sin(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*sin(c + d*x)**4, x)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,(dx+c)a + 4\,b\log\left(\tan\left(dx+c\right)^2+1\right) - \frac{5\,a\tan(dx+c)^3 - 8\,b\tan(dx+c)^2 + 3\,a\tan(dx+c) - 6\,b}{\tan(dx+c)^4 + 2\,\tan(dx+c)^2 + 1}}{8\,d}$$

input
```
integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/8*(3*(d*x + c)*a + 4*b*log(tan(d*x + c)^2 + 1) - (5*a*tan(d*x + c)^3 - 8
*b*tan(d*x + c)^2 + 3*a*tan(d*x + c) - 6*b)/(tan(d*x + c)^4 + 2*tan(d*x +
c)^2 + 1))/d
```

3.12.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(77) = 154$.

Time = 0.54 (sec) , antiderivative size = 976, normalized size of antiderivative = 11.76

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/32*(12*a*d*x*tan(d*x)^4*tan(c)^4 - 16*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*t
an(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan
(d*x)^4*tan(c)^4 + 24*a*d*x*tan(d*x)^4*tan(c)^2 + 24*a*d*x*tan(d*x)^2*tan(
c)^4 + 11*b*tan(d*x)^4*tan(c)^4 - 32*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(
d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*
x)^4*tan(c)^2 + 12*a*tan(d*x)^4*tan(c)^3 - 32*b*log(4*(tan(d*x)^2*tan(c)^2
 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1
))*tan(d*x)^2*tan(c)^4 + 12*a*tan(d*x)^3*tan(c)^4 + 12*a*d*x*tan(d*x)^4 +
48*a*d*x*tan(d*x)^2*tan(c)^2 + 6*b*tan(d*x)^4*tan(c)^2 - 32*b*tan(d*x)^3*t
an(c)^3 + 12*a*d*x*tan(c)^4 + 6*b*tan(d*x)^2*tan(c)^4 - 16*b*log(4*(tan(d*
x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 +
 tan(c)^2 + 1))*tan(d*x)^4 + 20*a*tan(d*x)^4*tan(c) - 64*b*log(4*(tan(d*x)
^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + t
an(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 24*a*tan(d*x)^3*tan(c)^2 + 24*a*tan(d*
x)^2*tan(c)^3 - 16*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(
tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(c)^4 + 20*a*tan(d*x)
*tan(c)^4 + 24*a*d*x*tan(d*x)^2 - 13*b*tan(d*x)^4 - 64*b*tan(d*x)^3*tan(c)
 + 24*a*d*x*tan(c)^2 - 36*b*tan(d*x)^2*tan(c)^2 - 64*b*tan(d*x)*tan(c)^3 -
 13*b*tan(c)^4 - 32*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/
(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2 - 20*a*ta...
```

3.12.9 Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.87

$$\int \sin^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,a\,x}{8}+\frac{b\,\ln\left(\tan(c+d\,x)^2+1\right)}{2\,d}+\frac{3\,b}{4\,d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}-\frac{5\,a\tan(c+d\,x)^3}{8\,d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}+\frac{b\tan(c+d\,x)^2}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}-\frac{3\,a\tan(c+d\,x)}{8\,d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}$$

input
```
int(sin(c + d*x)^4*(a + b*tan(c + d*x)),x)
```

output
```
(3*a*x)/8 + (b*log(tan(c + d*x)^2 + 1))/(2*d) + (3*b)/(4*d*(2*tan(c + d*x)
^2 + tan(c + d*x)^4 + 1)) - (5*a*tan(c + d*x)^3)/(8*d*(2*tan(c + d*x)^2 +
tan(c + d*x)^4 + 1)) + (b*tan(c + d*x)^2)/(d*(2*tan(c + d*x)^2 + tan(c + d
*x)^4 + 1)) - (3*a*tan(c + d*x))/(8*d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 +
 1))
```

3.13 $\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx$

3.13.1 Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{b\sin(c+dx)}{d} - \frac{b\sin^3(c+dx)}{3d}$$

output
```
b*arctanh(sin(d*x+c))/d-a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d-b*sin(d*x+c)/d
-1/3*b*sin(d*x+c)^3/d
```

3.13.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{3a\cos(c+dx)}{4d} + \frac{a\cos(3(c+dx))}{12d} - \frac{b\sin(c+dx)}{d} - \frac{b\sin^3(c+dx)}{3d}$$

input
```
Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

output
```
(b*ArcTanh[Sin[c + d*x]])/d - (3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x
)])/(12*d) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d)
```

3.13.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(c+dx)^3(a+b\tan(c+dx))dx$$

$$\Big\downarrow \; 4000$$

$$\int \left(a\sin^3(c+dx)+b\sin^3(c+dx)\tan(c+dx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}+\frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}$$

input
```
Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

output
```
(b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d + (a*Cos[c + d*x]^3)/(3*d
) - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d)
```

3.13.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.13.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{-\frac{a\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+b\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
default	$\frac{-\frac{a\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+b\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}$
risch	$\frac{5i e^{i(dx+c)}b}{8d}-\frac{3\,e^{i(dx+c)}a}{8d}-\frac{5i e^{-i(dx+c)}b}{8d}-\frac{3\,e^{-i(dx+c)}a}{8d}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a\cos(3dx-}{12d}$

input
```
int(sin(d*x+c)^3*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a*(2+sin(d*x+c)^2)*cos(d*x+c)+b*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln
(sec(d*x+c)+tan(d*x+c))))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{2\,a\cos\left(dx+c\right)^3-6\,a\cos\left(dx+c\right)+3\,b\log\left(\sin\left(dx+c\right)+1\right)-3\,b\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(b\cos\left(dx\right.\right.}{6\,d}$$

input
```
integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*a*cos(d*x + c)^3 - 6*a*cos(d*x + c) + 3*b*log(sin(d*x + c) + 1) - 3
*b*log(-sin(d*x + c) + 1) + 2*(b*cos(d*x + c)^2 - 4*b)*sin(d*x + c))/d
```

3.13.6 Sympy [F]

$$\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\sin^3(c+dx)\,dx$$

input
```
integrate(sin(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*sin(c + d*x)**3, x)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{2\left(\cos(dx+c)^3 - 3\cos(dx+c)\right)a - \left(2\sin(dx+c)^3 - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1)\right.}{6\,d}$$

input
```
integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/6*(2*(cos(d*x + c)^3 - 3*cos(d*x + c))*a - (2*sin(d*x + c)^3 - 3*log(sin
(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*b)/d
```

3.13.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4486 vs. 2(65) = 130.

Time = 1.00 (sec) , antiderivative size = 4486, normalized size of antiderivative = 65.01

$$\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 3*b*log(2*(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(
1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)
^6*tan(1/2*c)^6 + 4*a*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*b*log(2*(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)
^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(t
an(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d
*x)^6*tan(1/2*c)^4 - 9*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 12*b*tan(1
/2*d*x)^6*tan(1/2*c)^5 + 9*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 9*b*lo
g(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(...
```

3.13.9 Mupad [B] (verification not implemented)

Time = 4.71 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int \sin^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a\cos\left(c+d\,x\right)^3}{3\,d}-\frac{a\,\cos\left(c+d\,x\right)}{d}-\frac{4\,b\,\sin\left(c+d\,x\right)}{3\,d}+\frac{b\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(sin(c + d*x)^3*(a + b*tan(c + d*x)),x)
```

output
```
(2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a*cos(c + d*x)^3)/
(3*d) - (a*cos(c + d*x))/d - (4*b*sin(c + d*x))/(3*d) + (b*cos(c + d*x)^2*
sin(c + d*x))/(3*d)
```

3.14 $\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx$

3.14.1 Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{ax}{2}-\frac{b\log(\cos(c+dx))}{d}-\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))}{2d}$$

output
```
1/2*a*x-b*ln(cos(d*x+c))/d-1/2*cos(d*x+c)*sin(d*x+c)*(a+b*tan(d*x+c))/d
```

3.14.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{a(c+dx)}{2d}-\frac{b\left(-\frac{1}{2}\cos^2(c+dx)+\log(\cos(c+dx))\right)}{d}-\frac{a\sin(2(c+dx))}{4d}$$

input
```
Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

output
```
(a*(c + d*x))/(2*d) - (b*(-1/2*Cos[c + d*x]^2 + Log[Cos[c + d*x]]))/d - (a
*Sin[2*(c + d*x)])/(4*d)
```

3.14.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 4889, 530, 25, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)^2(a+b\tan(c+dx))dx$$

$$\Big\downarrow \text{4889}$$

$$\frac{\int \frac{\tan^2(c+dx)(a+b\tan(c+dx))}{(\tan^2(c+dx)+1)^2}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{530}$$

$$\frac{\frac{b-a\tan(c+dx)}{2(\tan^2(c+dx)+1)}-\frac{1}{2}\int -\frac{a+2b\tan(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\int \frac{a+2b\tan(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)+\frac{b-a\tan(c+dx)}{2(\tan^2(c+dx)+1)}}{d}$$

$$\Big\downarrow \text{452}$$

$$\frac{\frac{1}{2}\left(a\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)+2b\int \frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)\right)+\frac{b-a\tan(c+dx)}{2(\tan^2(c+dx)+1)}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}\left(2b\int \frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)+a\arctan(\tan(c+dx))\right)+\frac{b-a\tan(c+dx)}{2(\tan^2(c+dx)+1)}}{d}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{1}{2}\left(a\arctan(\tan(c+dx))+b\log\left(\tan^2(c+dx)+1\right)\right)+\frac{b-a\tan(c+dx)}{2(\tan^2(c+dx)+1)}}{d}$$

input
```
Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

output
```
((a*ArcTan[Tan[c + d*x]] + b*Log[1 + Tan[c + d*x]^2])/2 + (b - a*Tan[c + d
*x])/(2*(1 + Tan[c + d*x]^2)))/d
```

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.14.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$	52
default	$\frac{a\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$	52
risch	$ibx+\frac{ax}{2}+\frac{e^{2i(dx+c)}b}{8d}+\frac{ie^{2i(dx+c)}a}{8d}+\frac{e^{-2i(dx+c)}b}{8d}-\frac{ie^{-2i(dx+c)}a}{8d}+\frac{2ibc}{d}-\frac{b\ln\left(e^{2i(dx+c)}+1\right)}{d}$	99

input
```
int(sin(d*x+c)^2*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+b*(-1/2*sin(d*x+c)^2-ln(
cos(d*x+c))))
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{adx+b\cos(dx+c)^2-a\cos(dx+c)\sin(dx+c)-2\,b\log(-\cos(dx+c))}{2\,d}$$

input
```
integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*d*x + b*cos(d*x + c)^2 - a*cos(d*x + c)*sin(d*x + c) - 2*b*log(-cos
(d*x + c)))/d
```

3.14.6 Sympy [F]

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\sin^2(c+dx)\,dx$$

input
```
integrate(sin(d*x+c)**2*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*sin(c + d*x)**2, x)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{(dx+c)a + b\log\left(\tan(dx+c)^2+1\right) - \frac{a\tan(dx+c)-b}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*((d*x + c)*a + b*log(tan(d*x + c)^2 + 1) - (a*tan(d*x + c) - b)/(tan(d
*x + c)^2 + 1))/d
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(45) = 90.

Time = 0.38 (sec) , antiderivative size = 373, normalized size of antiderivative = 7.61

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{2\,adx\tan(dx)^2\tan(c)^2 - 2\,b\log\left(\frac{4\left(\tan(dx)^2\tan(c)^2-2\,\tan(dx)\tan(c)+1\right)}{\tan(dx)^2\tan(c)^2+\tan(dx)^2+\tan(c)^2+1}\right)\tan(dx)^2\tan(c)^2 + 2\,adx\tan(dx)^2 +}{}$$

input
```
integrate(sin(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(2*a*d*x*tan(d*x)^2*tan(c)^2 - 2*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(
d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*
x)^2*tan(c)^2 + 2*a*d*x*tan(d*x)^2 + 2*a*d*x*tan(c)^2 + b*tan(d*x)^2*tan(c
)^2 - 2*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*
tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2 + 2*a*tan(d*x)^2*tan(c)
- 2*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(c)^2 + 2*a*tan(d*x)*tan(c)^2 + 2*a*
d*x - b*tan(d*x)^2 - 4*b*tan(d*x)*tan(c) - b*tan(c)^2 - 2*b*log(4*(tan(d*x
)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 +
tan(c)^2 + 1)) - 2*a*tan(d*x) - 2*a*tan(c) + b)/(d*tan(d*x)^2*tan(c)^2 + d
*tan(d*x)^2 + d*tan(c)^2 + d)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \sin^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{\frac{b\cos(c+d\,x)^2}{2}-\frac{a\,\sin(c+d\,x)\,\cos(c+d\,x)}{2}+\frac{b\,\ln\left(\tan(c+d\,x)^2+1\right)}{2}+\frac{a\,d\,x}{2}}{d}$$

input
```
int(sin(c + d*x)^2*(a + b*tan(c + d*x)),x)
```

output
```
((b*log(tan(c + d*x)^2 + 1))/2 + (b*cos(c + d*x)^2)/2 - (a*cos(c + d*x)*si
n(c + d*x))/2 + (a*d*x)/2)/d
```

3.15 $\int \sin(c+dx)(a+b\tan(c+dx))\,dx$

3.15.1 Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \sin(c+dx)(a+b\tan(c+dx))\,dx=\frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin(c+dx)}{d}$$

output
```
b*arctanh(sin(d*x+c))/d-a*cos(d*x+c)/d-b*sin(d*x+c)/d
```

3.15.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \sin(c+dx)(a+b\tan(c+dx))\,dx=\frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{a\cos(c)\cos(dx)}{d}+\frac{a\sin(c)\sin(dx)}{d}-\frac{b\sin(c+dx)}{d}$$

input
```
Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x]),x]
```

output
```
(b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c]*Cos[d*x])/d + (a*Sin[c]*Sin[d*x])/
d - (b*Sin[c + d*x])/d
```

3.15.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)(a+b\tan(c+dx))dx$$

$$\Big\downarrow \text{4000}$$

$$\int (a\sin(c+dx)+b\sin(c+dx)\tan(c+dx))dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\cos(c+dx)}{d}+\frac{b\,\text{arctanh}(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}$$

input
```
Int[Sin[c + d*x]*(a + b*Tan[c + d*x]),x]
```

output
```
(b*ArcTanh[Sin[c + d*x]])/d - (a*Cos[c + d*x])/d - (b*Sin[c + d*x])/d
```

3.15.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.15.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{-a\cos(dx+c)+b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$	40
default	$\frac{-a\cos(dx+c)+b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$	40
risch	$\frac{i\,\mathrm{e}^{i(dx+c)}b}{2d}-\frac{\mathrm{e}^{i(dx+c)}a}{2d}-\frac{i\,\mathrm{e}^{-i(dx+c)}b}{2d}-\frac{\mathrm{e}^{-i(dx+c)}a}{2d}-\frac{b\ln(\mathrm{e}^{i(dx+c)}-i)}{d}+\frac{b\ln(\mathrm{e}^{i(dx+c)}+i)}{d}$	101

input
```
int(sin(d*x+c)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a*cos(d*x+c)+b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \sin(c+dx)(a+b\tan(c+dx))\,dx = -\frac{2\,a\cos(dx+c)-b\log(\sin(dx+c)+1)+b\log(-\sin(dx+c)+1)+2\,b\sin(dx+c)}{2\,d}$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*a*cos(d*x + c) - b*log(sin(d*x + c) + 1) + b*log(-sin(d*x + c) + 1
) + 2*b*sin(d*x + c))/d
```

3.15.6 Sympy [F]

$$\int \sin(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\sin(c+dx)\,dx$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*sin(c + d*x), x)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \sin(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\sin(dx+c))-2\,a\cos(dx+c)}{2\,d}$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) -
2*a*cos(d*x + c))/d
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1020 vs. $2(37) = 74$.

Time = 0.42 (sec) , antiderivative size = 1020, normalized size of antiderivative = 27.57

$$\int \sin(c+dx)(a+b\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - b*log(2*(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*
d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*a*tan(1/2*d*x)^2*tan(1/2*c)^2 + b*log(2*(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2
*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2
- b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*t
an(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
+ 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/
2*c)^2 + 1))*tan(1/2*d*x)^2 - 4*b*tan(1/2*d*x)^2*tan(1/2*c) + b*log(2*(tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c)
 + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*t
an(1/2*c)^2 - b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(
1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + ...
```

3.15.9 Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \sin(c+dx)(a+b\tan(c+dx))\,dx = \frac{2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{2\,a+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int(sin(c + d*x)*(a + b*tan(c + d*x)),x)
```

output
```
(2*b*atanh(tan(c/2 + (d*x)/2)))/d - (2*a + 2*b*tan(c/2 + (d*x)/2))/(d*(tan
(c/2 + (d*x)/2)^2 + 1))
```

3.16 $\int \csc(c+dx)(a+b\tan(c+dx))\,dx$

3.16.1 Optimal result

Integrand size = 17, antiderivative size = 26

$$\int \csc(c+dx)(a+b\tan(c+dx))\,dx=-\frac{a\text{arctanh}(\cos(c+dx))}{d}+\frac{b\text{arctanh}(\sin(c+dx))}{d}$$

output
```
-a*arctanh(cos(d*x+c))/d+b*arctanh(sin(d*x+c))/d
```

3.16.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.00

$$\int \csc(c+dx)(a+b\tan(c+dx))\,dx=\frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

input
```
Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x]),x]
```

output
```
(b*ArcTanh[Sin[c + d*x]])/d - (a*Log[Cos[c/2 + (d*x)/2]])/d + (a*Log[Sin[c
/2 + (d*x)/2]])/d
```

3.16.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)}{\sin(c+dx)}dx$$

$$\Big\downarrow \text{4000}$$

$$\int (a\csc(c+dx)+b\sec(c+dx))dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{a\text{arctanh}(\cos(c+dx))}{d}$$

input
```
Int[Csc[c + d*x]*(a + b*Tan[c + d*x]),x]
```

output
```
-((a*ArcTanh[Cos[c + d*x]])/d) + (b*ArcTanh[Sin[c + d*x]])/d
```

3.16.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.16.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

method	result	size
derivativedivides	$\frac{b\ln(\sec(dx+c)+\tan(dx+c))+a\ln(\csc(dx+c)-\cot(dx+c))}{d}$	40
default	$\frac{b\ln(\sec(dx+c)+\tan(dx+c))+a\ln(\csc(dx+c)-\cot(dx+c))}{d}$	40
risch	$-\frac{a\ln(e^{i(dx+c)}+1)}{d}+\frac{a\ln(e^{i(dx+c)}-1)}{d}+\frac{b\ln(e^{i(dx+c)}+i)}{d}-\frac{b\ln(e^{i(dx+c)}-i)}{d}$	74

input
```
int(csc(d*x+c)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*ln(sec(d*x+c)+tan(d*x+c))+a*ln(csc(d*x+c)-cot(d*x+c)))
```

3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(26) = 52.

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.23

$$\int \csc(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-a\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-b\log(\sin(dx+c)+1)+b\log(-\sin(dx+c)}{2\,d}$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(a*log(1/2*cos(d*x + c) + 1/2) - a*log(-1/2*cos(d*x + c) + 1/2) - b*l
og(sin(d*x + c) + 1) + b*log(-sin(d*x + c) + 1))/d
```

3.16.6 Sympy [F]

$$\int \csc(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\csc(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*csc(c + d*x), x)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \csc(c+dx)(a+b\tan(c+dx))\,dx = \frac{b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))-2\,a\log(\cot(dx+c)+\csc(dx+c))}{2\,d}$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 2*a*log(cot(d*x +
 c) + csc(d*x + c)))/d
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \csc(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{d}$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
(b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) + a*log(abs(tan(1/2*d*x + 1/2*c))))/d
```

3.16.9 Mupad [B] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.31

$$\int \csc(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{2\,b\,\mathrm{atanh}\left(\frac{b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int((a + b*tan(c + d*x))/sin(c + d*x),x)
```

output
```
(a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (2*b*atanh((b*cos(c/2 +
 (d*x)/2) - a*sin(c/2 + (d*x)/2))/(a*cos(c/2 + (d*x)/2) - b*sin(c/2 + (d*x
)/2))))/d
```

3.16. $\int \csc(c+dx)(a+b\tan(c+dx))\,dx$

3.17 $\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx$

3.17.1 Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx=-\frac{a\cot(c+dx)}{d}+\frac{b\log(\tan(c+dx))}{d}$$

output
```
-a*cot(d*x+c)/d+b*ln(tan(d*x+c))/d
```

3.17.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx=-\frac{a\cot(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}+\frac{b\log(\sin(c+dx))}{d}$$

input
```
Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

output
```
-((a*Cot[c + d*x])/d) - (b*Log[Cos[c + d*x]])/d + (b*Log[Sin[c + d*x]])/d
```

3.17.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4889, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{a+b\tan(c+dx)}{\sin(c+dx)^2}dx$$

$$\Big\downarrow \; 4889$$

$$\frac{\int \cot^2(c+dx)(a+b\tan(c+dx))d\tan(c+dx)}{d}$$

$$\Big\downarrow \; 49$$

$$\frac{\int \left(a\cot^2(c+dx)+b\cot(c+dx)\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\log(\tan(c+dx))-a\cot(c+dx)}{d}$$

input
```
Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

output
```
(-(a*Cot[c + d*x]) + b*Log[Tan[c + d*x]])/d
```

3.17.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.17.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\frac{a}{\tan(dx+c)}+b\ln(\tan(dx+c))}{d}$	26
default	$\frac{-\frac{a}{\tan(dx+c)}+b\ln(\tan(dx+c))}{d}$	26
risch	$-\frac{2ia}{d\left(e^{2i(dx+c)}-1\right)}+\frac{b\ln\left(e^{2i(dx+c)}-1\right)}{d}-\frac{b\ln\left(e^{2i(dx+c)}+1\right)}{d}$	57

input
```
int(csc(d*x+c)^2*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/tan(d*x+c)*a+b*ln(tan(d*x+c)))
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.48

$$\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\log\left(\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)-b\log\left(-\frac{1}{4}\,\cos\left(dx+c\right)^2+\frac{1}{4}\right)\sin\left(dx+c\right)+2\,a\cos\left(dx+c\right)}{2\,d\sin\left(dx+c\right)}$$

input
```
integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(b*log(cos(d*x + c)^2)*sin(d*x + c) - b*log(-1/4*cos(d*x + c)^2 + 1/4
)*sin(d*x + c) + 2*a*cos(d*x + c))/(d*sin(d*x + c))
```

3.17.6 Sympy [F]

$$\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\csc^2(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**2*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*csc(c + d*x)**2, x)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\log(\tan(dx+c)) - \frac{a}{\tan(dx+c)}}{d}$$

input
```
integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
(b*log(tan(d*x + c)) - a/tan(d*x + c))/d
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\log(|\tan(dx+c)|) - \frac{b\tan(dx+c)+a}{\tan(dx+c)}}{d}$$

input
```
integrate(csc(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
(b*log(abs(tan(d*x + c))) - (b*tan(d*x + c) + a)/tan(d*x + c))/d
```

3.17.9 Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \csc^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\,\ln(\tan(c+d\,x))}{d} - \frac{a\cot(c+d\,x)}{d}$$

input
```
int((a + b*tan(c + d*x))/sin(c + d*x)^2,x)
```

output
```
(b*log(tan(c + d*x)))/d - (a*cot(c + d*x))/d
```

3.18 $\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx$

3.18.1 Optimal result

Integrand size = 19, antiderivative size = 60

$$\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\text{arctanh}(\cos(c+dx))}{2d}+\frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{b\csc(c+dx)}{d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

output
```
-1/2*a*arctanh(cos(d*x+c))/d+b*arctanh(sin(d*x+c))/d-b*csc(d*x+c)/d-1/2*a*
cot(d*x+c)*csc(d*x+c)/d
```

3.18.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.78

$$\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(c+dx)\right)}{d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

input
```
Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

output
```
-1/8*(a*Csc[(c + d*x)/2]^2)/d - (b*Csc[c + d*x]*Hypergeometric2F1[-1/2, 1,
 1/2, Sin[c + d*x]^2])/d - (a*Log[Cos[(c + d*x)/2]])/(2*d) + (a*Log[Sin[(c
 + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

3.18.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)}{\sin(c+dx)^3}dx$$

$$\Big\downarrow \text{4000}$$

$$\int \left(a\csc^3(c+dx)+b\csc^2(c+dx)\sec(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\text{arctanh}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}+\frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{b\csc(c+dx)}{d}$$

input
```
Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

output
```
-1/2*(a*ArcTanh[Cos[c + d*x]])/d + (b*ArcTanh[Sin[c + d*x]])/d - (b*Csc[c
+ d*x])/d - (a*Cot[c + d*x]*Csc[c + d*x])/(2*d)
```

3.18.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.18.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
default	$\frac{b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
risch	$-\frac{i e^{i(dx+c)}\left(ia\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}+ia-2b\right)}{d\left(e^{2i(dx+c)}-1\right)^2}+\frac{a\ln\left(e^{i(dx+c)}-1\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{2d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{b\ln}{}$

input
```
int(csc(d*x+c)^3*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-1/2*csc(d*x+c)*cot(d*
x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c))))
```

3.18.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(56) = 112.

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.37

$$\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{2\,a\cos\left(dx+c\right)-\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a\cos\left(dx+c\right)^2-a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+\right.\right.}{4\,(}$$

input
```
integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*a*cos(d*x + c) - (a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2)
 + (a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) + 2*(b*cos(d*x + c)
^2 - b)*log(sin(d*x + c) + 1) - 2*(b*cos(d*x + c)^2 - b)*log(-sin(d*x + c)
 + 1) + 4*b*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

3.18.6 Sympy [F]

$$\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan{(c+dx)})\csc^3{(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*csc(c + d*x)**3, x)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.38

$$\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{a\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) - 2b\left(\frac{2}{\sin(dx+c)} - \log(\sin(dx+c)+1) + \right.}{4d}$$

input
```
integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*(a*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log(
cos(d*x + c) - 1)) - 2*b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin
(d*x + c) - 1)))/d
```

3.18.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(56) = 112$.

Time = 0.37 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.97

$$\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-8\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+4\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx\right.\right.\right.}{8\,d}$$

input
```
integrate(csc(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/8*(a*tan(1/2*d*x + 1/2*c)^2 + 8*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8
*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a*log(abs(tan(1/2*d*x + 1/2*c)))
 - 4*b*tan(1/2*d*x + 1/2*c) - (6*a*tan(1/2*d*x + 1/2*c)^2 + 4*b*tan(1/2*d*
x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c)^2)/d
```

3.18.9 Mupad [B] (verification not implemented)

Time = 4.88 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.48

$$\int \csc^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,d} - \frac{\frac{a}{2}+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2} - \frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,d} - \frac{2\,b\,\mathrm{atanh}\left(\frac{4\,b^2}{2\,a\,b-4\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{2\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a\,b-4\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d} + \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2\,d}$$

input
```
int((a + b*tan(c + d*x))/sin(c + d*x)^3,x)
```

output
```
(a*tan(c/2 + (d*x)/2)^2)/(8*d) - (a/2 + 2*b*tan(c/2 + (d*x)/2))/(4*d*tan(c
/2 + (d*x)/2)^2) - (b*tan(c/2 + (d*x)/2))/(2*d) - (2*b*atanh((4*b^2)/(2*a*
b - 4*b^2*tan(c/2 + (d*x)/2)) - (2*a*b*tan(c/2 + (d*x)/2))/(2*a*b - 4*b^2*
tan(c/2 + (d*x)/2))))/d + (a*log(tan(c/2 + (d*x)/2)))/(2*d)
```

3.19 $\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx$

3.19.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{b\log(\tan(c+dx))}{d}$$

output
```
-a*cot(d*x+c)/d-1/2*b*cot(d*x+c)^2/d-1/3*a*cot(d*x+c)^3/d+b*ln(tan(d*x+c))
/d
```

3.19.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.37

$$\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx = -\frac{2a\cot(c+dx)}{3d}-\frac{b\csc^2(c+dx)}{2d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}-\frac{b\log(\cos(c+dx))}{d}+\frac{b\log(\sin(c+dx))}{d}$$

input
```
Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

output
```
(-2*a*Cot[c + d*x])/(3*d) - (b*Csc[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]*Csc
[c + d*x]^2)/(3*d) - (b*Log[Cos[c + d*x]])/d + (b*Log[Sin[c + d*x]])/d
```

3.19.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4889, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)}{\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{4889}$$

$$\frac{\int \cot^4(c+dx)(a+b\tan(c+dx))\left(\tan^2(c+dx)+1\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(a\cot^4(c+dx)+b\cot^3(c+dx)+a\cot^2(c+dx)+b\cot(c+dx)\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}a\cot^3(c+dx)-a\cot(c+dx)-\frac{1}{2}b\cot^2(c+dx)+b\log(\tan(c+dx))}{d}$$

input
```
Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

output
```
(-(a*Cot[c + d*x]) - (b*Cot[c + d*x]^2)/2 - (a*Cot[c + d*x]^3)/3 + b*Log[T
an[c + d*x]])/d
```

3.19.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.19.4 Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)}{d}$	46
default	$\frac{b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)}{d}$	46
risch	$\frac{2b\,\mathrm{e}^{4i(dx+c)}+4ia\,\mathrm{e}^{2i(dx+c)}-2b\,\mathrm{e}^{2i(dx+c)}-\frac{4ia}{3}}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)^3}+\frac{b\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d}-\frac{b\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$	97

input
```
int(csc(d*x+c)^4*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*(-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a*(-2/3-1/3*csc(d*x+c)^2)*cot(d*
x+c))
```

3.19.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(53) = 106.

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.14

$$\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx =$$
$$-\frac{4\,a\cos\left(dx+c\right)^3+3\left(b\cos\left(dx+c\right)^2-b\right)\log\left(\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)-3\left(b\cos\left(dx+c\right)^2-b\right)\log\left(-\right.}{6\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right.}$$

input
```
integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(4*a*cos(d*x + c)^3 + 3*(b*cos(d*x + c)^2 - b)*log(cos(d*x + c)^2)*si
n(d*x + c) - 3*(b*cos(d*x + c)^2 - b)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d
*x + c) - 6*a*cos(d*x + c) - 3*b*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin
(d*x + c))
```

3.19.6 Sympy [F]

$$\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\csc^4(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*csc(c + d*x)**4, x)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{6\,b\log\left(\tan\left(dx+c\right)\right) - \frac{6\,a\tan\left(dx+c\right)^2+3\,b\tan\left(dx+c\right)+2\,a}{\tan\left(dx+c\right)^3}}{6\,d}$$

input
```
integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/6*(6*b*log(tan(d*x + c)) - (6*a*tan(d*x + c)^2 + 3*b*tan(d*x + c) + 2*a)
/tan(d*x + c)^3)/d
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{6\,b\log\left(|\tan\left(dx+c\right)|\right)-\frac{11\,b\tan(dx+c)^3+6\,a\tan(dx+c)^2+3\,b\tan(dx+c)+2\,a}{\tan(dx+c)^3}}{6\,d}$$

input
```
integrate(csc(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(6*b*log(abs(tan(d*x + c))) - (11*b*tan(d*x + c)^3 + 6*a*tan(d*x + c)^
2 + 3*b*tan(d*x + c) + 2*a)/tan(d*x + c)^3)/d
```

3.19.9 Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \csc^4(c+dx)(a+b\tan(c+dx))\,dx=\frac{b\,\ln\left(\tan(c+d\,x)\right)}{d}-\frac{a\tan(c+d\,x)^2+\frac{b\tan(c+d\,x)}{2}+\frac{a}{3}}{d\tan\left(c+d\,x\right)^3}$$

input
```
int((a + b*tan(c + d*x))/sin(c + d*x)^4,x)
```

output
```
(b*log(tan(c + d*x)))/d - (a/3 + (b*tan(c + d*x))/2 + a*tan(c + d*x)^2)/(d
*tan(c + d*x)^3)
```

3.20 $\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx$

3.20.1 Optimal result

Integrand size = 19, antiderivative size = 98

$$\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx = -\frac{3a\text{arctanh}(\cos(c+dx))}{8d}+\frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{b\csc(c+dx)}{d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\csc^3(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}$$

output
```
-3/8*a*arctanh(cos(d*x+c))/d+b*arctanh(sin(d*x+c))/d-b*csc(d*x+c)/d-3/8*a*
cot(d*x+c)*csc(d*x+c)/d-1/3*b*csc(d*x+c)^3/d-1/4*a*cot(d*x+c)*csc(d*x+c)^3
/d
```

3.20.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.54

$$\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx$$
$$= -\frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{b\csc^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\sin^2(c+dx)\right)}{3d} - \frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

input
```
Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

output
```
(-3*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (b*Csc[
c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[c + d*x]^2])/(3*d) - (3*a*
Log[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) + (3*a*Se
c[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

3.20.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{a+b\tan(c+dx)}{\sin(c+dx)^5}dx$$
$$\Big\downarrow \text{4000}$$

$$\int \left(a\csc^5(c+dx)+b\csc^4(c+dx)\sec(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3a\text{arctanh}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}+\frac{b\text{arctanh}(\sin(c+dx))}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}$$

input
```
Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

output
```
(-3*a*ArcTanh[Cos[c + d*x]])/(8*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Csc[
c + d*x])/d - (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (b*Csc[c + d*x]^3)/(
3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

3.20.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.20.4 Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c}{8}\right.}{d}$
default	$\frac{b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(\left(-\frac{\left(\csc^3(dx+c)\right)}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c}{8}\right.}{d}$
risch	$-\frac{i e^{i(dx+c)}\left(9ia\,e^{6i(dx+c)}+24b\,e^{6i(dx+c)}-33ia\,e^{4i(dx+c)}-104b\,e^{4i(dx+c)}-33ia\,e^{2i(dx+c)}+104b\,e^{2i(dx+c)}+9ia-24b\right)}{12d\left(e^{2i(dx+c)}-1\right)^4}-$

input
```
int(csc(d*x+c)^5*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*((-1/4
*csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(csc(d*x+c)-cot(d*x+c))))
```

3.20.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(90) = 180.

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.17

$$\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{18\,a\cos{(dx+c)}^3-30\,a\cos{(dx+c)}-9\left(a\cos{(dx+c)}^4-2\,a\cos{(dx+c)}^2+a\right)\log\left(\frac{1}{2}\cos{(dx+c)}+\frac{1}{2}\right)}{}$$

input
```
integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/48*(18*a*cos(d*x + c)^3 - 30*a*cos(d*x + c) - 9*(a*cos(d*x + c)^4 - 2*a*
cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2) + 9*(a*cos(d*x + c)^4 - 2*
a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2) + 24*(b*cos(d*x + c)^4
- 2*b*cos(d*x + c)^2 + b)*log(sin(d*x + c) + 1) - 24*(b*cos(d*x + c)^4 - 2
*b*cos(d*x + c)^2 + b)*log(-sin(d*x + c) + 1) + 16*(3*b*cos(d*x + c)^2 - 4
*b)*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

3.20.6 Sympy [F]

$$\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\csc^5(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**5*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*csc(c + d*x)**5, x)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.26

$$\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{3\,a\left(\frac{2\,\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-8\,b\left(\frac{2\,\left(3\,\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}\right.}{48\,d}$$

input
```
integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/48*(3*a*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 8*b*
(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log
(sin(d*x + c) - 1)))/d
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.81

$$\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+192\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-}{}$$

input
```
integrate(csc(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 - 8*b*tan(1/2*d*x + 1/2*c)^3 + 24*a*tan(
1/2*d*x + 1/2*c)^2 + 192*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 192*b*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) + 72*a*log(abs(tan(1/2*d*x + 1/2*c))) - 120
*b*tan(1/2*d*x + 1/2*c) - (150*a*tan(1/2*d*x + 1/2*c)^4 + 120*b*tan(1/2*d*
x + 1/2*c)^3 + 24*a*tan(1/2*d*x + 1/2*c)^2 + 8*b*tan(1/2*d*x + 1/2*c) + 3*
a)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.20.9 Mupad [B] (verification not implemented)

Time = 4.63 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.15

$$\int \csc^5(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d} - \frac{5b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d} - \frac{2b\,\mathrm{atanh}\left(\frac{4b^2}{\frac{3ab}{2}-4b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)} - \frac{3ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\left(\frac{3ab}{2}-4b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}\right)}{d} + \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64d} - \frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24d} + \frac{3a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8d} - \frac{10b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 + 2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + \frac{2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3} + \frac{a}{4}}{16d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}$$

input
```
int((a + b*tan(c + d*x))/sin(c + d*x)^5,x)
```

output
```
(a*tan(c/2 + (d*x)/2)^2)/(8*d) - (5*b*tan(c/2 + (d*x)/2))/(8*d) - (2*b*ata
nh((4*b^2)/((3*a*b)/2 - 4*b^2*tan(c/2 + (d*x)/2)) - (3*a*b*tan(c/2 + (d*x)
/2))/(2*((3*a*b)/2 - 4*b^2*tan(c/2 + (d*x)/2)))))/d + (a*tan(c/2 + (d*x)/2
)^4)/(64*d) - (b*tan(c/2 + (d*x)/2)^3)/(24*d) + (3*a*log(tan(c/2 + (d*x)/2
)))/(8*d) - (a/4 + (2*b*tan(c/2 + (d*x)/2))/3 + 2*a*tan(c/2 + (d*x)/2)^2 +
 10*b*tan(c/2 + (d*x)/2)^3)/(16*d*tan(c/2 + (d*x)/2)^4)
```

3.21 $\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx$

3.21.1 Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx = -\frac{a\cot(c+dx)}{d}-\frac{b\cot^2(c+dx)}{d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{b\cot^4(c+dx)}{4d}-\frac{a\cot^5(c+dx)}{5d}+\frac{b\log(\tan(c+dx))}{d}$$

output
```
-a*cot(d*x+c)/d-b*cot(d*x+c)^2/d-2/3*a*cot(d*x+c)^3/d-1/4*b*cot(d*x+c)^4/d
-1/5*a*cot(d*x+c)^5/d+b*ln(tan(d*x+c))/d
```

3.21.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.33

$$\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx = -\frac{8a\cot(c+dx)}{15d}-\frac{b\csc^2(c+dx)}{2d}-\frac{4a\cot(c+dx)\csc^2(c+dx)}{15d}-\frac{b\csc^4(c+dx)}{4d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}-\frac{b\log(\cos(c+dx))}{d}+\frac{b\log(\sin(c+dx))}{d}$$

input
```
Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

output
```
(-8*a*Cot[c + d*x])/(15*d) - (b*Csc[c + d*x]^2)/(2*d) - (4*a*Cot[c + d*x]*
Csc[c + d*x]^2)/(15*d) - (b*Csc[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]*Csc[c
+ d*x]^4)/(5*d) - (b*Log[Cos[c + d*x]])/d + (b*Log[Sin[c + d*x]])/d
```

3.21.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4889, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)}{\sin(c+dx)^6}dx$$

$$\Big\downarrow \text{4889}$$

$$\frac{\int \cot^6(c+dx)(a+b\tan(c+dx))\left(\tan^2(c+dx)+1\right)^2d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(a\cot^6(c+dx)+b\cot^5(c+dx)+2a\cot^4(c+dx)+2b\cot^3(c+dx)+a\cot^2(c+dx)+b\cot(c+dx)\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{5}a\cot^5(c+dx)-\frac{2}{3}a\cot^3(c+dx)-a\cot(c+dx)-\frac{1}{4}b\cot^4(c+dx)-b\cot^2(c+dx)+b\log(\tan(c+dx))}{d}$$

input
```
Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

output
```
(-(a*Cot[c + d*x]) - b*Cot[c + d*x]^2 - (2*a*Cot[c + d*x]^3)/3 - (b*Cot[c
+ d*x]^4)/4 - (a*Cot[c + d*x]^5)/5 + b*Log[Tan[c + d*x]])/d
```

3.21.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.21.4 Maple [A] (verified)

Time = 5.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{b\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx+c)}{d}$
default	$\frac{b\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx+c)}{d}$
risch	$\frac{2b\,e^{8i(dx+c)}-10b\,e^{6i(dx+c)}-\frac{32ia\,e^{4i(dx+c)}}{3}+10b\,e^{4i(dx+c)}+\frac{16ia\,e^{2i(dx+c)}}{3}-2b\,e^{2i(dx+c)}-\frac{16ia}{15}}{d\left(e^{2i(dx+c)}-1\right)^5}+\frac{b\ln\left(e^{2i(dx+c)}-1\right)}{d}-$

input
```
int(csc(d*x+c)^6*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*(-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a*(-8/15-1/5*cs
c(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c))
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(81) = 162$.

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.00

$$\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx =$$
$$-\frac{32\,a\cos\left(dx+c\right)^5-80\,a\cos\left(dx+c\right)^3+30\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+b\right)\log\left(\cos\left(dx+c\right)^2\right)\,\mathrm{s}}{6}$$

input
```
integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/60*(32*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 30*(b*cos(d*x + c)^4 -
2*b*cos(d*x + c)^2 + b)*log(cos(d*x + c)^2)*sin(d*x + c) - 30*(b*cos(d*x +
 c)^4 - 2*b*cos(d*x + c)^2 + b)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c
) + 60*a*cos(d*x + c) - 15*(2*b*cos(d*x + c)^2 - 3*b)*sin(d*x + c))/((d*co
s(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

3.21.6 Sympy [F]

$$\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a+b\tan\left(c+dx\right)\right)\csc^6\left(c+dx\right)dx$$

input
```
integrate(csc(d*x+c)**6*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*csc(c + d*x)**6, x)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{60\,b\log\left(\tan\left(dx+c\right)\right)-\frac{60\,a\tan\left(dx+c\right)^4+60\,b\tan\left(dx+c\right)^3+40\,a\tan\left(dx+c\right)^2+15\,b\tan\left(dx+c\right)+12\,a}{\tan\left(dx+c\right)^5}}{60\,d}$$

input
```
integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/60*(60*b*log(tan(d*x + c)) - (60*a*tan(d*x + c)^4 + 60*b*tan(d*x + c)^3
+ 40*a*tan(d*x + c)^2 + 15*b*tan(d*x + c) + 12*a)/tan(d*x + c)^5)/d
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{60\,b\log\left(\left|\tan\left(dx+c\right)\right|\right) - \frac{137\,b\tan(dx+c)^5+60\,a\tan(dx+c)^4+60\,b\tan(dx+c)^3+40\,a\tan(dx+c)^2+15\,b\tan(dx+c)+12\,a}{\tan(dx+c)^5}}{60\,d}$$

input
```
integrate(csc(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/60*(60*b*log(abs(tan(d*x + c))) - (137*b*tan(d*x + c)^5 + 60*a*tan(d*x +
 c)^4 + 60*b*tan(d*x + c)^3 + 40*a*tan(d*x + c)^2 + 15*b*tan(d*x + c) + 12
*a)/tan(d*x + c)^5)/d
```

3.21.9 Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \csc^6(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{b\,\ln\left(\tan(c+d\,x)\right)}{d} - \frac{a\tan(c+d\,x)^4+b\tan(c+d\,x)^3+\frac{2\,a\tan(c+d\,x)^2}{3}+\frac{b\tan(c+d\,x)}{4}+\frac{a}{5}}{d\tan\left(c+d\,x\right)^5}$$

input
```
int((a + b*tan(c + d*x))/sin(c + d*x)^6,x)
```

output
```
(b*log(tan(c + d*x)))/d - (a/5 + (b*tan(c + d*x))/4 + (2*a*tan(c + d*x)^2)
/3 + a*tan(c + d*x)^4 + b*tan(c + d*x)^3)/(d*tan(c + d*x)^5)
```

3.22 $\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx$

3.22.1 Optimal result

Integrand size = 21, antiderivative size = 113

$$\begin{aligned}\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx &= \frac{3}{8}\left(a^2-5b^2\right)x-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}\\ &\quad+\frac{\cos^2(c+dx)(7b-5a\tan(c+dx))(a+b\tan(c+dx))}{8d}\\ &\quad+\frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{4d}\end{aligned}$$

output
```
3/8*(a^2-5*b^2)*x-2*a*b*ln(cos(d*x+c))/d+b^2*tan(d*x+c)/d+1/8*cos(d*x+c)^2
*(7*b-5*a*tan(d*x+c))*(a+b*tan(d*x+c))/d+1/4*cos(d*x+c)^3*sin(d*x+c)*(a+b*
tan(d*x+c))^2/d
```

3.22.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 250 vs. 2(113) = 226.

Time = 3.12 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.21

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{b\left(\frac{3(a^2-b^2)\arctan(\tan(c+dx))}{b}+\frac{4(-2a^2+3b^2)\arctan(\tan(c+dx))}{b}+16a\cos^2(c+dx)-4a\cos^4(c+dx)+4\left(2a+\frac{a^2-3b}{\sqrt{-b^2}}\right.\right.}{}$$

input
```
Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

output
```
(b*((3*(a^2 - b^2)*ArcTan[Tan[c + d*x]])/b + (4*(-2*a^2 + 3*b^2)*ArcTan[Ta
n[c + d*x]])/b + 16*a*Cos[c + d*x]^2 - 4*a*Cos[c + d*x]^4 + 4*(2*a + (a^2
- 3*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 4*(2*a + (-a^2 + 3
*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (2*(a^2 - b^2)*Cos[c
+ d*x]^3*Sin[c + d*x])/b + (3*(a - b)*(a + b)*Sin[2*(c + d*x)])/(2*b) + (2
*(-2*a^2 + 3*b^2)*Sin[2*(c + d*x)])/b + 8*b*Tan[c + d*x]))/(8*d)
```

3.22.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.51, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3999, 531, 2176, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)^4(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3999}$$

$$\frac{b\int \frac{b^4\tan^4(c+dx)(a+b\tan(c+dx))^2}{\left(\tan^2(c+dx)b^2+b^2\right)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{531}$$

$$\frac{b\left(\frac{b^3\tan(c+dx)(a+b\tan(c+dx))^2}{4(b^2\tan^2(c+dx)+b^2)^2}-\frac{\int \frac{(a+b\tan(c+dx))\left(-4\tan^3(c+dx)b^5+3\tan(c+dx)b^5-4a\tan^2(c+dx)b^4+ab^4\right)}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2}\right)}{d}$$

$$\Big\downarrow \text{2176}$$

$$\frac{b\left(\frac{b^3\tan(c+dx)(a+b\tan(c+dx))^2}{4(b^2\tan^2(c+dx)+b^2)^2}-\frac{-\frac{\int\frac{8\tan^2(c+dx)b^6+16a\tan(c+dx)b^5+\left(3a^2-7b^2\right)b^4}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2}-\frac{b^2(a+b\tan(c+dx))\left(7b^2-5ab\tan(c+dx)\right)}{2\left(b^2\tan^2(c+dx)+b^2\right)}}{4b^2}\right)}{d}$$

$\Big\downarrow$ 2341

$$\frac{b\left(\frac{b^3\tan(c+dx)(a+b\tan(c+dx))^2}{4(b^2\tan^2(c+dx)+b^2)^2}-\frac{-\frac{\int\left(8b^4+\frac{16a\tan(c+dx)b^5+3\left(a^2-5b^2\right)b^4}{\tan^2(c+dx)b^2+b^2}\right)d(b\tan(c+dx))}{2b^2}-\frac{b^2(a+b\tan(c+dx))\left(7b^2-5ab\tan(c+dx)\right)}{2\left(b^2\tan^2(c+dx)+b^2\right)}}{4b^2}\right)}{d}$$

$\Big\downarrow$ 2009

$$\frac{b\left(\frac{b^3\tan(c+dx)(a+b\tan(c+dx))^2}{4(b^2\tan^2(c+dx)+b^2)^2}-\frac{-\frac{3b^3\left(a^2-5b^2\right)\arctan(\tan(c+dx))+8ab^4\log\left(b^2\tan^2(c+dx)+b^2\right)+8b^5\tan(c+dx)}{2b^2}-\frac{b^2(a+b\tan(c+dx))\left(7b^2-5ab\tan(c\right.}{2\left(b^2\tan^2(c+dx)+b^2\right)}}{4b^2}\right.}{d}$$

input
```
Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

output
```
(b*((b^3*Tan[c + d*x]*(a + b*Tan[c + d*x])^2)/(4*(b^2 + b^2*Tan[c + d*x]^2
)^2) - (-1/2*(3*b^3*(a^2 - 5*b^2)*ArcTan[Tan[c + d*x]] + 8*a*b^4*Log[b^2 +
 b^2*Tan[c + d*x]^2] + 8*b^5*Tan[c + d*x])/b^2 - (b^2*(a + b*Tan[c + d*x])
*(7*b^2 - 5*a*b*Tan[c + d*x]))/(2*(b^2 + b^2*Tan[c + d*x]^2)))/(4*b^2)))/d
```

3.22.3.1 Defintions of rubi rules used

rule 531
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomi
alRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
 + b*x^2, x], x, 1]}, Simp[(c + d*x)^n*(a*f - b*e*x)*((a + b*x^2)^(p + 1)/(
2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b
*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*(c + d*x)*Qx - a*d*f*n + b*c*e*(2*p
 + 3) + b*d*e*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGt
Q[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && GtQ[n, 1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.22.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin^7(}{\cos(}\right.}{d}$
default	$\frac{a^2\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin^7(}{\cos(}\right.}{d}$
risch	$2iabx+\frac{3a^2x}{8}-\frac{15b^2x}{8}+\frac{3\,e^{2i(dx+c)}ab}{8d}-\frac{i e^{-2i(dx+c)}a^2}{8d}-\frac{i e^{2i(dx+c)}b^2}{4d}+\frac{3\,e^{-2i(dx+c)}ab}{8d}+\frac{i e^{2i(dx+c)}a^2}{8d}$

input
```
int(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+2*a
*b*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+b^2*(sin(d*x+c)^7/c
os(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)-15/8*
d*x-15/8*c))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{8\,ab\cos\left(dx+c\right)^5 - 32\,ab\cos\left(dx+c\right)^3 + 32\,ab\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right) - (6\,(a^2-5\,b^2)dx - 13}{16\,d\cos\left(dx+\right.}$$

input
```
integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/16*(8*a*b*cos(d*x + c)^5 - 32*a*b*cos(d*x + c)^3 + 32*a*b*cos(d*x + c)*
log(-cos(d*x + c)) - (6*(a^2 - 5*b^2)*d*x - 13*a*b)*cos(d*x + c) - 2*(2*(a
^2 - b^2)*cos(d*x + c)^4 - (5*a^2 - 9*b^2)*cos(d*x + c)^2 + 8*b^2)*sin(d*x
 + c))/(d*cos(d*x + c))
```

3.22.6 Sympy [F]

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^2\sin^4\left(c+dx\right)dx$$

input
```
integrate(sin(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**4, x)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.13

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{8\,ab\log\left(\tan\left(dx+c\right)^2+1\right)+8\,b^2\tan\left(dx+c\right)+3\left(a^2-5\,b^2\right)(dx+c)+\frac{16\,ab\tan(dx+c)^2-\left(5\,a^2-9\,b^2\right)\tan(dx+c)^3+}{\tan(dx+c)^4+2\,\tan(dx}}{8\,d}$$

input
```
integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/8*(8*a*b*log(tan(d*x + c)^2 + 1) + 8*b^2*tan(d*x + c) + 3*(a^2 - 5*b^2)*
(d*x + c) + (16*a*b*tan(d*x + c)^2 - (5*a^2 - 9*b^2)*tan(d*x + c)^3 + 12*a
*b - (3*a^2 - 7*b^2)*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1)
)/d
```

3.22.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5604 vs. 2(107) = 214.

Time = 2.74 (sec) , antiderivative size = 5604, normalized size of antiderivative = 49.59

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/64*(3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2
*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^5 + 24*a^2*d
*x*tan(d*x)^5*tan(c)^5 - 120*b^2*d*x*tan(d*x)^5*tan(c)^5 + 3*pi*b^2*sgn(-2
*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)
^5*tan(c)^5 + 6*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*ta
n(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^3 -
3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d
*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 6*pi*b^2*sgn(2
*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 +
 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^5 + 6*b^2*arctan((tan(d*x) + tan
(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^5*tan(c)^5 - 6*b^2*arctan(-(tan(d*x)
- tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^5*tan(c)^5 - 64*a*b*log(4*(tan(d
*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2
+ tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 + 48*a^2*d*x*tan(d*x)^5*tan(c)^3 - 24
0*b^2*d*x*tan(d*x)^5*tan(c)^3 + 6*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(
d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^3 - 24*a^2*d*x*ta
n(d*x)^4*tan(c)^4 + 120*b^2*d*x*tan(d*x)^4*tan(c)^4 - 3*pi*b^2*sgn(-2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*ta
n(c)^4 + 48*a^2*d*x*tan(d*x)^3*tan(c)^5 - 240*b^2*d*x*tan(d*x)^3*tan(c)^5
+ 6*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) ...
```

3.22.9 Mupad [B] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= x\left(\frac{3a^2}{8}-\frac{15b^2}{8}\right)+\frac{b^2\tan(c+d\,x)}{d}$$
$$+\frac{\left(\frac{9b^2}{8}-\frac{5a^2}{8}\right)\tan(c+d\,x)^3+2\,a\,b\tan(c+d\,x)^2+\left(\frac{7b^2}{8}-\frac{3a^2}{8}\right)\tan(c+d\,x)+\frac{3\,a\,b}{2}}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}$$
$$+\frac{a\,b\,\ln\left(\tan(c+d\,x)^2+1\right)}{d}$$

input
```
int(sin(c + d*x)^4*(a + b*tan(c + d*x))^2,x)
```

output
```
x*((3*a^2)/8 - (15*b^2)/8) + (b^2*tan(c + d*x))/d + ((3*a*b)/2 - tan(c + d
*x)*((3*a^2)/8 - (7*b^2)/8) - tan(c + d*x)^3*((5*a^2)/8 - (9*b^2)/8) + 2*a
*b*tan(c + d*x)^2)/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1)) + (a*b*log(
tan(c + d*x)^2 + 1))/d
```

3.22. $\int \sin^4(c+dx)(a+b\tan(c+dx))^2\,dx$

3.23 $\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx$

3.23.1 Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2ab\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a^2\cos(c+dx)}{d} + \frac{2b^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} - \frac{b^2\cos^3(c+dx)}{3d} + \frac{b^2\sec(c+dx)}{d} - \frac{2ab\sin(c+dx)}{d} - \frac{2ab\sin^3(c+dx)}{3d}$$

output
```
2*a*b*arctanh(sin(d*x+c))/d-a^2*cos(d*x+c)/d+2*b^2*cos(d*x+c)/d+1/3*a^2*co
s(d*x+c)^3/d-1/3*b^2*cos(d*x+c)^3/d+b^2*sec(d*x+c)/d-2*a*b*sin(d*x+c)/d-2/
3*a*b*sin(d*x+c)^3/d
```

3.23.2 Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.25

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{\sec(c+dx)\left(-9a^2+45b^2+(-8a^2+20b^2)\cos(2(c+dx))+(a^2-b^2)\cos(4(c+dx))-48ab\cos(c+dx)\,\mathrm{l}\right.}{}$$

input
```
Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*(-9*a^2 + 45*b^2 + (-8*a^2 + 20*b^2)*Cos[2*(c + d*x)] + (a^2
 - b^2)*Cos[4*(c + d*x)] - 48*a*b*Cos[c + d*x]*Log[Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]] + 48*a*b*Cos[c + d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]] - 28*a*b*Sin[2*(c + d*x)] + 2*a*b*Sin[4*(c + d*x)]))/(24*d)
```

3.23.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)^3(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{4000}$$

$$\int \left(a^2\sin^3(c+dx)+2ab\sin^3(c+dx)\tan(c+dx)+b^2\sin^3(c+dx)\tan^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos(c+dx)}{d}+\frac{2ab\text{arctanh}(\sin(c+dx))}{d}-\frac{2ab\sin^3(c+dx)}{3d}-\frac{2ab\sin(c+dx)}{d}-\frac{b^2\cos^3(c+dx)}{3d}+\frac{2b^2\cos(c+dx)}{d}+\frac{b^2\sec(c+dx)}{d}$$

input
```
Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

output
```
(2*a*b*ArcTanh[Sin[c + d*x]])/d - (a^2*Cos[c + d*x])/d + (2*b^2*Cos[c + d*
x])/d + (a^2*Cos[c + d*x]^3)/(3*d) - (b^2*Cos[c + d*x]^3)/(3*d) + (b^2*Sec
[c + d*x])/d - (2*a*b*Sin[c + d*x])/d - (2*a*b*Sin[c + d*x]^3)/(3*d)
```

3.23.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.23.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{a^2\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+2ab\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin\right.\right.}{d}$
default	$\frac{-\frac{a^2\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+2ab\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+b^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin\right.\right.}{d}$
risch	$\frac{5i\mathrm{e}^{i(dx+c)}ab}{4d}-\frac{3\,\mathrm{e}^{i(dx+c)}a^2}{8d}+\frac{7\,\mathrm{e}^{i(dx+c)}b^2}{8d}-\frac{5i\mathrm{e}^{-i(dx+c)}ab}{4d}-\frac{3\,\mathrm{e}^{-i(dx+c)}a^2}{8d}+\frac{7\,\mathrm{e}^{-i(dx+c)}b^2}{8d}+\frac{2b^2\mathrm{e}^{i(dx+c}}{d\left(\mathrm{e}^{2i(dx+c)}+\right.}$

input
```
int(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a^2*(2+sin(d*x+c)^2)*cos(d*x+c)+2*a*b*(-1/3*sin(d*x+c)^3-sin(d*x
+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c
)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{(a^2-b^2)\cos(dx+c)^4+3\,ab\cos(dx+c)\log(\sin(dx+c)+1)-3\,ab\cos(dx+c)\log(-\sin(dx+c)+1)}{3\,d\cos(dx+c)}$$

input
```
integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*((a^2 - b^2)*cos(d*x + c)^4 + 3*a*b*cos(d*x + c)*log(sin(d*x + c) + 1)
 - 3*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - 3*(a^2 - 2*b^2)*cos(d*x + c
)^2 + 3*b^2 + 2*(a*b*cos(d*x + c)^3 - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c))
```

3.23.6 Sympy [F]

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx=\int (a+b\tan(c+dx))^2\sin^3(c+dx)\,dx$$

input
```
integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**3, x)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{\left(\cos(dx+c)^3-3\cos(dx+c)\right)a^2-\left(2\sin(dx+c)^3-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right.}{3\,d}$$

input
```
integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/3*((cos(d*x + c)^3 - 3*cos(d*x + c))*a^2 - (2*sin(d*x + c)^3 - 3*log(sin
(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a*b - (cos(d*x
+ c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*b^2)/d
```

3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32694 vs. $2(116) = 232$.

Time = 16.95 (sec) , antiderivative size = 32694, normalized size of antiderivative = 267.98

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/192*(15*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/
2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 + 15*pi*b^2*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - ta
n(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2
*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^8*tan(1/2*c)^8 - 15*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 + 15*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 - 30*pi*b^2*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 +
 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 + 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/
2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2
*c)^6 + 30*pi*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/...
```

3.23.9 Mupad [B] (verification not implemented)

Time = 7.45 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.43

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{4\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{8\,a^2}{3}-\frac{32\,b^2}{3}\right)-4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{4\,a^2}{3}-\frac{16\,b^2}{3}+\frac{28\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-\frac{28\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-4\,a}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int(sin(c + d*x)^3*(a + b*tan(c + d*x))^2,x)
```

output
```
(4*a*b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^2*((8*a^2)/3 - (
32*b^2)/3) - 4*a^2*tan(c/2 + (d*x)/2)^4 + (4*a^2)/3 - (16*b^2)/3 + (28*a*b
*tan(c/2 + (d*x)/2)^3)/3 - (28*a*b*tan(c/2 + (d*x)/2)^5)/3 - 4*a*b*tan(c/2
 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*t
an(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + 1))
```

3.24 $\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx$

3.24.1 Optimal result

Integrand size = 21, antiderivative size = 76

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{1}{2}\left(a^2-3b^2\right)x-\frac{2ab\log(\cos(c+dx))}{d}+\frac{3b^2\tan(c+dx)}{2d}-\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^2}{2d}$$

output
```
1/2*(a^2-3*b^2)*x-2*a*b*ln(cos(d*x+c))/d+3/2*b^2*tan(d*x+c)/d-1/2*cos(d*x+
c)*sin(d*x+c)*(a+b*tan(d*x+c))^2/d
```

3.24.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 162 vs. $2(76) = 152$.

Time = 2.70 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.13

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\left(\frac{(-a^2+b^2)\arctan(\tan(c+dx))}{b}+2a\cos^2(c+dx)+\left(2a+\frac{a^2-2b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+\left(2a+\frac{-a^2+2b^2}{\sqrt{-b^2}}\right.\right.}{2d}$$

input
```
Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

output
```
(b*(((-a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + 2*a*Cos[c + d*x]^2 + (2*a + (a
^2 - 2*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + (2*a + (-a^2 +
2*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + ((-a^2 + b^2)*Sin[2*
(c + d*x)])/(2*b) + 2*b*Tan[c + d*x]))/(2*d)
```

3.24.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3999, 531, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(c+dx)^2(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{ 3999}$$

$$\frac{b\int \frac{b^2\tan^2(c+dx)(a+b\tan(c+dx))^2}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 531}$$

$$\frac{b\left(-\frac{\int -\frac{b^2(a+b\tan(c+dx))(a+3b\tan(c+dx))}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2}-\frac{b\tan(c+dx)(a+b\tan(c+dx))^2}{2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\left(\frac{\int \frac{b^2(a+b\tan(c+dx))(a+3b\tan(c+dx))}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2}-\frac{b\tan(c+dx)(a+b\tan(c+dx))^2}{2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\left(\frac{1}{2}\int \frac{(a+b\tan(c+dx))(a+3b\tan(c+dx))}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))-\frac{b\tan(c+dx)(a+b\tan(c+dx))^2}{2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{ 657}$$

$$\frac{b\left(\frac{1}{2}\int\left(\frac{a^2+4b\tan(c+dx)a-3b^2}{\tan^2(c+dx)b^2+b^2}+3\right)d(b\tan(c+dx))-\frac{b\tan(c+dx)(a+b\tan(c+dx))^2}{2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\left(\frac{1}{2}\left(\frac{(a^2-3b^2)\arctan(\tan(c+dx))}{b}+2a\log\left(b^2\tan^2(c+dx)+b^2\right)+3b\tan(c+dx)\right)-\frac{b\tan(c+dx)(a+b\tan(c+dx))^2}{2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

input
```
Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

output
```
(b*((((a^2 - 3*b^2)*ArcTan[Tan[c + d*x]])/b + 2*a*Log[b^2 + b^2*Tan[c + d*
x]^2] + 3*b*Tan[c + d*x])/2 - (b*Tan[c + d*x]*(a + b*Tan[c + d*x])^2)/(2*(
b^2 + b^2*Tan[c + d*x]^2))))/d
```

3.24.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 531
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomi
alRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
 + b*x^2, x], x, 1]}, Simp[(c + d*x)^n*(a*f - b*e*x)*((a + b*x^2)^(p + 1)/(
2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b
*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*(c + d*x)*Qx - a*d*f*n + b*c*e*(2*p
 + 3) + b*d*e*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGt
Q[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && GtQ[n, 1] && IntegerQ[2*p]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.24.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{a^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\right.}{d}$
default	$\frac{a^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\right.}{d}$
risch	$2iabx+\frac{a^2x}{2}-\frac{3b^2x}{2}+\frac{e^{2i(dx+c)}ab}{4d}+\frac{ie^{2i(dx+c)}a^2}{8d}-\frac{ie^{2i(dx+c)}b^2}{8d}+\frac{e^{-2i(dx+c)}ab}{4d}-\frac{ie^{-2i(dx+c)}a^2}{8d}+$

input
```
int(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+2*a*b*(-1/2*sin(d*x+c)
^2-ln(cos(d*x+c)))+b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+
c))*cos(d*x+c)-3/2*d*x-3/2*c))
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.33

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{2\,ab\cos\left(dx+c\right)^3-4\,ab\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)+\left(\left(a^2-3\,b^2\right)dx-ab\right)\cos\left(dx+c\right)-\left(\left(a^2-b^2\right)\right.}{2\,d\cos\left(dx+c\right)}$$

input
```
integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*cos(d*x + c)^3 - 4*a*b*cos(d*x + c)*log(-cos(d*x + c)) + ((a^2
- 3*b^2)*d*x - a*b)*cos(d*x + c) - ((a^2 - b^2)*cos(d*x + c)^2 - 2*b^2)*si
n(d*x + c))/(d*cos(d*x + c))
```

3.24.6 Sympy [F]

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\sin^2(c+dx)\,dx$$

input
```
integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**2, x)
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2\,ab\log\left(\tan(dx+c)^2+1\right)+2\,b^2\tan(dx+c)+(a^2-3\,b^2)(dx+c)+\frac{2\,ab-(a^2-b^2)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*(2*a*b*log(tan(d*x + c)^2 + 1) + 2*b^2*tan(d*x + c) + (a^2 - 3*b^2)*(d
*x + c) + (2*a*b - (a^2 - b^2)*tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

3.24.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 981 vs. 2(70) = 140.

Time = 0.62 (sec) , antiderivative size = 981, normalized size of antiderivative = 12.91

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(a^2*d*x*tan(d*x)^3*tan(c)^3 - 3*b^2*d*x*tan(d*x)^3*tan(c)^3 - 2*a*b*l
og(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 +
tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + a^2*d*x*tan(d*x)^3*tan(c
) - 3*b^2*d*x*tan(d*x)^3*tan(c) - a^2*d*x*tan(d*x)^2*tan(c)^2 + 3*b^2*d*x*
tan(d*x)^2*tan(c)^2 + a^2*d*x*tan(d*x)*tan(c)^3 - 3*b^2*d*x*tan(d*x)*tan(c
)^3 + a*b*tan(d*x)^3*tan(c)^3 - 2*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d
*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x
)^3*tan(c) + 2*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(ta
n(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + a^2*
tan(d*x)^3*tan(c)^2 - 3*b^2*tan(d*x)^3*tan(c)^2 - 2*a*b*log(4*(tan(d*x)^2*
tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(
c)^2 + 1))*tan(d*x)*tan(c)^3 + a^2*tan(d*x)^2*tan(c)^3 - 3*b^2*tan(d*x)^2*
tan(c)^3 - a^2*d*x*tan(d*x)^2 + 3*b^2*d*x*tan(d*x)^2 + a^2*d*x*tan(d*x)*ta
n(c) - 3*b^2*d*x*tan(d*x)*tan(c) - a*b*tan(d*x)^3*tan(c) - a^2*d*x*tan(c)^
2 + 3*b^2*d*x*tan(c)^2 - 5*a*b*tan(d*x)^2*tan(c)^2 - a*b*tan(d*x)*tan(c)^3
 + 2*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*t
an(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2 - 2*b^2*tan(d*x)^3 - 2*a*
b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2
 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)*tan(c) - 2*a^2*tan(d*x)^2*tan(c) +
 2*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*...
```

3.24.9 Mupad [B] (verification not implemented)

Time = 4.62 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{\cos(c+d\,x)^2\left(a\,b-\tan(c+d\,x)\,\left(\frac{a^2}{2}-\frac{b^2}{2}\right)\right)+b^2\tan(c+d\,x)+a\,b\,\ln\left(\tan(c+d\,x)^2+1\right)+d\,x\,\left(\frac{a^2}{2}-\frac{3}{\cdots}\right.}{d}$$

input
```
int(sin(c + d*x)^2*(a + b*tan(c + d*x))^2,x)
```

output
```
(cos(c + d*x)^2*(a*b - tan(c + d*x)*(a^2/2 - b^2/2)) + b^2*tan(c + d*x) +
a*b*log(tan(c + d*x)^2 + 1) + d*x*(a^2/2 - (3*b^2)/2))/d
```

3.25 $\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx$

3.25.1 Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx=\frac{2ab\text{arctanh}(\sin(c+dx))}{d}-\frac{a^2\cos(c+dx)}{d}+\frac{b^2\cos(c+dx)}{d}+\frac{b^2\sec(c+dx)}{d}-\frac{2ab\sin(c+dx)}{d}$$

output
```
2*a*b*arctanh(sin(d*x+c))/d-a^2*cos(d*x+c)/d+b^2*cos(d*x+c)/d+b^2*sec(d*x+
c)/d-2*a*b*sin(d*x+c)/d
```

3.25.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.63

$$\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{\sec(c+dx)\left(a^2-3b^2+(a^2-b^2)\cos(2(c+dx))+4ab\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+d\right.\right.\right.\right.}{2d}$$

input
```
Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

output
```
-1/2*(Sec[c + d*x]*(a^2 - 3*b^2 + (a^2 - b^2)*Cos[2*(c + d*x)] + 4*a*b*Cos
[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]) + 2*a*b*Sin[2*(c + d*x)]))/d
```

3.25.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(c+dx)(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \ 4000$$

$$\int \left(a^2\sin(c+dx)+2ab\sin(c+dx)\tan(c+dx)+b^2\sin(c+dx)\tan^2(c+dx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^2\cos(c+dx)}{d}+\frac{2ab\text{arctanh}(\sin(c+dx))}{d}-\frac{2ab\sin(c+dx)}{d}+\frac{b^2\cos(c+dx)}{d}+\frac{b^2\sec(c+dx)}{d}$$

input
```
Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

output
```
(2*a*b*ArcTanh[Sin[c + d*x]])/d - (a^2*Cos[c + d*x])/d + (b^2*Cos[c + d*x]
)/d + (b^2*Sec[c + d*x])/d - (2*a*b*Sin[c + d*x])/d
```

3.25.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.25.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{-a^2\cos(dx+c)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$
default	$\frac{-a^2\cos(dx+c)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$
risch	$\frac{ie^{i(dx+c)}ab}{d}-\frac{e^{i(dx+c)}a^2}{2d}+\frac{e^{i(dx+c)}b^2}{2d}-\frac{ie^{-i(dx+c)}ab}{d}-\frac{e^{-i(dx+c)}a^2}{2d}+\frac{e^{-i(dx+c)}b^2}{2d}+\frac{2b^2e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}-$

input
```
int(sin(d*x+c)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*cos(d*x+c)+2*a*b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^2*(si
n(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)))
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.32

$$\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{ab\cos(dx+c)\log(\sin(dx+c)+1)-ab\cos(dx+c)\log(-\sin(dx+c)+1)-2\,ab\cos(dx+c)\sin(dx+}{d\cos(dx+c)}$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
(a*b*cos(d*x + c)*log(sin(d*x + c) + 1) - a*b*cos(d*x + c)*log(-sin(d*x +
c) + 1) - 2*a*b*cos(d*x + c)*sin(d*x + c) - (a^2 - b^2)*cos(d*x + c)^2 + b
^2)/(d*cos(d*x + c))
```

3.25.6 Sympy [F]

$$\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\sin(c+dx)\,dx$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sin(c + d*x), x)
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

$$\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{b^2\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)+ab(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\sin(dx+c))-a^2\cos}{d}$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
(b^2*(1/cos(d*x + c) + cos(d*x + c)) + a*b*(log(sin(d*x + c) + 1) - log(si
n(d*x + c) - 1) - 2*sin(d*x + c)) - a^2*cos(d*x + c))/d
```

3.25.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2405 vs. 2(68) = 136.

Time = 1.08 (sec) , antiderivative size = 2405, normalized size of antiderivative = 35.37

$$\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-(a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2
*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x
) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - a*b*log(2*(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*
d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*t
an(1/2*c)^4 + a^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 2*b^2*tan(1/2*d*x)^4*tan(1
/2*c)^4 - 4*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(
1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*t
an(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 4*a*b*log(2*(tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1
/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) +
1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(
1/2*d*x)^3*tan(1/2*c)^3 - 4*a*b*tan(1/2*d*x)^4*tan(1/2*c)^3 - 4*a*b*tan(1/
2*d*x)^3*tan(1/2*c)^4 - 2*a^2*tan(1/2*d*x)^4*tan(1/2*c)^2 - 8*a^2*tan(1/2*
d*x)^3*tan(1/2*c)^3 + 8*b^2*tan(1/2*d*x)^3*tan(1/2*c)^3 - 2*a^2*tan(1/2*d*
x)^2*tan(1/2*c)^4 - a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2...
```

3.25.9 Mupad [B] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.37

$$\int \sin(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{4\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\,a^2+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4\,b^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}$$

input
```
int(sin(c + d*x)*(a + b*tan(c + d*x))^2,x)
```

output
```
(4*a*b*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2)^2 - 2*a^2
+ 4*b^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 - 4*a*b*tan(c/2 + (d*x)/2))/(d*(tan(c
/2 + (d*x)/2)^4 - 1))
```

3.26 $\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx$

3.26.1 Optimal result

Integrand size = 19, antiderivative size = 43

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{a^2\text{arctanh}(\cos(c+dx))}{d} + \frac{2ab\text{arctanh}(\sin(c+dx))}{d} + \frac{b^2\sec(c+dx)}{d}$$

output
```
-a^2*arctanh(cos(d*x+c))/d+2*a*b*arctanh(sin(d*x+c))/d+b^2*sec(d*x+c)/d
```

3.26.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 97 vs. $2(43) = 86$.

Time = 1.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.26

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{a\left(-a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2b\log}{d}$$

input
```
Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

output
```
(a*(-(a*Log[Cos[(c + d*x)/2]]) - 2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]] + a*Log[Sin[(c + d*x)/2]] + 2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]]) + b^2*Sec[c + d*x])/d
```

3.26.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sin(c+dx)}dx$$

$$\Big\downarrow \text{4000}$$

$$\int \left(a^2\csc(c+dx)+2ab\sec(c+dx)+b^2\tan(c+dx)\sec(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\text{arctanh}(\cos(c+dx))}{d}+\frac{2ab\text{arctanh}(\sin(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

input
```
Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

output
```
-((a^2*ArcTanh[Cos[c + d*x]])/d) + (2*a*b*ArcTanh[Sin[c + d*x]])/d + (b^2*
Sec[c + d*x])/d
```

3.26.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.26.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$\frac{\frac{b^2}{\cos(dx+c)}+2ab\ln(\sec(dx+c)+\tan(dx+c))+a^2\ln(\csc(dx+c)-\cot(dx+c))}{d}$	56
default	$\frac{\frac{b^2}{\cos(dx+c)}+2ab\ln(\sec(dx+c)+\tan(dx+c))+a^2\ln(\csc(dx+c)-\cot(dx+c))}{d}$	56
risch	$\frac{2b^2 e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}+\frac{2ab\ln(e^{i(dx+c)}+i)}{d}-\frac{2ab\ln(e^{i(dx+c)}-i)}{d}+\frac{a^2\ln(e^{i(dx+c)}-1)}{d}-\frac{a^2\ln(e^{i(dx+c)}+1)}{d}$	111

input
```
int(csc(d*x+c)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2/cos(d*x+c)+2*a*b*ln(sec(d*x+c)+tan(d*x+c))+a^2*ln(csc(d*x+c)-cot(
d*x+c)))
```

3.26.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(43) = 86.

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.37

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{a^2\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-a^2\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2\,ab\cos(dx+c)\,l}{2\,d\cos(dx+c)}$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(a^2*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - a^2*cos(d*x + c)*log(
-1/2*cos(d*x + c) + 1/2) - 2*a*b*cos(d*x + c)*log(sin(d*x + c) + 1) + 2*a*
b*cos(d*x + c)*log(-sin(d*x + c) + 1) - 2*b^2)/(d*cos(d*x + c))
```

3.26.6 Sympy [F]

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan{(c+dx)})^2\csc{(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*csc(c + d*x), x)
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{ab(\log{(\sin{(dx+c)}+1)} - \log{(\sin{(dx+c)}-1)}) - a^2\log{(\cot{(dx+c)}+\csc{(dx+c)})} + \frac{b^2}{\cos(dx+c)}}{d}$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
(a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - a^2*log(cot(d*x + c
) + csc(d*x + c)) + b^2/cos(d*x + c))/d
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{2\,ab\log{\left(\left|\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+1\right|\right)} - 2\,ab\log{\left(\left|\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-1\right|\right)} + a^2\log{\left(\left|\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right|\right)} - \frac{2}{\tan(\frac{1}{2}\,dx}}{d}$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
(2*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*b*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) + a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*b^2/(tan(1/2*d*x + 1/2
*c)^2 - 1))/d
```

3.26.9 Mupad [B] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.91

$$\int \csc(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{2b^2}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$
$$-\frac{4ab\,\mathrm{atanh}\left(\frac{16a^2b^2}{8a^3b-16a^2b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{8a^3b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^3b-16a^2b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}$$

input
```
int((a + b*tan(c + d*x))^2/sin(c + d*x),x)
```

output
```
(a^2*log(tan(c/2 + (d*x)/2)))/d - (2*b^2)/(d*(tan(c/2 + (d*x)/2)^2 - 1)) -
 (4*a*b*atanh((16*a^2*b^2)/(8*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2)) - (8*
a^3*b*tan(c/2 + (d*x)/2))/(8*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2))))/d
```

3.27 $\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx$

3.27.1 Optimal result

Integrand size = 21, antiderivative size = 42

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{a^2\cot(c+dx)}{d}+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

output
```
-a^2*cot(d*x+c)/d+2*a*b*ln(tan(d*x+c))/d+b^2*tan(d*x+c)/d
```

3.27.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 91 vs. 2(42) = 84.

Time = 1.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.17

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{\cos(c+dx)\left(a\cos(c+dx)(a\cot(c+dx)+2b(\log(\cos(c+dx))-\log(\sin(c+dx))))-b^2\sin(c+dx)\right)(a}{d(a\cos(c+dx)+b\sin(c+dx))^2}$$

input
```
Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

output
```
-((Cos[c + d*x]*(a*Cos[c + d*x]*(a*Cot[c + d*x] + 2*b*(Log[Cos[c + d*x]] -
 Log[Sin[c + d*x]])) - b^2*Sin[c + d*x])*(a + b*Tan[c + d*x])^2)/(d*(a*Cos
[c + d*x] + b*Sin[c + d*x])^2))
```

3.27.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3999, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3999}$$

$$\frac{b\int \frac{\cot^2(c+dx)(a+b\tan(c+dx))^2}{b^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{49}$$

$$\frac{b\int \left(\frac{a^2\cot^2(c+dx)}{b^2}+\frac{2a\cot(c+dx)}{b}+1\right)d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{a^2\cot(c+dx)}{b}+2a\log(b\tan(c+dx))+b\tan(c+dx)\right)}{d}$$

input
```
Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

output
```
(b*(-((a^2*Cot[c + d*x])/b) + 2*a*Log[b*Tan[c + d*x]] + b*Tan[c + d*x]))/d
```

3.27.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.27.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{b^2\tan(dx+c)+2ab\ln(\tan(dx+c))-a^2\cot(dx+c)}{d}$	38
default	$\frac{b^2\tan(dx+c)+2ab\ln(\tan(dx+c))-a^2\cot(dx+c)}{d}$	38
risch	$-\frac{2i(a^2 e^{2i(dx+c)}-b^2 e^{2i(dx+c)}+a^2+b^2)}{d(e^{2i(dx+c)}-1)(e^{2i(dx+c)}+1)}+\frac{2ab\ln(e^{2i(dx+c)}-1)}{d}-\frac{2ab\ln(e^{2i(dx+c)}+1)}{d}$	106

input
```
int(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*tan(d*x+c)+2*a*b*ln(tan(d*x+c))-a^2*cot(d*x+c))
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(42) = 84.

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.29

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{ab\cos(dx+c)\log\left(\cos(dx+c)^2\right)\sin(dx+c)-ab\cos(dx+c)\log\left(-\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\right)\sin(dx+c)}{d\cos(dx+c)\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(a*b*cos(d*x + c)*log(cos(d*x + c)^2)*sin(d*x + c) - a*b*cos(d*x + c)*log
(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + (a^2 + b^2)*cos(d*x + c)^2 - b^
2)/(d*cos(d*x + c)*sin(d*x + c))
```

3.27.6 Sympy [F]

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2 \csc^2(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**2, x)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2\,ab\log(\tan(dx+c)) + b^2\tan(dx+c) - \frac{a^2}{\tan(dx+c)}}{d}$$

input
```
integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
(2*a*b*log(tan(d*x + c)) + b^2*tan(d*x + c) - a^2/tan(d*x + c))/d
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2\,ab\log(|\tan(dx+c)|) + b^2\tan(dx+c) - \frac{2\,ab\tan(dx+c)+a^2}{\tan(dx+c)}}{d}$$

input
```
integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
(2*a*b*log(abs(tan(d*x + c))) + b^2*tan(d*x + c) - (2*a*b*tan(d*x + c) + a
^2)/tan(d*x + c))/d
```

3.27.9 Mupad [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b^2\tan(c+d\,x)}{d} - \frac{a^2}{d\tan\left(c+d\,x\right)} + \frac{2\,a\,b\,\ln\left(\tan(c+d\,x)\right)}{d}$$

input
```
int((a + b*tan(c + d*x))^2/sin(c + d*x)^2,x)
```

output
```
(b^2*tan(c + d*x))/d - a^2/(d*tan(c + d*x)) + (2*a*b*log(tan(c + d*x)))/d
```

3.28 $\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx$

3.28.1 Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx=-\frac{a^2\text{arctanh}(\cos(c+dx))}{2d}-\frac{b^2\text{arctanh}(\cos(c+dx))}{d}+\frac{2ab\text{arctanh}(\sin(c+dx))}{d}-\frac{2ab\csc(c+dx)}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}+\frac{b^2\sec(c+dx)}{d}$$

output
```
-1/2*a^2*arctanh(cos(d*x+c))/d-b^2*arctanh(cos(d*x+c))/d+2*a*b*arctanh(sin
(d*x+c))/d-2*a*b*csc(d*x+c)/d-1/2*a^2*cot(d*x+c)*csc(d*x+c)/d+b^2*sec(d*x+
c)/d
```

3.28.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 250 vs. 2(95) = 190.

Time = 2.78 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.63

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{8b^2-8ab\cot\left(\frac{1}{2}(c+dx)\right)-a^2\csc^2\left(\frac{1}{2}(c+dx)\right)-4(a^2+2b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-16ab\log\left(\cos\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

output
```
(8*b^2 - 8*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 - 4*(a^2 + 2*b^2)
*Log[Cos[(c + d*x)/2]] - 16*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 4*(a^2 + 2*b^2)*Log[Sin[(c + d*x)/2]] + 16*a*b*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 + (8*b^2*Sin[(c + d*x)/2])/(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]) - (8*b^2*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]) - 8*a*b*Tan[(c + d*x)/2])/(8*d)
```

3.28.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sin(c+dx)^3}dx$$

$$\Big\downarrow \text{4000}$$

$$\int \left(a^2\csc^3(c+dx)+2ab\csc^2(c+dx)\sec(c+dx)+b^2\csc(c+dx)\sec^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\text{arctanh}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}+\frac{2ab\text{arctanh}(\sin(c+dx))}{d}-\frac{2ab\csc(c+dx)}{d}-\frac{b^2\text{arctanh}(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

input
```
Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

output
```
-1/2*(a^2*ArcTanh[Cos[c + d*x]])/d - (b^2*ArcTanh[Cos[c + d*x]])/d + (2*a*
b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Csc[c + d*x])/d - (a^2*Cot[c + d*x]*Cs
c[c + d*x])/(2*d) + (b^2*Sec[c + d*x])/d
```

3.28.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.28.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{b^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+2ab\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a^2\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\right.}{d}$
default	$\frac{b^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+2ab\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a^2\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\right.}{d}$
risch	$\frac{e^{i(dx+c)}\left(a^2e^{4i(dx+c)}+2b^2e^{4i(dx+c)}-4iab\,e^{4i(dx+c)}+2a^2e^{2i(dx+c)}-4b^2e^{2i(dx+c)}+a^2+2b^2+4iab\right)}{d\left(e^{2i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)}+\frac{a^2\ln\left(e^{i(dx+c)}-1\right.}{2d}$

input
```
int(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+2*a*b*(-1/sin(d*x+c)+ln(
sec(d*x+c)+tan(d*x+c)))+a^2*(-1/2*csc(d*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-
cot(d*x+c))))
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(91) = 182.

Time = 0.31 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.42

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{8\,ab\cos(dx+c)\sin(dx+c)+2\,(a^2+2\,b^2)\cos(dx+c)^2-4\,b^2-\left((a^2+2\,b^2)\cos(dx+c)^3-(a^2+2\,b^2)\cos\right.}{}$$

input
```
integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/4*(8*a*b*cos(d*x + c)*sin(d*x + c) + 2*(a^2 + 2*b^2)*cos(d*x + c)^2 - 4*
b^2 - ((a^2 + 2*b^2)*cos(d*x + c)^3 - (a^2 + 2*b^2)*cos(d*x + c))*log(1/2*
cos(d*x + c) + 1/2) + ((a^2 + 2*b^2)*cos(d*x + c)^3 - (a^2 + 2*b^2)*cos(d*
x + c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(a*b*cos(d*x + c)^3 - a*b*cos(d*x
 + c))*log(sin(d*x + c) + 1) - 4*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*l
og(-sin(d*x + c) + 1))/(d*cos(d*x + c)^3 - d*cos(d*x + c))
```

3.28.6 Sympy [F]

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\csc^3(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**3, x)
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.28

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{a^2\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)+2\,b^2\left(\frac{2}{\cos(dx+c)}-\log(\cos(dx+c)+1)\right.}{4\,d}$$

input
```
integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*(a^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + lo
g(cos(d*x + c) - 1)) + 2*b^2*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log
(cos(d*x + c) - 1)) - 4*a*b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)))/d
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.81

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$= \frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+16\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-16\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-8\,ab\tan\left(\frac{1}{2}\right.}{}$$

input
```
integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 + 16*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) - 16*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 8*a*b*tan(1/2*d*x + 1/2*c)
 + 4*(a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) - 16*b^2/(tan(1/2*d*x +
1/2*c)^2 - 1) - (6*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*tan(1/2*d*x + 1/2*c
)^2 + 8*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c)^2)/d
```

3.28.9 Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.07

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{a^2}{2}+b^2\right)}{d}$$
$$+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^2}{2}+8\,b^2\right)-\frac{a^2}{2}+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}$$
$$+\frac{4\,a\,b\,\mathrm{atanh}\left(\frac{8\,a\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^3\,b-16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+8\,a\,b^3}-\frac{16\,a^2\,b^2}{4\,a^3\,b-16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+8\,a\,b^3}+\frac{4\,a^3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^3\,b-16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+8\,a\,b^3}\right)}{d}$$
$$-\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$

input
```
int((a + b*tan(c + d*x))^2/sin(c + d*x)^3,x)
```

output
```
(a^2*tan(c/2 + (d*x)/2)^2)/(8*d) + (log(tan(c/2 + (d*x)/2))*(a^2/2 + b^2))
/d + (tan(c/2 + (d*x)/2)^2*(a^2/2 + 8*b^2) - a^2/2 + 4*a*b*tan(c/2 + (d*x)
/2)^3 - 4*a*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 +
 (d*x)/2)^4)) + (4*a*b*atanh((8*a*b^3*tan(c/2 + (d*x)/2))/(8*a*b^3 + 4*a^3
*b - 16*a^2*b^2*tan(c/2 + (d*x)/2)) - (16*a^2*b^2)/(8*a*b^3 + 4*a^3*b - 16
*a^2*b^2*tan(c/2 + (d*x)/2)) + (4*a^3*b*tan(c/2 + (d*x)/2))/(8*a*b^3 + 4*a
^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2))))/d - (a*b*tan(c/2 + (d*x)/2))/d
```

3.28. $\int \csc^3(c+dx)(a+b\tan(c+dx))^2\,dx$

3.29 $\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx$

3.29.1 Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{(a^2+b^2)\cot(c+dx)}{d} - \frac{ab\cot^2(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} + \frac{2ab\log(\tan(c+dx))}{d} + \frac{b^2\tan(c+dx)}{d}$$

output
```
-(a^2+b^2)*cot(d*x+c)/d-a*b*cot(d*x+c)^2/d-1/3*a^2*cot(d*x+c)^3/d+2*a*b*ln
(tan(d*x+c))/d+b^2*tan(d*x+c)/d
```

3.29.2 Mathematica [A] (verified)

Time = 2.75 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.61

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{(3ab\cot^2(c+dx) + a^2\cot^3(c+dx) + \cos^2(c+dx)\left((2a^2+3b^2)\cot(c+dx) + 6ab(\log(\cos(c+dx)) - \text{l}\right.}{3d(a\cos(c+dx)+b\sin(c+dx))^2}$$

input
```
Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

output
```
-1/3*((3*a*b*Cot[c + d*x]^2 + a^2*Cot[c + d*x]^3 + Cos[c + d*x]^2*((2*a^2
+ 3*b^2)*Cot[c + d*x] + 6*a*b*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])) - (
3*b^2*Sin[2*(c + d*x)])/2)*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c + d*x] + b*
Sin[c + d*x])^2)
```

3.29.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3999, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{3999}$$

$$\frac{b\int \frac{\cot^4(c+dx)(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)}{b^4}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{b\int \left(\frac{a^2\cot^4(c+dx)}{b^2}+\frac{2a\cot^3(c+dx)}{b}+\frac{(a^2+b^2)\cot^2(c+dx)}{b^2}+\frac{2a\cot(c+dx)}{b}+1\right)d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{(a^2+b^2)\cot(c+dx)}{b}-\frac{a^2\cot^3(c+dx)}{3b}+2a\log(b\tan(c+dx))-a\cot^2(c+dx)+b\tan(c+dx)\right)}{d}$$

input
```
Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

output
```
(b*(-(((a^2 + b^2)*Cot[c + d*x])/b) - a*Cot[c + d*x]^2 - (a^2*Cot[c + d*x]
^3)/(3*b) + 2*a*Log[b*Tan[c + d*x]] + b*Tan[c + d*x]))/d
```

3.29.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.29.4 Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+2ab\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^2\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)}{d}$
default	$\frac{b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+2ab\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^2\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)}{d}$
risch	$\frac{4ab\,e^{6i(dx+c)}+4ia^2e^{4i(dx+c)}-4ib^2e^{4i(dx+c)}+\frac{8ia^2e^{2i(dx+c)}}{3}+8ib^2e^{2i(dx+c)}-4ab\,e^{2i(dx+c)}-\frac{4ia^2}{3}-4ib^2}{d\left(e^{2i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}+1\right)}+\frac{2ab\ln(e^{2i(dx}}{d}$

input
```
int(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+2*a*b*(-1/2/sin(d*x+c)^2+l
n(tan(d*x+c)))+a^2*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c))
```

3.29.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(77) = 154$.

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.20

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{2\left(a^2+3\,b^2\right)\cos\left(dx+c\right)^4 - 3\,ab\cos\left(dx+c\right)\sin\left(dx+c\right) - 3\left(a^2+3\,b^2\right)\cos\left(dx+c\right)^2 + 3\left(ab\cos\left(dx+\right.\right.}{3\left(d\,c\right.}$$

input
```
integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/3*(2*(a^2 + 3*b^2)*cos(d*x + c)^4 - 3*a*b*cos(d*x + c)*sin(d*x + c) - 3
*(a^2 + 3*b^2)*cos(d*x + c)^2 + 3*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*
log(cos(d*x + c)^2)*sin(d*x + c) - 3*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c
))*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + 3*b^2)/((d*cos(d*x + c)^3
 - d*cos(d*x + c))*sin(d*x + c))
```

3.29.6 Sympy [F]

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^2\csc^4\left(c+dx\right)\,dx$$

input
```
integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**4, x)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.87

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{6\,ab\log\left(\tan\left(dx+c\right)\right) + 3\,b^2\tan\left(dx+c\right) - \frac{3\,ab\tan(dx+c)+3\left(a^2+b^2\right)\tan(dx+c)^2+a^2}{\tan(dx+c)^3}}{3\,d}$$

input
```
integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/3*(6*a*b*log(tan(d*x + c)) + 3*b^2*tan(d*x + c) - (3*a*b*tan(d*x + c) +
3*(a^2 + b^2)*tan(d*x + c)^2 + a^2)/tan(d*x + c)^3)/d
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{6\,ab\log(|\tan(dx+c)|)+3\,b^2\tan(dx+c)-\frac{11\,ab\tan(dx+c)^3+3\,a^2\tan(dx+c)^2+3\,b^2\tan(dx+c)^2+3\,ab\tan(dx+c)+a^2}{\tan(dx+c)^3}}{3\,d}$$

input
```
integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3*(6*a*b*log(abs(tan(d*x + c))) + 3*b^2*tan(d*x + c) - (11*a*b*tan(d*x +
 c)^3 + 3*a^2*tan(d*x + c)^2 + 3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) +
 a^2)/tan(d*x + c)^3)/d
```

3.29.9 Mupad [B] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b^2\tan(c+d\,x)}{d} - \frac{\tan(c+d\,x)^2\,(a^2+b^2)+\frac{a^2}{3}+a\,b\tan(c+d\,x)}{d\tan(c+d\,x)^3} + \frac{2\,a\,b\,\ln(\tan(c+d\,x))}{d}$$

input
```
int((a + b*tan(c + d*x))^2/sin(c + d*x)^4,x)
```

output
```
(b^2*tan(c + d*x))/d - (tan(c + d*x)^2*(a^2 + b^2) + a^2/3 + a*b*tan(c + d
*x))/(d*tan(c + d*x)^3) + (2*a*b*log(tan(c + d*x)))/d
```

3.30 $\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx$

3.30.1 Optimal result

Integrand size = 21, antiderivative size = 165

$$\begin{aligned}\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx = &-\frac{3a^2\text{arctanh}(\cos(c+dx))}{8d} \\ &-\frac{3b^2\text{arctanh}(\cos(c+dx))}{2d} \\ &+\frac{2ab\text{arctanh}(\sin(c+dx))}{d}-\frac{2ab\csc(c+dx)}{d} \\ &-\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d} \\ &-\frac{2ab\csc^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d} \\ &+\frac{3b^2\sec(c+dx)}{2d}-\frac{b^2\csc^2(c+dx)\sec(c+dx)}{2d}\end{aligned}$$

output
```
-3/8*a^2*arctanh(cos(d*x+c))/d-3/2*b^2*arctanh(cos(d*x+c))/d+2*a*b*arctanh
(sin(d*x+c))/d-2*a*b*csc(d*x+c)/d-3/8*a^2*cot(d*x+c)*csc(d*x+c)/d-2/3*a*b*
csc(d*x+c)^3/d-1/4*a^2*cot(d*x+c)*csc(d*x+c)^3/d+3/2*b^2*sec(d*x+c)/d-1/2*
b^2*csc(d*x+c)^2*sec(d*x+c)/d
```

3.30.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 994 vs. $2(165) = 330$.

Time = 7.25 (sec) , antiderivative size = 994, normalized size of antiderivative = 6.02

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{b^2\cos^2(c+dx)(a+b\tan(c+dx))^2}{d(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{7ab\cos^2(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{6d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{(-3a^2-4b^2)\cos^2(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{32d(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{ab\cos^2(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{12d(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{a^2\cos^2(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{64d(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{3(a^2+4b^2)\cos^2(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2}{8d(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{2ab\cos^2(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2}{d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{3(a^2+4b^2)\cos^2(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2}{8d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{2ab\cos^2(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2}{d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{(3a^2+4b^2)\cos^2(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{32d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{a^2\cos^2(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{64d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{b^2\cos^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{b^2\cos^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{7ab\cos^2(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{6d(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{ab\cos^2(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^2}{12d(a\cos(c+dx)+b\sin(c+dx))^2}$$

input
```
Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

output
```
(b^2*Cos[c + d*x]^2*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^2) - (7*a*b*Cos[c + d*x]^2*Cot[(c + d*x)/2]*(a + b*Tan[c + d*x])^2)
/(6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + ((-3*a^2 - 4*b^2)*Cos[c + d*x
]^2*Csc[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^2)/(32*d*(a*Cos[c + d*x] + b*S
in[c + d*x])^2) - (a*b*Cos[c + d*x]^2*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*
(a + b*Tan[c + d*x])^2)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (a^2*
Cos[c + d*x]^2*Csc[(c + d*x)/2]^4*(a + b*Tan[c + d*x])^2)/(64*d*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^2) - (3*(a^2 + 4*b^2)*Cos[c + d*x]^2*Log[Cos[(c +
d*x)/2]]*(a + b*Tan[c + d*x])^2)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
 - (2*a*b*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*T
an[c + d*x])^2)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (3*(a^2 + 4*b^2)
*Cos[c + d*x]^2*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^2)/(8*d*(a*Cos[
c + d*x] + b*Sin[c + d*x])^2) + (2*a*b*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^2)/(d*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^2) + ((3*a^2 + 4*b^2)*Cos[c + d*x]^2*Sec[(c + d*x)/2]^2*(a + b*Tan
[c + d*x])^2)/(32*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (a^2*Cos[c + d*
x]^2*Sec[(c + d*x)/2]^4*(a + b*Tan[c + d*x])^2)/(64*d*(a*Cos[c + d*x] + b*
Sin[c + d*x])^2) + (b^2*Cos[c + d*x]^2*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x
])^2)/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^2) - (b^2*Cos[c + d*x]^2*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^2...
```

3.30.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sin(c+dx)^5}dx$$

$$\Big\downarrow \text{ 4000}$$

$$\int \left(a^2\csc^5(c+dx)+2ab\csc^4(c+dx)\sec(c+dx)+b^2\csc^3(c+dx)\sec^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3a^2\text{arctanh}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d}+\frac{2ab\text{arctanh}(\sin(c+dx))}{d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}-\frac{3b^2\text{arctanh}(\cos(c+dx))}{2d}+\frac{3b^2\sec(c+dx)}{2d}-\frac{b^2\csc^2(c+dx)\sec(c+dx)}{2d}$$

input
```
Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-3*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (3*b^2*ArcTanh[Cos[c + d*x]])/(2*d)
 + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Csc[c + d*x])/d - (3*a^2*Cot[c
 + d*x]*Csc[c + d*x])/(8*d) - (2*a*b*Csc[c + d*x]^3)/(3*d) - (a^2*Cot[c +
d*x]*Csc[c + d*x]^3)/(4*d) + (3*b^2*Sec[c + d*x])/(2*d) - (b^2*Csc[c + d*x
]^2*Sec[c + d*x])/(2*d)
```

3.30.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.30.4 Maple [A] (verified)

Time = 6.33 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{b^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2ab\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan}{d}$
default	$\frac{b^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2ab\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan}{d}$
risch	$\frac{e^{i(dx+c)}\left(9a^2e^{8i(dx+c)}+36b^2e^{8i(dx+c)}-48iab\,e^{8i(dx+c)}-24a^2e^{6i(dx+c)}-96b^2e^{6i(dx+c)}+160iab\,e^{6i(dx+c)}-66a^2e^{4i(dx+c}}{12d\left(e^{2i(dx+c)}-1\right)^4\left(e^{2i(dx+c)}+1\right.}$

input
```
int(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c)-co
t(d*x+c)))+2*a*b*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))
)+a^2*((-1/4*csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(csc(d*x+c)-cot
(d*x+c))))
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(151) = 302.

Time = 0.30 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.02

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{18\left(a^2+4b^2\right)\cos\left(dx+c\right)^4-30\left(a^2+4b^2\right)\cos\left(dx+c\right)^2+48b^2-9\left(\left(a^2+4b^2\right)\cos\left(dx+c\right)^5-2\left(a^2+4\right.\right.}{}$$

input
```
integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/48*(18*(a^2 + 4*b^2)*cos(d*x + c)^4 - 30*(a^2 + 4*b^2)*cos(d*x + c)^2 +
48*b^2 - 9*((a^2 + 4*b^2)*cos(d*x + c)^5 - 2*(a^2 + 4*b^2)*cos(d*x + c)^3
+ (a^2 + 4*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 9*((a^2 + 4*b^
2)*cos(d*x + c)^5 - 2*(a^2 + 4*b^2)*cos(d*x + c)^3 + (a^2 + 4*b^2)*cos(d*x
 + c))*log(-1/2*cos(d*x + c) + 1/2) + 48*(a*b*cos(d*x + c)^5 - 2*a*b*cos(d
*x + c)^3 + a*b*cos(d*x + c))*log(sin(d*x + c) + 1) - 48*(a*b*cos(d*x + c)
^5 - 2*a*b*cos(d*x + c)^3 + a*b*cos(d*x + c))*log(-sin(d*x + c) + 1) + 32*
(3*a*b*cos(d*x + c)^3 - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^
5 - 2*d*cos(d*x + c)^3 + d*cos(d*x + c))
```

3.30.6 Sympy [F]

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\csc^5(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**5*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**5, x)
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{3a^2\left(\frac{2\left(3\cos(dx+c)^3-5\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1} - 3\log(\cos(dx+c)+1) + 3\log(\cos(dx+c)-1)\right) + 12b^2\left(\frac{2\left(3\cos(dx+c)\right.}{\cos(dx+c)^3-\cos}\right.}{}$$

input
```
integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/48*(3*a^2*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos
(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 12
*b^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(cos
(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 16*a*b*(2*(3*sin(d*x + c)^2 +
1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.63

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+384\,a\cdots}{\cdots}$$

input
```
integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 - 16*a*b*tan(1/2*d*x + 1/2*c)^3 + 24*a
^2*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 384*a*b*log(ab
s(tan(1/2*d*x + 1/2*c) + 1)) - 384*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1))
- 240*a*b*tan(1/2*d*x + 1/2*c) + 72*(a^2 + 4*b^2)*log(abs(tan(1/2*d*x + 1/
2*c))) - 384*b^2/(tan(1/2*d*x + 1/2*c)^2 - 1) - (150*a^2*tan(1/2*d*x + 1/2
*c)^4 + 600*b^2*tan(1/2*d*x + 1/2*c)^4 + 240*a*b*tan(1/2*d*x + 1/2*c)^3 +
24*a^2*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 16*a*b*tan
(1/2*d*x + 1/2*c) + 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.30.9 Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.29

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{3\,a^2}{8}+\frac{3\,b^2}{2}\right)}{d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64\,d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{7\,a^2}{4}+2\,b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(2\,a^2+34\,b^2\right)+\frac{a^2}{4}+\frac{56\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}-20\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\cdots}{d\left(16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\right)}$$
$$+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a^2}{8}+\frac{b^2}{8}\right)}{d}-\frac{a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{12\,d}$$
$$+\frac{4\,a\,b\,\mathrm{atanh}\left(\frac{12\,a\,b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3\,a^3\,b-16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2\,b^2+12\,a\,b^3}-\frac{16\,a^2\,b^2}{3\,a^3\,b-16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2\,b^2+12\,a\,b^3}+\frac{3\,a^3\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3\,a^3\,b-16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2\,b^2+12\,a\,b^3}\right)}{d}$$
$$-\frac{5\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,d}$$

input
```
int((a + b*tan(c + d*x))^2/sin(c + d*x)^5,x)
```

output
```
(log(tan(c/2 + (d*x)/2))*((3*a^2)/8 + (3*b^2)/2))/d + (a^2*tan(c/2 + (d*x)
/2)^4)/(64*d) - (tan(c/2 + (d*x)/2)^2*((7*a^2)/4 + 2*b^2) - tan(c/2 + (d*x
)/2)^4*(2*a^2 + 34*b^2) + a^2/4 + (56*a*b*tan(c/2 + (d*x)/2)^3)/3 - 20*a*b
*tan(c/2 + (d*x)/2)^5 + (4*a*b*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*
x)/2)^4 - 16*tan(c/2 + (d*x)/2)^6)) + (tan(c/2 + (d*x)/2)^2*(a^2/8 + b^2/8
))/d - (a*b*tan(c/2 + (d*x)/2)^3)/(12*d) + (4*a*b*atanh((12*a*b^3*tan(c/2
+ (d*x)/2))/(12*a*b^3 + 3*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2)) - (16*a^2
*b^2)/(12*a*b^3 + 3*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2)) + (3*a^3*b*tan(
c/2 + (d*x)/2))/(12*a*b^3 + 3*a^3*b - 16*a^2*b^2*tan(c/2 + (d*x)/2))))/d -
 (5*a*b*tan(c/2 + (d*x)/2))/(4*d)
```

3.30. $\int \csc^5(c+dx)(a+b\tan(c+dx))^2\,dx$

3.31 $\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx$

3.31.1 Optimal result

Integrand size = 21, antiderivative size = 122

$$\begin{aligned}\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx = &-\frac{(a^2+2b^2)\cot(c+dx)}{d}-\frac{2ab\cot^2(c+dx)}{d}\\&-\frac{(2a^2+b^2)\cot^3(c+dx)}{3d}\\&-\frac{ab\cot^4(c+dx)}{2d}-\frac{a^2\cot^5(c+dx)}{5d}\\&+\frac{2ab\log(\tan(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}\end{aligned}$$

output
```
-(a^2+2*b^2)*cot(d*x+c)/d-2*a*b*cot(d*x+c)^2/d-1/3*(2*a^2+b^2)*cot(d*x+c)^
3/d-1/2*a*b*cot(d*x+c)^4/d-1/5*a^2*cot(d*x+c)^5/d+2*a*b*ln(tan(d*x+c))/d+b
^2*tan(d*x+c)/d
```

3.31.2 Mathematica [A] (verified)

Time = 2.82 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{2\cot(c+dx)\left(8a^2+25b^2+(4a^2+5b^2)\csc^2(c+dx)+3a^2\csc^4(c+dx)\right)+15b(2a\csc^2(c+dx)+a\csc^4}{30d}$$

input
```
Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

output
```
-1/30*(2*Cot[c + d*x]*(8*a^2 + 25*b^2 + (4*a^2 + 5*b^2)*Csc[c + d*x]^2 + 3
*a^2*Csc[c + d*x]^4) + 15*b*(2*a*Csc[c + d*x]^2 + a*Csc[c + d*x]^4 + 4*a*L
og[Cos[c + d*x]] - 4*a*Log[Sin[c + d*x]] - 2*b*Tan[c + d*x]))/d
```

3.31.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3999, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sin(c+dx)^6}dx$$

$$\Big\downarrow \text{3999}$$

$$\frac{b\int \frac{\cot^6(c+dx)(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)^2}{b^6}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{b\int \left(\frac{a^2\cot^6(c+dx)}{b^2}+\frac{2a\cot^5(c+dx)}{b}+\frac{(b^4+2a^2b^2)\cot^4(c+dx)}{b^4}+\frac{4a\cot^3(c+dx)}{b}+\frac{(a^2+2b^2)\cot^2(c+dx)}{b^2}+\frac{2a\cot(c+dx)}{b}+1\right)d(b\tan$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\Big(-\frac{(2a^2+b^2)\cot^3(c+dx)}{3b}-\frac{(a^2+2b^2)\cot(c+dx)}{b}-\frac{a^2\cot^5(c+dx)}{5b}+2a\log(b\tan(c+dx))-\frac{1}{2}a\cot^4(c+dx)-2a\cot^2(c+d}{d}$$

input
```
Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

output
```
(b*(-(((a^2 + 2*b^2)*Cot[c + d*x])/b) - 2*a*Cot[c + d*x]^2 - ((2*a^2 + b^2
)*Cot[c + d*x]^3)/(3*b) - (a*Cot[c + d*x]^4)/2 - (a^2*Cot[c + d*x]^5)/(5*b
) + 2*a*Log[b*Tan[c + d*x]] + b*Tan[c + d*x]))/d
```

3.31.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.31.4 Maple [A] (verified)

Time = 12.76 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{b^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+2ab\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^2}{d}$
default	$\frac{b^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+2ab\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^2}{d}$
risch	$\frac{4ab\,e^{10i(dx+c)}-16ab\,e^{8i(dx+c)}-\frac{32ia^2e^{6i(dx+c)}}{3}+\frac{32ib^2e^{6i(dx+c)}}{3}-\frac{16ia^2e^{4i(dx+c)}}{3}-\frac{80ib^2e^{4i(dx+c)}}{3}+16ab\,e^{4i(dx+c)}+\frac{64ia^2}{}}{d\left(e^{2i(dx+c)}-1\right)^5\left(e^{2i(dx+c)}+1\right)}$

input
```
int(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(-1/3/sin(d*x+c)^3/cos(d*x+c)+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d
*x+c))+2*a*b*(-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a^2*(-8/1
5-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c))
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(116) = 232$.

Time = 0.27 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.97

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx =$$
$$-\frac{16\,(a^2+5\,b^2)\cos{(dx+c)}^6-40\,(a^2+5\,b^2)\cos{(dx+c)}^4+30\,(a^2+5\,b^2)\cos{(dx+c)}^2+30\,(ab\cos{(dx}}{}$$

input
```
integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/30*(16*(a^2 + 5*b^2)*cos(d*x + c)^6 - 40*(a^2 + 5*b^2)*cos(d*x + c)^4 +
 30*(a^2 + 5*b^2)*cos(d*x + c)^2 + 30*(a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x
+ c)^3 + a*b*cos(d*x + c))*log(cos(d*x + c)^2)*sin(d*x + c) - 30*(a*b*cos(
d*x + c)^5 - 2*a*b*cos(d*x + c)^3 + a*b*cos(d*x + c))*log(-1/4*cos(d*x + c
)^2 + 1/4)*sin(d*x + c) - 30*b^2 - 15*(2*a*b*cos(d*x + c)^3 - 3*a*b*cos(d*
x + c))*sin(d*x + c))/((d*cos(d*x + c)^5 - 2*d*cos(d*x + c)^3 + d*cos(d*x
+ c))*sin(d*x + c))
```

3.31.6 Sympy [F]

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan{(c+dx)})^2\csc^6{(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**6*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*csc(c + d*x)**6, x)
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{60\,ab\log\left(\tan\left(dx+c\right)\right)+30\,b^2\tan\left(dx+c\right)-\frac{60\,ab\tan(dx+c)^3+30\,(a^2+2\,b^2)\tan(dx+c)^4+15\,ab\tan(dx+c)+10\,(2\,a^2+b^2)\tan(}{\tan(dx+c)^5}}{30\,d}$$

input
```
integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/30*(60*a*b*log(tan(d*x + c)) + 30*b^2*tan(d*x + c) - (60*a*b*tan(d*x + c
)^3 + 30*(a^2 + 2*b^2)*tan(d*x + c)^4 + 15*a*b*tan(d*x + c) + 10*(2*a^2 +
b^2)*tan(d*x + c)^2 + 6*a^2)/tan(d*x + c)^5)/d
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{60\,ab\log\left(\left|\tan\left(dx+c\right)\right|\right)+30\,b^2\tan\left(dx+c\right)-\frac{137\,ab\tan(dx+c)^5+30\,a^2\tan(dx+c)^4+60\,b^2\tan(dx+c)^4+60\,ab\tan(dx+c)^3+20}{\tan(dx+c)^5}}{30\,d}$$

input
```
integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/30*(60*a*b*log(abs(tan(d*x + c))) + 30*b^2*tan(d*x + c) - (137*a*b*tan(d
*x + c)^5 + 30*a^2*tan(d*x + c)^4 + 60*b^2*tan(d*x + c)^4 + 60*a*b*tan(d*x
 + c)^3 + 20*a^2*tan(d*x + c)^2 + 10*b^2*tan(d*x + c)^2 + 15*a*b*tan(d*x +
 c) + 6*a^2)/tan(d*x + c)^5)/d
```

3.31.9 Mupad [B] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b^2\tan(c+d\,x)}{d} - \frac{\tan(c+d\,x)^4\,(a^2+2\,b^2)+\frac{a^2}{5}+\tan(c+d\,x)^2\,\left(\frac{2\,a^2}{3}+\frac{b^2}{3}\right)+\frac{a\,b\tan(c+d\,x)}{2}+2\,a\,b\tan(c+d\,x)^3}{d\tan\left(c+d\,x\right)^5} + \frac{2\,a\,b\,\ln\left(\tan(c+d\,x)\right)}{d}$$

input
```
int((a + b*tan(c + d*x))^2/sin(c + d*x)^6,x)
```

output
```
(b^2*tan(c + d*x))/d - (tan(c + d*x)^4*(a^2 + 2*b^2) + a^2/5 + tan(c + d*x
)^2*((2*a^2)/3 + b^2/3) + (a*b*tan(c + d*x))/2 + 2*a*b*tan(c + d*x)^3)/(d*
tan(c + d*x)^5) + (2*a*b*log(tan(c + d*x)))/d
```

3.32 $\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx$

3.32.1 Optimal result

Integrand size = 21, antiderivative size = 205

$$\begin{aligned}\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{3a^2b\,\text{arctanh}(\sin(c+dx))}{d} \\ &\quad -\frac{5b^3\text{arctanh}(\sin(c+dx))}{2d}-\frac{a^3\cos(c+dx)}{d} \\ &\quad +\frac{6ab^2\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d} \\ &\quad -\frac{ab^2\cos^3(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d} \\ &\quad -\frac{3a^2b\sin(c+dx)}{d}+\frac{5b^3\sin(c+dx)}{2d} \\ &\quad -\frac{a^2b\sin^3(c+dx)}{d}+\frac{5b^3\sin^3(c+dx)}{6d} \\ &\quad +\frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d}\end{aligned}$$

output
```
3*a^2*b*arctanh(sin(d*x+c))/d-5/2*b^3*arctanh(sin(d*x+c))/d-a^3*cos(d*x+c)
/d+6*a*b^2*cos(d*x+c)/d+1/3*a^3*cos(d*x+c)^3/d-a*b^2*cos(d*x+c)^3/d+3*a*b^
2*sec(d*x+c)/d-3*a^2*b*sin(d*x+c)/d+5/2*b^3*sin(d*x+c)/d-a^2*b*sin(d*x+c)^
3/d+5/6*b^3*sin(d*x+c)^3/d+1/2*b^3*sin(d*x+c)^3*tan(d*x+c)^2/d
```

3.32.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 771 vs. 2(205) = 410.

Time = 7.29 (sec) , antiderivative size = 771, normalized size of antiderivative = 3.76

$$
\begin{aligned}
&\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx \\
&= \frac{3ab^2\cos^3(c+dx)(a+b\tan(c+dx))^3}{d(a\cos(c+dx)+b\sin(c+dx))^3} - \frac{3a(a^2-7b^2)\cos^4(c+dx)(a+b\tan(c+dx))^3}{4d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad + \frac{a(a^2-3b^2)\cos^3(c+dx)\cos(3(c+dx))(a+b\tan(c+dx))^3}{12d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad + \frac{(-6a^2b+5b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad + \frac{(6a^2b-5b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad + \frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad + \frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad - \frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad - \frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad - \frac{3b(5a^2-3b^2)\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{4d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad + \frac{b(3a^2-b^2)\cos^3(c+dx)\sin(3(c+dx))(a+b\tan(c+dx))^3}{12d(a\cos(c+dx)+b\sin(c+dx))^3}
\end{aligned}
$$

input
```
Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

output
```
(3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^3) - (3*a*(a^2 - 7*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3)/
(4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (a*(a^2 - 3*b^2)*Cos[c + d*x]^
3*Cos[3*(c + d*x)]*(a + b*Tan[c + d*x])^3)/(12*d*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^3) + ((-6*a^2*b + 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d
*x])^3) + ((6*a^2*b - 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^
3) + (b^3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*a*b^2*Cos[c
+ d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b^3*Cos[c + d*x]^
3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*
Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a*b^2*Cos[c + d*x]^3*Sin[(c + d*x)/
2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos
[c + d*x] + b*Sin[c + d*x])^3) - (3*b*(5*a^2 - 3*b^2)*Cos[c + d*x]^3*Sin[c
 + d*x]*(a + b*Tan[c + d*x])^3)/(4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
+ (b*(3*a^2 - b^2)*Cos[c + d*x]^3*Sin[3*(c + d*x)]*(a + b*Tan[c + d*x])^3)
/(12*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

3.32.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(c+dx)^3(a+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{ 4000}$$

$$\int \left(a^3\sin^3(c+dx)+3a^2b\sin^3(c+dx)\tan(c+dx)+3ab^2\sin^3(c+dx)\tan^2(c+dx)+b^3\sin^3(c+dx)\tan^3(c+dx\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{a^3\cos(c+dx)}{d}+\frac{3a^2b\text{arctanh}(\sin(c+dx))}{d}-\frac{a^2b\sin^3(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}-\frac{ab^2\cos^3(c+dx)}{d}+\frac{6ab^2\cos(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}-\frac{5b^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{5b^3\sin^3(c+dx)}{6d}+\frac{5b^3\sin(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d}$$

input
```
Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

output
```
(3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (5*b^3*ArcTanh[Sin[c + d*x]])/(2*d) -
(a^3*Cos[c + d*x])/d + (6*a*b^2*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*
d) - (a*b^2*Cos[c + d*x]^3)/d + (3*a*b^2*Sec[c + d*x])/d - (3*a^2*b*Sin[c
+ d*x])/d + (5*b^3*Sin[c + d*x])/(2*d) - (a^2*b*Sin[c + d*x]^3)/d + (5*b^3
*Sin[c + d*x]^3)/(6*d) + (b^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/(2*d)
```

3.32.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.32.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{a^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+3a^2b\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+3a\,b^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}\right.\right.}{}$
default	$\frac{-\frac{a^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+3a^2b\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+3a\,b^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}\right.\right.}{}$
risch	$\frac{i\mathrm{e}^{-3i(dx+c)}b\,a^2}{8d}+\frac{15i\mathrm{e}^{i(dx+c)}b\,a^2}{8d}+\frac{\mathrm{e}^{3i(dx+c)}a^3}{24d}-\frac{\mathrm{e}^{3i(dx+c)}a\,b^2}{8d}-\frac{ib^2\mathrm{e}^{i(dx+c)}\left(6ia\,\mathrm{e}^{2i(dx+c)}+b\,\mathrm{e}^{2i(dx+c)}+6ia-\right.}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}$

input
```
int(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a^3*(2+sin(d*x+c)^2)*cos(d*x+c)+3*a^2*b*(-1/3*sin(d*x+c)^3-sin(d
*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3*a*b^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin
(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+b^3*(1/2*sin(d*x+c)^7/cos(d*x+c)^2
+1/2*sin(d*x+c)^5+5/6*sin(d*x+c)^3+5/2*sin(d*x+c)-5/2*ln(sec(d*x+c)+tan(d*
x+c))))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.92

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{4\,(a^3-3\,ab^2)\cos{(dx+c)}^5+36\,ab^2\cos{(dx+c)}-12\,(a^3-6\,ab^2)\cos{(dx+c)}^3+3\,(6\,a^2b-5\,b^3)\cos{(dx+}}{}$$

input
```
integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(4*(a^3 - 3*a*b^2)*cos(d*x + c)^5 + 36*a*b^2*cos(d*x + c) - 12*(a^3 -
 6*a*b^2)*cos(d*x + c)^3 + 3*(6*a^2*b - 5*b^3)*cos(d*x + c)^2*log(sin(d*x
+ c) + 1) - 3*(6*a^2*b - 5*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*
(2*(3*a^2*b - b^3)*cos(d*x + c)^4 + 3*b^3 - 2*(12*a^2*b - 7*b^3)*cos(d*x +
 c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.32.6 Sympy [F]

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\sin^3{(c+dx)}\,dx$$

input
```
integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*sin(c + d*x)**3, x)
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.84

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{4\left(\cos\left(dx+c\right)^3 - 3\cos\left(dx+c\right)\right)a^3 - 6\left(2\sin\left(dx+c\right)^3 - 3\log\left(\sin\left(dx+c\right)+1\right) + 3\log\left(\sin\left(dx+c\right) - \right.\right.}{}$$

input
```
integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/12*(4*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^3 - 6*(2*sin(d*x + c)^3 - 3*lo
g(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^2*b - 12
*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a*b^2 + (4*sin(d*x + c
)^3 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*
log(sin(d*x + c) - 1) + 24*sin(d*x + c))*b^3)/d
```

3.32.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66584 vs. 2(195) = 390.

Time = 147.41 (sec) , antiderivative size = 66584, normalized size of antiderivative = 324.80

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/192*(45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b^2*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)
^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
 - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*
tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2
*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 - 360*pi*a*b^2*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b^2*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan
(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*ta
n(1/2*d*x)^10*tan(1/2*c)^8 + 45*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c
) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - ...
```

3.32.9 Mupad [B] (verification not implemented)

Time = 7.57 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.42

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(6\,a^2\,b-5\,b^3\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(6\,a^2\,b-5\,b^3\right)+4\,a^3\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-16\,a\,b^2-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,\left(16\,a\,b^2-\frac{4\,a^3}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\mathrm{t}\right.}$$

input
```
int(sin(c + d*x)^3*(a + b*tan(c + d*x))^3,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(6*a^2*b - 5*b^3))/d - (tan(c/2 + (d*x)/2)*(6*a
^2*b - 5*b^3) + 4*a^3*tan(c/2 + (d*x)/2)^6 - 16*a*b^2 - tan(c/2 + (d*x)/2)
^2*(16*a*b^2 - (4*a^3)/3) + tan(c/2 + (d*x)/2)^4*(32*a*b^2 - (20*a^3)/3) +
 tan(c/2 + (d*x)/2)^9*(6*a^2*b - 5*b^3) + tan(c/2 + (d*x)/2)^3*(8*a^2*b -
(20*b^3)/3) + tan(c/2 + (d*x)/2)^7*(8*a^2*b - (20*b^3)/3) - tan(c/2 + (d*x
)/2)^5*(28*a^2*b - (22*b^3)/3) + (4*a^3)/3)/(d*(tan(c/2 + (d*x)/2)^2 - 2*t
an(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + tan(
c/2 + (d*x)/2)^10 + 1))
```

3.32. $\int \sin^3(c+dx)(a+b\tan(c+dx))^3\,dx$

3.33 $\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx$

3.33.1 Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{1}{2}a\left(a^2-9b^2\right)x-\frac{b(3a^2-2b^2)\log(\cos(c+dx))}{d}+\frac{9ab^2\tan(c+dx)}{2d}+\frac{b^3\tan^2(c+dx)}{d}-\frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{2d}$$

output
```
1/2*a*(a^2-9*b^2)*x-b*(3*a^2-2*b^2)*ln(cos(d*x+c))/d+9/2*a*b^2*tan(d*x+c)/
d+b^3*tan(d*x+c)^2/d-1/2*cos(d*x+c)*sin(d*x+c)*(a+b*tan(d*x+c))^3/d
```

3.33.2 Mathematica [A] (verified)

Time = 4.79 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.97

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b\left(-\frac{a(a^2-3b^2)\arctan(\tan(c+dx))}{b}+(3a^2-b^2)\cos^2(c+dx)+\left(3a^2-2b^2+\frac{a^3-6ab^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)\right.}{}$$

input
```
Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*(-((a*(a^2 - 3*b^2)*ArcTan[Tan[c + d*x]])/b) + (3*a^2 - b^2)*Cos[c + d*
x]^2 + (3*a^2 - 2*b^2 + (a^3 - 6*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan
[c + d*x]] + (3*a^2 - 2*b^2 + (-a^3 + 6*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2]
+ b*Tan[c + d*x]] - (a*(a^2 - 3*b^2)*Sin[2*(c + d*x)])/(2*b) + 6*a*b*Tan[c
 + d*x] + b^2*Tan[c + d*x]^2))/(2*d)
```

3.33.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3999, 531, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(c+dx)^2(a+b\tan(c+dx))^3dx$$

$$\Big\downarrow \; 3999$$

$$\frac{b\int \frac{b^2\tan^2(c+dx)(a+b\tan(c+dx))^3}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 531$$

$$\frac{b\left(-\frac{\int -\frac{b^2(a+b\tan(c+dx))^2(a+4b\tan(c+dx))}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2}-\frac{b\tan(c+dx)(a+b\tan(c+dx))^3}{2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{\int \frac{b^2(a+b\tan(c+dx))^2(a+4b\tan(c+dx))}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2}-\frac{b\tan(c+dx)(a+b\tan(c+dx))^3}{2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{1}{2}\int \frac{(a+b\tan(c+dx))^2(a+4b\tan(c+dx))}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))-\frac{b\tan(c+dx)(a+b\tan(c+dx))^3}{2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \; 657$$

$$\frac{b\left(\frac{1}{2}\int\left(9a+4b\tan(c+dx)+\frac{a^3-9b^2a+2b(3a^2-2b^2)\tan(c+dx)}{\tan^2(c+dx)b^2+b^2}\right)d(b\tan(c+dx))-\frac{b\tan(c+dx)(a+b\tan(c+dx))^3}{2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$\downarrow$ 2009

$$\frac{b\left(\frac{1}{2}\left(\frac{a(a^2-9b^2)\arctan(\tan(c+dx))}{b}+(3a^2-2b^2)\log\left(b^2\tan^2(c+dx)+b^2\right)+9ab\tan(c+dx)+2b^2\tan^2(c+dx)\right)-\frac{b\text{t}}{}\right.}{d}$$

input
```
Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*(-1/2*(b*Tan[c + d*x]*(a + b*Tan[c + d*x])^3)/(b^2 + b^2*Tan[c + d*x]^2
) + ((a*(a^2 - 9*b^2)*ArcTan[Tan[c + d*x]])/b + (3*a^2 - 2*b^2)*Log[b^2 +
b^2*Tan[c + d*x]^2] + 9*a*b*Tan[c + d*x] + 2*b^2*Tan[c + d*x]^2)/2))/d
```

3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 531
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomi
alRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
 + b*x^2, x], x, 1]}, Simp[(c + d*x)^n*(a*f - b*e*x)*((a + b*x^2)^(p + 1)/(
2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b
*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*(c + d*x)*Qx - a*d*f*n + b*c*e*(2*p
 + 3) + b*d*e*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGt
Q[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && GtQ[n, 1] && IntegerQ[2*p]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.33.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.58

method	result
derivativedivides	$\frac{a^3\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a\,b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+}{2}\right.\right.}{d}$
default	$\frac{a^3\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a\,b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+}{2}\right.\right.}{d}$
risch	$\frac{3i\mathrm{e}^{-2i(dx+c)}a\,b^2}{8d}-\frac{4ib^3c}{d}+\frac{a^3x}{2}-\frac{9xa\,b^2}{2}+\frac{3\,\mathrm{e}^{2i(dx+c)}b\,a^2}{8d}-\frac{\mathrm{e}^{2i(dx+c)}b^3}{8d}+3ixb\,a^2-\frac{i\mathrm{e}^{-2i(dx+c)}a^3}{8d}+$

input
```
int(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*(-1/2*sin(d*x+
c)^2-ln(cos(d*x+c)))+3*a*b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*si
n(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+b^3*(1/2*sin(d*x+c)^6/cos(d*x+c)^2+1/2
*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(cos(d*x+c))))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.45

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{2\,(3\,a^2b-b^3)\cos{(dx+c)}^4-4\,(3\,a^2b-2\,b^3)\cos{(dx+c)}^2\log{(-\cos{(dx+c)})}+2\,b^3-(3\,a^2b-b^3-2\,(a^3}{4\,d\cos{(dx+c)}^2}$$

input
```
integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(2*(3*a^2*b - b^3)*cos(d*x + c)^4 - 4*(3*a^2*b - 2*b^3)*cos(d*x + c)^2
*log(-cos(d*x + c)) + 2*b^3 - (3*a^2*b - b^3 - 2*(a^3 - 9*a*b^2)*d*x)*cos(
d*x + c)^2 + 2*(6*a*b^2*cos(d*x + c) - (a^3 - 3*a*b^2)*cos(d*x + c)^3)*sin
(d*x + c))/(d*cos(d*x + c)^2)
```

3.33.6 Sympy [F]

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\sin^2{(c+dx)}\,dx$$

input
```
integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*sin(c + d*x)**2, x)
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{b^3\tan{(dx+c)}^2 + 6\,ab^2\tan{(dx+c)} + (a^3-9\,ab^2)(dx+c) + (3\,a^2b-2\,b^3)\log{\left(\tan{(dx+c)}^2+1\right)} + \frac{3\,a^2b\cdots}{\cdots}}{2\,d}$$

input
```
integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(b^3*tan(d*x + c)^2 + 6*a*b^2*tan(d*x + c) + (a^3 - 9*a*b^2)*(d*x + c)
 + (3*a^2*b - 2*b^3)*log(tan(d*x + c)^2 + 1) + (3*a^2*b - b^3 - (a^3 - 3*a
*b^2)*tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

3.33.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2370 vs. $2(97) = 194$.

Time = 1.18 (sec) , antiderivative size = 2370, normalized size of antiderivative = 23.01

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/4*(2*a^3*d*x*tan(d*x)^4*tan(c)^4 - 18*a*b^2*d*x*tan(d*x)^4*tan(c)^4 - 6*
a^2*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 4*b^3*log(4*(tan(
d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2
 + tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 2*a^3*d*x*tan(d*x)^4*tan(c)^2 - 18
*a*b^2*d*x*tan(d*x)^4*tan(c)^2 - 4*a^3*d*x*tan(d*x)^3*tan(c)^3 + 36*a*b^2*
d*x*tan(d*x)^3*tan(c)^3 + 2*a^3*d*x*tan(d*x)^2*tan(c)^4 - 18*a*b^2*d*x*tan
(d*x)^2*tan(c)^4 + 3*a^2*b*tan(d*x)^4*tan(c)^4 + b^3*tan(d*x)^4*tan(c)^4 -
 6*a^2*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*t
an(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^4*tan(c)^2 + 4*b^3*log(4*(t
an(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x
)^2 + tan(c)^2 + 1))*tan(d*x)^4*tan(c)^2 + 12*a^2*b*log(4*(tan(d*x)^2*tan(
c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2
 + 1))*tan(d*x)^3*tan(c)^3 - 8*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)
*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3
*tan(c)^3 + 2*a^3*tan(d*x)^4*tan(c)^3 - 18*a*b^2*tan(d*x)^4*tan(c)^3 - 6*a
^2*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c
)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^4 + 4*b^3*log(4*(tan(d
*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2
+ tan(c)^2 + 1))*tan(d*x)^2*tan(c)^4 + 2*a^3*tan(d*x)^3*tan(c)^4 - 18*a...
```

3.33.9 Mupad [B] (verification not implemented)

Time = 4.68 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.47

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{b^3\tan(c+d\,x)^2}{2\,d}+\frac{\cos\left(c+d\,x\right)^2\,\left(\frac{3\,a^2\,b}{2}-\frac{b^3}{2}+\tan(c+d\,x)\,\left(\frac{3\,a\,b^2}{2}-\frac{a^3}{2}\right)\right)}{d}$$
$$+\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{3\,a^2\,b}{2}-b^3\right)}{d}+\frac{3\,a\,b^2\tan(c+d\,x)}{d}$$
$$-\frac{a\,\mathrm{atan}\left(\frac{a\tan(c+d\,x)\,(a-3\,b)\,(a+3\,b)}{2\,\left(\frac{9\,a\,b^2}{2}-\frac{a^3}{2}\right)}\right)\,(a-3\,b)\,(a+3\,b)}{2\,d}$$

input
```
int(sin(c + d*x)^2*(a + b*tan(c + d*x))^3,x)
```

output
```
(b^3*tan(c + d*x)^2)/(2*d) + (cos(c + d*x)^2*((3*a^2*b)/2 - b^3/2 + tan(c
+ d*x)*((3*a*b^2)/2 - a^3/2)))/d + (log(tan(c + d*x)^2 + 1)*((3*a^2*b)/2 -
 b^3))/d + (3*a*b^2*tan(c + d*x))/d - (a*atan((a*tan(c + d*x)*(a - 3*b)*(a
 + 3*b))/(2*((9*a*b^2)/2 - a^3/2)))*(a - 3*b)*(a + 3*b))/(2*d)
```

3.34 $\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx$

3.34.1 Optimal result

Integrand size = 19, antiderivative size = 133

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3a^2b\text{arctanh}(\sin(c+dx))}{d} - \frac{3b^3\text{arctanh}(\sin(c+dx))}{2d} - \frac{a^3\cos(c+dx)}{d} + \frac{3ab^2\cos(c+dx)}{d} + \frac{3ab^2\sec(c+dx)}{d} - \frac{3a^2b\sin(c+dx)}{d} + \frac{3b^3\sin(c+dx)}{2d} + \frac{b^3\sin(c+dx)\tan^2(c+dx)}{2d}$$

output
```
3*a^2*b*arctanh(sin(d*x+c))/d-3/2*b^3*arctanh(sin(d*x+c))/d-a^3*cos(d*x+c)
/d+3*a*b^2*cos(d*x+c)/d+3*a*b^2*sec(d*x+c)/d-3*a^2*b*sin(d*x+c)/d+3/2*b^3*
sin(d*x+c)/d+1/2*b^3*sin(d*x+c)*tan(d*x+c)^2/d
```

3.34.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 637 vs. 2(133) = 266.

Time = 6.79 (sec) , antiderivative size = 637, normalized size of antiderivative = 4.79

$$\begin{aligned}
&\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx \\
&= \frac{3ab^2\cos^3(c+dx)(a+b\tan(c+dx))^3}{d(a\cos(c+dx)+b\sin(c+dx))^3} - \frac{a(a^2-3b^2)\cos^4(c+dx)(a+b\tan(c+dx))^3}{d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad - \frac{3(2a^2b-b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad + \frac{3(2a^2b-b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad + \frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad + \frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad - \frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad - \frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3} \\
&\quad - \frac{b(3a^2-b^2)\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{d(a\cos(c+dx)+b\sin(c+dx))^3}
\end{aligned}$$

input
```
Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

output
```
(3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^3) - (a*(a^2 - 3*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3)/(d
*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*(2*a^2*b - b^3)*Cos[c + d*x]^3*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*C
os[c + d*x] + b*Sin[c + d*x])^3) + (3*(2*a^2*b - b^3)*Cos[c + d*x]^3*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^3) + (b^3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/
(4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d
*x])^3) + (3*a*b^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)
/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x]
)^3) - (b^3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*a*b^2*Cos[
c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (b*(3*a^2 - b^2)
*Cos[c + d*x]^3*Sin[c + d*x]*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] +
b*Sin[c + d*x])^3)
```

3.34. $\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx$

3.34.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)(a+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{4000}$$

$$\int \left(a^3\sin(c+dx)+3a^2b\sin(c+dx)\tan(c+dx)+3ab^2\sin(c+dx)\tan^2(c+dx)+b^3\sin(c+dx)\tan^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\cos(c+dx)}{d}+\frac{3a^2b\text{arctanh}(\sin(c+dx))}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{3ab^2\cos(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}-\frac{3b^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{3b^3\sin(c+dx)}{2d}+\frac{b^3\sin(c+dx)\tan^2(c+dx)}{2d}$$

input
```
Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

output
```
(3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d) -
(a^3*Cos[c + d*x])/d + (3*a*b^2*Cos[c + d*x])/d + (3*a*b^2*Sec[c + d*x])/d
 - (3*a^2*b*Sin[c + d*x])/d + (3*b^3*Sin[c + d*x])/(2*d) + (b^3*Sin[c + d*
x]*Tan[c + d*x]^2)/(2*d)
```

3.34.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.34.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{-a^3\cos(dx+c)+3a^2b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+3a\,b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+b^3\left(\right.}{d}$
default	$\frac{-a^3\cos(dx+c)+3a^2b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+3a\,b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+b^3\left(\right.}{d}$
risch	$\frac{3i\mathrm{e}^{i(dx+c)}b\,a^2}{2d}-\frac{i\mathrm{e}^{i(dx+c)}b^3}{2d}-\frac{\mathrm{e}^{i(dx+c)}a^3}{2d}+\frac{3\,\mathrm{e}^{i(dx+c)}a\,b^2}{2d}-\frac{3i\mathrm{e}^{-i(dx+c)}b\,a^2}{2d}+\frac{i\mathrm{e}^{-i(dx+c)}b^3}{2d}-\frac{\mathrm{e}^{-i(dx+c)}a^3}{2d}$

input
```
int(sin(d*x+c)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^3*cos(d*x+c)+3*a^2*b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3*a*b
^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+b^3*(1/2*sin(d*x+
c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(sec(d*x+c)+tan(d*
x+c))))
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.08

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{12\,ab^2\cos(dx+c)-4\,(a^3-3\,ab^2)\cos(dx+c)^3+3\,(2\,a^2b-b^3)\cos(dx+c)^2\log(\sin(dx+c)+1)-3\,(2}{4\,d\cos(dx+}$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(12*a*b^2*cos(d*x + c) - 4*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 3*(2*a^2*b
 - b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*(2*a^2*b - b^3)*cos(d*x +
 c)^2*log(-sin(d*x + c) + 1) + 2*(b^3 - 2*(3*a^2*b - b^3)*cos(d*x + c)^2)*
sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.34.6 Sympy [F]

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3 \sin(c+dx)\,dx$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*sin(c + d*x), x)
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+3\log(\sin(dx+c)+1)-3\log(\sin(dx+c)-1)-4\sin(dx+c)\right)-12ab^2\left(\frac{1}{\cos(dx+c)}\right.}{}$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/4*(b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 12*a*b^2*(1/cos(d*x + c) + co
s(d*x + c)) - 6*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*s
in(d*x + c)) + 4*a^3*cos(d*x + c))/d
```

3.34.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12476 vs. $2(127) = 254$.

Time = 8.35 (sec) , antiderivative size = 12476, normalized size of antiderivative = 93.80

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/4*(9*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 6*a^2
*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*b^3*log(2*(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan
(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d
*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*ta
n(1/2*c)^6 + 6*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2
*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 3*b^3*log(2*(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*
tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*
c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))
*tan(1/2*d*x)^6*tan(1/2*c)^6 - 9*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d
*x)^6*tan(1/2*c)^4 - 72*pi*a*b^2*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2
*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^5*...
```

3.34.9 Mupad [B] (verification not implemented)

Time = 6.43 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.45

$$\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(6\,a^2\,b-3\,b^3\right)+2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-12\,a\,b^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(12\,a\,b^2-4\,a^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{d\,\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-\right.}$$
$$+\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(6\,a^2\,b-3\,b^3\right)}{d}$$

input
```
int(sin(c + d*x)*(a + b*tan(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)*(6*a^2*b - 3*b^3) + 2*a^3*tan(c/2 + (d*x)/2)^4 - 12*a*
b^2 + tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 4*a^3) + tan(c/2 + (d*x)/2)^5*(6*a^
2*b - 3*b^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b - 2*b^3) + 2*a^3)/(d*(tan(c/
2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 - 1)) + (atan
h(tan(c/2 + (d*x)/2))*(6*a^2*b - 3*b^3))/d
```

3.34. $\int \sin(c+dx)(a+b\tan(c+dx))^3\,dx$

3.35 $\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx$

3.35.1 Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx=-\frac{a^3\text{arctanh}(\cos(c+dx))}{d}+\frac{3a^2b\text{arctanh}(\sin(c+dx))}{d}-\frac{b^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
-a^3*arctanh(cos(d*x+c))/d+3*a^2*b*arctanh(sin(d*x+c))/d-1/2*b^3*arctanh(s
in(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)*tan(d*x+c)/d
```

3.35.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 241 vs. 2(86) = 172.

Time = 3.87 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.80

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{12ab^2-4a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-12a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2b^3\log\left(\cos\left(\frac{1}{2}(c+d\right.\right.}{}$$

input
```
Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

output
```
(12*a*b^2 - 4*a^3*Log[Cos[(c + d*x)/2]] - 12*a^2*b*Log[Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]] + 2*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*a^3
*Log[Sin[(c + d*x)/2]] + 12*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
 - 2*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^3/(Cos[(c + d*x)/2]
- Sin[(c + d*x)/2])^2 + 24*a*b^2*Sec[c + d*x]*Sin[(c + d*x)/2]^2 - b^3/(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(4*d)
```

3.35.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sin(c+dx)}dx$$

$$\Big\downarrow \text{4000}$$

$$\int \left(a^3\csc(c+dx)+3a^2b\sec(c+dx)+3ab^2\tan(c+dx)\sec(c+dx)+b^3\tan^2(c+dx)\sec(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\text{arctanh}(\cos(c+dx))}{d}+\frac{3a^2b\text{arctanh}(\sin(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}-\frac{b^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{b^3\tan(c+dx)\sec(c+dx)}{2d}$$

input
```
Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

output
```
-((a^3*ArcTanh[Cos[c + d*x]])/d) + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (b^
3*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d
*x]*Tan[c + d*x])/(2*d)
```

3.35.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.35.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{b^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{3ab^2}{\cos(dx+c)}+3a^2b\ln(\sec(dx+c)+\tan(dx+c))+a^3\ln(\csc(dx+c)-}{d}$
default	$\frac{b^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{3ab^2}{\cos(dx+c)}+3a^2b\ln(\sec(dx+c)+\tan(dx+c))+a^3\ln(\csc(dx+c)-}{d}$
risch	$-\frac{ib^2 e^{i(dx+c)}\left(6ia\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}+6ia-b\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{3b\ln\left(e^{i(dx+c)}-i\right)a^2}{d}+\frac{b^3\ln\left(e^{i(dx+c)}-i\right)}{2d}+\frac{3b\ln\left(e^{i(dx+c)}+i\right)a}{d}$

input
```
int(csc(d*x+c)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+t
an(d*x+c)))+3*a*b^2/cos(d*x+c)+3*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+a^3*ln(cs
c(d*x+c)-cot(d*x+c)))
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.72

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{2\,a^3\cos\left(dx+c\right)^2\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-2\,a^3\cos\left(dx+c\right)^2\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-12\,ab^2\cos\left(d\right.}{}$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a^3*cos(d*x + c)^2*log(1/2*cos(d*x + c) + 1/2) - 2*a^3*cos(d*x + c
)^2*log(-1/2*cos(d*x + c) + 1/2) - 12*a*b^2*cos(d*x + c) - (6*a^2*b - b^3)
*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (6*a^2*b - b^3)*cos(d*x + c)^2*log
(-sin(d*x + c) + 1) - 2*b^3*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.35.6 Sympy [F]

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^3\csc\left(c+dx\right)\,dx$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*csc(c + d*x), x)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.29

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{b^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-6\,a^2b(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\right.}{4\,d}$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/4*(b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - l
og(sin(d*x + c) - 1)) - 6*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
- 1)) + 4*a^3*log(cot(d*x + c) + csc(d*x + c)) - 12*a*b^2/cos(d*x + c))/d
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.67

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\left(6\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(6\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.\right.}{2\,d}$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(2*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + (6*a^2*b - b^3)*log(abs(tan(1/
2*d*x + 1/2*c) + 1)) - (6*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
+ 2*(b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 + b^3*tan
(1/2*d*x + 1/2*c) + 6*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.35.9 Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.23

$$\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{2\left(\frac{a^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}-\frac{b^3\,\mathrm{atan}\left(\frac{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3-6\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}{2\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3-6\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b+1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}\right)\,1\mathrm{i}}{2}+a^2\,b\,\mathrm{atan}\left(\frac{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3-6\,\cos\left(\right.}{2\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3-6\mathrm{i}\,\sin\left(\right.}\right.\right.}{d}$$
$$+\frac{\frac{\sin(c+d\,x)\,b^3}{2}+3\,a\,\cos\left(c+d\,x\right)\,b^2}{d\,\left(\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{1}{2}\right)}$$

input
```
int((a + b*tan(c + d*x))^3/sin(c + d*x),x)
```

output
```
(2*((a^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - (b^3*atan((b^3*co
s(c/2 + (d*x)/2) + 2*a^3*sin(c/2 + (d*x)/2) - 6*a^2*b*cos(c/2 + (d*x)/2))/
(a^3*cos(c/2 + (d*x)/2)*2i + b^3*sin(c/2 + (d*x)/2)*1i - a^2*b*sin(c/2 + (
d*x)/2)*6i))*1i)/2 + a^2*b*atan((b^3*cos(c/2 + (d*x)/2) + 2*a^3*sin(c/2 +
(d*x)/2) - 6*a^2*b*cos(c/2 + (d*x)/2))/(a^3*cos(c/2 + (d*x)/2)*2i + b^3*si
n(c/2 + (d*x)/2)*1i - a^2*b*sin(c/2 + (d*x)/2)*6i))*3i))/d + ((b^3*sin(c +
 d*x))/2 + 3*a*b^2*cos(c + d*x))/(d*(cos(2*c + 2*d*x)/2 + 1/2))
```

3.35. $\int \csc(c+dx)(a+b\tan(c+dx))^3\,dx$

3.36 $\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx$

3.36.1 Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx=-\frac{a^3\cot(c+dx)}{d}+\frac{3a^2b\log(\tan(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\tan^2(c+dx)}{2d}$$

output
```
-a^3*cot(d*x+c)/d+3*a^2*b*ln(tan(d*x+c))/d+3*a*b^2*tan(d*x+c)/d+1/2*b^3*ta
n(d*x+c)^2/d
```

3.36.2 Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.97

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{\csc(c+dx)\sec^2(c+dx)\left(3a(a^2-b^2)\cos(c+dx)+(a^3+3ab^2)\cos(3(c+dx))-2b(b^2-3a^2\log(\cos(c\right.}{4d}$$

input
```
Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

output
```
-1/4*(Csc[c + d*x]*Sec[c + d*x]^2*(3*a*(a^2 - b^2)*Cos[c + d*x] + (a^3 + 3
*a*b^2)*Cos[3*(c + d*x)] - 2*b*(b^2 - 3*a^2*Log[Cos[c + d*x]] - 3*a^2*Cos[
2*(c + d*x)]*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]]) + 3*a^2*Log[Sin[c + d
*x]])*Sin[c + d*x]))/d
```

3.36.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3999, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3999}$$

$$\frac{b\int \frac{\cot^2(c+dx)(a+b\tan(c+dx))^3}{b^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{49}$$

$$\frac{b\int \left(\frac{\cot^2(c+dx)a^3}{b^2}+\frac{3\cot(c+dx)a^2}{b}+3a+b\tan(c+dx)\right)d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{a^3\cot(c+dx)}{b}+3a^2\log(b\tan(c+dx))+3ab\tan(c+dx)+\frac{1}{2}b^2\tan^2(c+dx)\right)}{d}$$

input
```
Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*(-((a^3*Cot[c + d*x])/b) + 3*a^2*Log[b*Tan[c + d*x]] + 3*a*b*Tan[c + d*
x] + (b^2*Tan[c + d*x]^2)/2))/d
```

3.36.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.36.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{b^{3}}{2 \cos \left(d x+c\right)^{2}}+3 a \,b^{2} \tan \left(d x+c\right)+3 a^{2} b \ln \left(\tan \left(d x+c\right)\right)-a^{3} \cot \left(d x+c\right)}{d}$
default	$\frac{\frac{b^{3}}{2 \cos \left(d x+c\right)^{2}}+3 a \,b^{2} \tan \left(d x+c\right)+3 a^{2} b \ln \left(\tan \left(d x+c\right)\right)-a^{3} \cot \left(d x+c\right)}{d}$
risch	$\frac{-2 i a^{3} {\mathrm e}^{4 i \left(d x+c\right)}+6 i a \,b^{2} {\mathrm e}^{4 i \left(d x+c\right)}+2 b^{3} {\mathrm e}^{4 i \left(d x+c\right)}-4 i a^{3} {\mathrm e}^{2 i \left(d x+c\right)}-2 b^{3} {\mathrm e}^{2 i \left(d x+c\right)}-2 i a^{3}-6 i a \,b^{2}}{d \left({\mathrm e}^{2 i \left(d x+c\right)}+1\right)^{2} \left({\mathrm e}^{2 i \left(d x+c\right)}-1\right)}+\frac{3 a^{2} b \ln \left({\mathrm e}^{2 i \left(d x+c\right)}-1\right)}{d}-$

input
```
int(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*b^3/cos(d*x+c)^2+3*a*b^2*tan(d*x+c)+3*a^2*b*ln(tan(d*x+c))-a^3*co
t(d*x+c))
```

3.36.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(62) = 124.

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.98

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{3\,a^2b\cos(dx+c)^2\log\left(\cos(dx+c)^2\right)\sin(dx+c) - 3\,a^2b\cos(dx+c)^2\log\left(-\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\right)\sin(d}{2\,d\cos(dx+c)^2\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2*(3*a^2*b*cos(d*x + c)^2*log(cos(d*x + c)^2)*sin(d*x + c) - 3*a^2*b*co
s(d*x + c)^2*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 6*a*b^2*cos(d*x
 + c) + 2*(a^3 + 3*a*b^2)*cos(d*x + c)^3 - b^3*sin(d*x + c))/(d*cos(d*x +
c)^2*sin(d*x + c))
```

3.36.6 Sympy [F]

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\csc^2(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**2, x)
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b^3\tan(dx+c)^2 + 6\,a^2b\log(\tan(dx+c)) + 6\,ab^2\tan(dx+c) - \frac{2\,a^3}{\tan(dx+c)}}{2\,d}$$

input
```
integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(b^3*tan(d*x + c)^2 + 6*a^2*b*log(tan(d*x + c)) + 6*a*b^2*tan(d*x + c)
 - 2*a^3/tan(d*x + c))/d
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b^3\tan(dx+c)^2+6\,a^2b\log(|\tan(dx+c)|)+6\,ab^2\tan(dx+c)-\frac{2\,(3\,a^2b\tan(dx+c)+a^3)}{\tan(dx+c)}}{2\,d}$$

input
```
integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(b^3*tan(d*x + c)^2 + 6*a^2*b*log(abs(tan(d*x + c))) + 6*a*b^2*tan(d*x
 + c) - 2*(3*a^2*b*tan(d*x + c) + a^3)/tan(d*x + c))/d
```

3.36.9 Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b^3\tan(c+d\,x)^2}{2\,d}-\frac{a^3\cot(c+d\,x)}{d}+\frac{3\,a^2\,b\,\ln(\tan(c+d\,x))}{d}+\frac{3\,a\,b^2\tan(c+d\,x)}{d}$$

input
```
int((a + b*tan(c + d*x))^3/sin(c + d*x)^2,x)
```

output
```
(b^3*tan(c + d*x)^2)/(2*d) - (a^3*cot(c + d*x))/d + (3*a^2*b*log(tan(c + d
*x)))/d + (3*a*b^2*tan(c + d*x))/d
```

3.37 $\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx$

3.37.1 Optimal result

Integrand size = 21, antiderivative size = 141

$$\begin{aligned}\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx = & -\frac{a^3\text{arctanh}(\cos(c+dx))}{2d} \\ & -\frac{3ab^2\text{arctanh}(\cos(c+dx))}{d} \\ & +\frac{3a^2b\text{arctanh}(\sin(c+dx))}{d} \\ & +\frac{b^3\text{arctanh}(\sin(c+dx))}{2d}-\frac{3a^2b\csc(c+dx)}{d} \\ & -\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}+\frac{3ab^2\sec(c+dx)}{d} \\ & +\frac{b^3\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

output
```
-1/2*a^3*arctanh(cos(d*x+c))/d-3*a*b^2*arctanh(cos(d*x+c))/d+3*a^2*b*arcta
nh(sin(d*x+c))/d+1/2*b^3*arctanh(sin(d*x+c))/d-3*a^2*b*csc(d*x+c)/d-1/2*a^
3*cot(d*x+c)*csc(d*x+c)/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)*tan(d*x+
c)/d
```

3.37.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 897 vs. 2(141) = 282.

Time = 7.38 (sec) , antiderivative size = 897, normalized size of antiderivative = 6.36

$$\begin{aligned}
&\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx \\
&= \frac{3ab^2\cos^3(c+dx)(a+b\tan(c+dx))^3}{d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&- \frac{3a^2b\cos^3(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&- \frac{a^3\cos^3(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{8d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&+ \frac{(-a^3-6ab^2)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&+ \frac{(-6a^2b-b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&+ \frac{(a^3+6ab^2)\cos^3(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&+ \frac{(6a^2b+b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&+ \frac{a^3\cos^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{8d(a\cos(c+dx)+b\sin(c+dx))^3} \\
&+ \frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3} \\
&+ \frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3} \\
&- \frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3} \\
&- \frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3} \\
&- \frac{3a^2b\cos^3(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3}
\end{aligned}$$

input
```
Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

output
```
(3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^3) - (3*a^2*b*Cos[c + d*x]^3*Cot[(c + d*x)/2]*(a + b*Tan[c + d*
x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^3*Cos[c + d*x]^3*Csc
[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^3)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d
*x])^3) + ((-a^3 - 6*a*b^2)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2]]*(a + b*Ta
n[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + ((-6*a^2*b - b^
3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c +
d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + ((a^3 + 6*a*b^2)*Cos[
c + d*x]^3*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d
*x] + b*Sin[c + d*x])^3) + ((6*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*x] + b
*Sin[c + d*x])^3) + (a^3*Cos[c + d*x]^3*Sec[(c + d*x)/2]^2*(a + b*Tan[c +
d*x])^3)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (b^3*Cos[c + d*x]^3*(
a + b*Tan[c + d*x])^3)/(4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos
[c + d*x] + b*Sin[c + d*x])^3) + (3*a*b^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*
(a + b*Tan[c + d*x])^3)/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^3) - (b^3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/
(4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d
*x])^3) - (3*a*b^2*Cos[c + d*x]^3*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)
/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d...
```

3.37.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sin(c+dx)^3}dx$$

$$\Big\downarrow \text{ 4000}$$

$$\int \left(a^3\csc^3(c+dx)+3a^2b\csc^2(c+dx)\sec(c+dx)+3ab^2\csc(c+dx)\sec^2(c+dx)+b^3\sec^3(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3\operatorname{arctanh}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}+\frac{3a^2b\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{3a^2b\csc(c+dx)}{d}-\frac{3ab^2\operatorname{arctanh}(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{b^3\tan(c+dx)\sec(c+dx)}{2d}$$

input
```
Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

output
```
-1/2*(a^3*ArcTanh[Cos[c + d*x]])/d - (3*a*b^2*ArcTanh[Cos[c + d*x]])/d + (
3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (3*
a^2*b*Csc[c + d*x])/d - (a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (3*a*b^2*S
ec[c + d*x])/d + (b^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

3.37.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.37.4 Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a\,b^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a^2b\left(-\frac{1}{\sin(dx+c)}+\ln\right.}{d}$
default	$\frac{b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a\,b^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a^2b\left(-\frac{1}{\sin(dx+c)}+\ln\right.}{d}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}\left(3ia^3\mathrm{e}^{2i(dx+c)}+3ia^3\mathrm{e}^{4i(dx+c)}+6a^2b\,\mathrm{e}^{6i(dx+c)}+b^3\mathrm{e}^{6i(dx+c)}+ia^3\mathrm{e}^{6i(dx+c)}+6ia\,b^2\mathrm{e}^{6i(dx+c)}+6a^2b\,\mathrm{e}^{4i(dx+c)}\right.}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2\left(\mathrm{e}^{2i(dx\right.}}$

input
```
int(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a*b^2
*(1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+3*a^2*b*(-1/sin(d*x+c)+ln(sec(d*
x+c)+tan(d*x+c)))+a^3*(-1/2*csc(d*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-cot(d*
x+c))))
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(133) = 266$.

Time = 0.32 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.12

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx =$$

$$-\frac{12\,ab^2\cos{(dx+c)} - 2\,(a^3+6\,ab^2)\cos{(dx+c)}^3 + \left((a^3+6\,ab^2)\cos{(dx+c)}^4 - (a^3+6\,ab^2)\cos{(dx+c}\right.}{}$$

input
```
integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/4*(12*a*b^2*cos(d*x + c) - 2*(a^3 + 6*a*b^2)*cos(d*x + c)^3 + ((a^3 + 6
*a*b^2)*cos(d*x + c)^4 - (a^3 + 6*a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x +
 c) + 1/2) - ((a^3 + 6*a*b^2)*cos(d*x + c)^4 - (a^3 + 6*a*b^2)*cos(d*x + c
)^2)*log(-1/2*cos(d*x + c) + 1/2) - ((6*a^2*b + b^3)*cos(d*x + c)^4 - (6*a
^2*b + b^3)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((6*a^2*b + b^3)*cos(d
*x + c)^4 - (6*a^2*b + b^3)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(b^
3 - (6*a^2*b + b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4 - d*co
s(d*x + c)^2)
```

3.37.6 Sympy [F]

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\csc^3{(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**3, x)
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.21

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{a^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1} - \log\left(\cos\left(dx+c\right)+1\right) + \log\left(\cos\left(dx+c\right)-1\right)\right) - b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right)\right.}{}$$

input
```
integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/4*(a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + lo
g(cos(d*x + c) - 1)) - b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a*b^2*(2/cos(d*x + c) - log(cos
(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 6*a^2*b*(2/sin(d*x + c) - log(si
n(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

3.37.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(133) = 266.

Time = 0.75 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.16

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 4\left(6\,a^2b+b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 4\left(6\,a^2b+b^3\right)}{}$$

input
```
integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c) + 4*(6*a^2
*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*(6*a^2*b + b^3)*log(abs(t
an(1/2*d*x + 1/2*c) - 1)) + 4*(a^3 + 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c)
)) - (2*a^3*tan(1/2*d*x + 1/2*c)^6 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*
a^2*b*tan(1/2*d*x + 1/2*c)^5 - 8*b^3*tan(1/2*d*x + 1/2*c)^5 - 3*a^3*tan(1/
2*d*x + 1/2*c)^4 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b*tan(1/2*d*x
+ 1/2*c)^3 - 8*b^3*tan(1/2*d*x + 1/2*c)^3 - 36*a*b^2*tan(1/2*d*x + 1/2*c)^
2 + 12*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2
*d*x + 1/2*c))^2)/d
```

3.37.9 Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 581, normalized size of antiderivative = 4.12

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,d}$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{a^3}{2}+24\,a\,b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(a^3+24\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(6\,a^2\,b-4\,b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right.}{d\left(4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}$$

$$+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{a^3}{2}+3\,a\,b^2\right)}{d}$$

$$-\frac{\operatorname{atan}\left(-\frac{\left(3\,a^2\,b+\frac{b^3}{2}\right)\left(6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3\,a^2\,b+\frac{b^3}{2}\right)+6\,a^2\,b+b^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(a^3+6\,a\,b^2\right)\right)1\mathrm{i}-\left(3\,a^2\,b+\frac{b^3}{2}\right)\left(\right.}{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(36\,a^4\,b^2+12\,a^2\,b^4+b^6\right)-\left(3\,a^2\,b+\frac{b^3}{2}\right)\left(6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3\,a^2\,b+\frac{b^3}{2}\right)+6\,a^2\,b+b^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(a^3+6\,a\,b^2\right)\right)-\left(3\,a^2\,b\right.}\right.}{d}$$

$$-\frac{3\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,d}$$

input
```
int((a + b*tan(c + d*x))^3/sin(c + d*x)^3,x)
```

output
```
(a^3*tan(c/2 + (d*x)/2)^2)/(8*d) - (tan(c/2 + (d*x)/2)^4*(24*a*b^2 + a^3/2
) - tan(c/2 + (d*x)/2)^2*(24*a*b^2 + a^3) + tan(c/2 + (d*x)/2)^5*(6*a^2*b
- 4*b^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b + 4*b^3) + a^3/2 + 6*a^2*b*tan(c
/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 - 8*tan(c/2 + (d*x)/2)^4 + 4*tan
(c/2 + (d*x)/2)^6)) + (log(tan(c/2 + (d*x)/2))*(3*a*b^2 + a^3/2))/d - (ata
n(-((3*a^2*b + b^3/2)*(6*tan(c/2 + (d*x)/2)*(3*a^2*b + b^3/2) + 6*a^2*b +
b^3 - tan(c/2 + (d*x)/2)*(6*a*b^2 + a^3))*1i - (3*a^2*b + b^3/2)*(6*tan(c/
2 + (d*x)/2)*(3*a^2*b + b^3/2) - 6*a^2*b - b^3 + tan(c/2 + (d*x)/2)*(6*a*b
^2 + a^3))*1i)/(2*tan(c/2 + (d*x)/2)*(b^6 + 12*a^2*b^4 + 36*a^4*b^2) - (3*
a^2*b + b^3/2)*(6*tan(c/2 + (d*x)/2)*(3*a^2*b + b^3/2) + 6*a^2*b + b^3 - t
an(c/2 + (d*x)/2)*(6*a*b^2 + a^3)) - (3*a^2*b + b^3/2)*(6*tan(c/2 + (d*x)/
2)*(3*a^2*b + b^3/2) - 6*a^2*b - b^3 + tan(c/2 + (d*x)/2)*(6*a*b^2 + a^3))
 + 6*a*b^5 + 6*a^5*b + 37*a^3*b^3))*(a^2*b*6i + b^3*1i))/d - (3*a^2*b*tan(
c/2 + (d*x)/2))/(2*d)
```

3.38 $\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx$

3.38.1 Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{a(a^2+3b^2)\cot(c+dx)}{d} - \frac{3a^2b\cot^2(c+dx)}{2d} - \frac{a^3\cot^3(c+dx)}{3d} + \frac{b(3a^2+b^2)\log(\tan(c+dx))}{d} + \frac{3ab^2\tan(c+dx)}{d} + \frac{b^3\tan^2(c+dx)}{2d}$$

output
```
-a*(a^2+3*b^2)*cot(d*x+c)/d-3/2*a^2*b*cot(d*x+c)^2/d-1/3*a^3*cot(d*x+c)^3/
d+b*(3*a^2+b^2)*ln(tan(d*x+c))/d+3*a*b^2*tan(d*x+c)/d+1/2*b^3*tan(d*x+c)^2
/d
```

3.38.2 Mathematica [A] (verified)

Time = 4.19 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.88

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{(b+a\cot(c+dx))^3\sec^2(c+dx)\left(-16a^3\cos(c+dx)-2\sin(c+dx)\left(18a^2b-6b^3+6(3a^2b+b^3)\cos(2(c-\right.\right.}{}$$

input
```
Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

output
```
((b + a*Cot[c + d*x])^3*Sec[c + d*x]^2*(-16*a^3*Cos[c + d*x] - 2*Sin[c + d
*x]*(18*a^2*b - 6*b^3 + 6*(3*a^2*b + b^3)*Cos[2*(c + d*x)] + 9*a^2*b*Log[C
os[c + d*x]] + 3*b^3*Log[Cos[c + d*x]] - 3*b*(3*a^2 + b^2)*Cos[4*(c + d*x)
]*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]]) - 9*a^2*b*Log[Sin[c + d*x]] - 3*
b^3*Log[Sin[c + d*x]] + 2*a^3*Sin[4*(c + d*x)] + 18*a*b^2*Sin[4*(c + d*x)]
)))/(48*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

3.38.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3999, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sin(c+dx)^4}dx$$

$$\Big\downarrow \ 3999$$

$$\frac{b\int \frac{\cot^4(c+dx)(a+b\tan(c+dx))^3(\tan^2(c+dx)b^2+b^2)}{b^4}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \ 522$$

$$\frac{b\int \left(\frac{a^3\cot^4(c+dx)}{b^2}+\frac{3a^2\cot^3(c+dx)}{b}+\frac{(a^3+3b^2a)\cot^2(c+dx)}{b^2}+\frac{(3a^2+b^2)\cot(c+dx)}{b}+3a+b\tan(c+dx)\right)d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{b\left(-\frac{a^3\cot^3(c+dx)}{3b}-\frac{a(a^2+3b^2)\cot(c+dx)}{b}+(3a^2+b^2)\log(b\tan(c+dx))-\frac{3}{2}a^2\cot^2(c+dx)+3ab\tan(c+dx)+\frac{1}{2}b^2 \text{t}\right.}{d}$$

input
```
Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*(-((a*(a^2 + 3*b^2)*Cot[c + d*x])/b) - (3*a^2*Cot[c + d*x]^2)/2 - (a^3*
Cot[c + d*x]^3)/(3*b) + (3*a^2 + b^2)*Log[b*Tan[c + d*x]] + 3*a*b*Tan[c +
d*x] + (b^2*Tan[c + d*x]^2)/2))/d
```

3.38.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.38.4 Maple [A] (verified)

Time = 6.97 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{b^3\left(\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+3a\,b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a^2b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^3}{d}$
default	$\frac{b^3\left(\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+3a\,b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a^2b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^3}{d}$
risch	$\frac{6a^2b\,e^{8i(dx+c)}+2b^3e^{8i(dx+c)}+\frac{4ia^3e^{2i(dx+c)}}{3}+12ia\,b^2e^{2i(dx+c)}+6a^2b\,e^{6i(dx+c)}-6b^3e^{6i(dx+c)}-12ia\,b^2e^{6i(dx+c)}+\frac{20ia^3e}{}}{d\left(e^{2i(dx+c)}+1\right)^2}$

input
```
int(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(1/2/cos(d*x+c)^2+ln(tan(d*x+c)))+3*a*b^2*(1/sin(d*x+c)/cos(d*x+c
)-2*cot(d*x+c))+3*a^2*b*(-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a^3*(-2/3-1/3*c
sc(d*x+c)^2)*cot(d*x+c))
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(107) = 214$.

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.10

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{4\,(a^3+9\,ab^2)\cos\left(dx+c\right)^5+18\,ab^2\cos\left(dx+c\right)-6\,(a^3+9\,ab^2)\cos\left(dx+c\right)^3+3\,\left((3\,a^2b+b^3)\cos\left(dx\right.\right.}{}$$

input
```
integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/6*(4*(a^3 + 9*a*b^2)*cos(d*x + c)^5 + 18*a*b^2*cos(d*x + c) - 6*(a^3 +
9*a*b^2)*cos(d*x + c)^3 + 3*((3*a^2*b + b^3)*cos(d*x + c)^4 - (3*a^2*b + b
^3)*cos(d*x + c)^2)*log(cos(d*x + c)^2)*sin(d*x + c) - 3*((3*a^2*b + b^3)*
cos(d*x + c)^4 - (3*a^2*b + b^3)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c)^2 +
 1/4)*sin(d*x + c) + 3*(b^3 - (3*a^2*b + b^3)*cos(d*x + c)^2)*sin(d*x + c)
)/((d*cos(d*x + c)^4 - d*cos(d*x + c)^2)*sin(d*x + c))
```

3.38.6 Sympy [F]

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^3\csc^4\left(c+dx\right)dx$$

input
```
integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**4, x)
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{3\,b^3\tan\left(dx+c\right)^2 + 18\,ab^2\tan\left(dx+c\right) + 6\left(3\,a^2b+b^3\right)\log\left(\tan\left(dx+c\right)\right) - \frac{9\,a^2b\tan(dx+c)+2\,a^3+6\,(a^3+3\,ab^2)\tan(}{\tan(dx+c)^3}}{6\,d}$$

input
```
integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/6*(3*b^3*tan(d*x + c)^2 + 18*a*b^2*tan(d*x + c) + 6*(3*a^2*b + b^3)*log(
tan(d*x + c)) - (9*a^2*b*tan(d*x + c) + 2*a^3 + 6*(a^3 + 3*a*b^2)*tan(d*x
+ c)^2)/tan(d*x + c)^3)/d
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{3\,b^3\tan\left(dx+c\right)^2 + 18\,ab^2\tan\left(dx+c\right) + 6\left(3\,a^2b+b^3\right)\log\left(\left|\tan\left(dx+c\right)\right|\right) - \frac{33\,a^2b\tan(dx+c)^3+11\,b^3\tan(dx+c)^3+}{}}{6\,d}$$

input
```
integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(3*b^3*tan(d*x + c)^2 + 18*a*b^2*tan(d*x + c) + 6*(3*a^2*b + b^3)*log(
abs(tan(d*x + c))) - (33*a^2*b*tan(d*x + c)^3 + 11*b^3*tan(d*x + c)^3 + 6*
a^3*tan(d*x + c)^2 + 18*a*b^2*tan(d*x + c)^2 + 9*a^2*b*tan(d*x + c) + 2*a^
3)/tan(d*x + c)^3)/d
```

3.38.9 Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\ln(\tan(c+d\,x))\,\left(3\,a^2\,b+b^3\right)}{d} - \frac{\cot(c+d\,x)^3\,\left(\frac{a^3}{3}+\tan(c+d\,x)^2\,\left(a^3+3\,a\,b^2\right)+\frac{3\,a^2\,b\tan(c+d\,x)}{2}\right)}{d} + \frac{b^3\tan(c+d\,x)^2}{2\,d} + \frac{3\,a\,b^2\tan(c+d\,x)}{d}$$

input
```
int((a + b*tan(c + d*x))^3/sin(c + d*x)^4,x)
```

output
```
(log(tan(c + d*x))*(3*a^2*b + b^3))/d - (cot(c + d*x)^3*(a^3/3 + tan(c + d
*x)^2*(3*a*b^2 + a^3) + (3*a^2*b*tan(c + d*x))/2))/d + (b^3*tan(c + d*x)^2
)/(2*d) + (3*a*b^2*tan(c + d*x))/d
```

3.39 $\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx$

3.39.1 Optimal result

Integrand size = 21, antiderivative size = 229

$$\begin{aligned}\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx = &-\frac{3a^3\text{arctanh}(\cos(c+dx))}{8d}\\&-\frac{9ab^2\text{arctanh}(\cos(c+dx))}{2d}\\&+\frac{3a^2b\text{arctanh}(\sin(c+dx))}{d}\\&+\frac{3b^3\text{arctanh}(\sin(c+dx))}{2d}-\frac{3a^2b\csc(c+dx)}{d}\\&-\frac{3b^3\csc(c+dx)}{2d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{8d}\\&-\frac{a^2b\csc^3(c+dx)}{d}-\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}\\&+\frac{9ab^2\sec(c+dx)}{2d}-\frac{3ab^2\csc^2(c+dx)\sec(c+dx)}{2d}\\&+\frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d}\end{aligned}$$

output
```
-3/8*a^3*arctanh(cos(d*x+c))/d-9/2*a*b^2*arctanh(cos(d*x+c))/d+3*a^2*b*arc
tanh(sin(d*x+c))/d+3/2*b^3*arctanh(sin(d*x+c))/d-3*a^2*b*csc(d*x+c)/d-3/2*
b^3*csc(d*x+c)/d-3/8*a^3*cot(d*x+c)*csc(d*x+c)/d-a^2*b*csc(d*x+c)^3/d-1/4*
a^3*cot(d*x+c)*csc(d*x+c)^3/d+9/2*a*b^2*sec(d*x+c)/d-3/2*a*b^2*csc(d*x+c)^
2*sec(d*x+c)/d+1/2*b^3*csc(d*x+c)*sec(d*x+c)^2/d
```

3.39.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1229 vs. $2(229) = 458$.

Time = 7.64 (sec) , antiderivative size = 1229, normalized size of antiderivative = 5.37

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3ab^2\cos^3(c+dx)(a+b\tan(c+dx))^3}{d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$+\frac{\left(-7a^2b\cos\left(\frac{1}{2}(c+dx)\right)-2b^3\cos\left(\frac{1}{2}(c+dx)\right)\right)\cos^3(c+dx)\csc\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{4d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$-\frac{3(a^3+4ab^2)\cos^3(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{32d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$-\frac{a^2b\cos^3(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{8d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$-\frac{a^3\cos^3(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{64d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$-\frac{3(a^3+12ab^2)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{8d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$-\frac{3(2a^2b+b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$+\frac{3(a^3+12ab^2)\cos^3(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{8d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$+\frac{3(2a^2b+b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{2d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$+\frac{3(a^3+4ab^2)\cos^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{32d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$+\frac{a^3\cos^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{64d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$+\frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$+\frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$-\frac{b^3\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$-\frac{3ab^2\cos^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$+\frac{\cos^3(c+dx)\sec\left(\frac{1}{2}(c+dx)\right)\left(-7a^2b\sin\left(\frac{1}{2}(c+dx)\right)-2b^3\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3}{4d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$-\frac{a^2b\cos^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^3}{8d(a\cos(c+dx)+b\sin(c+dx))^3}$$

input
```
Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

output
```
(3*a*b^2*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3)/(d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^3) + ((-7*a^2*b*Cos[(c + d*x)/2] - 2*b^3*Cos[(c + d*x)/2])*Cos[
c + d*x]^3*Csc[(c + d*x)/2]*(a + b*Tan[c + d*x])^3)/(4*d*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^3) - (3*(a^3 + 4*a*b^2)*Cos[c + d*x]^3*Csc[(c + d*x)/2]^2
*(a + b*Tan[c + d*x])^3)/(32*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^2
*b*Cos[c + d*x]^3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(a + b*Tan[c + d*x])
^3)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a^3*Cos[c + d*x]^3*Csc[(c
 + d*x)/2]^4*(a + b*Tan[c + d*x])^3)/(64*d*(a*Cos[c + d*x] + b*Sin[c + d*x
])^3) - (3*(a^3 + 12*a*b^2)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2]]*(a + b*Ta
n[c + d*x])^3)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (3*(2*a^2*b + b
^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c +
 d*x])^3)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*(a^3 + 12*a*b^2)*
Cos[c + d*x]^3*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(8*d*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^3) + (3*(2*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3)/(2*d*(a*Cos[c + d*
x] + b*Sin[c + d*x])^3) + (3*(a^3 + 4*a*b^2)*Cos[c + d*x]^3*Sec[(c + d*x)/
2]^2*(a + b*Tan[c + d*x])^3)/(32*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) +
(a^3*Cos[c + d*x]^3*Sec[(c + d*x)/2]^4*(a + b*Tan[c + d*x])^3)/(64*d*(a*Co
s[c + d*x] + b*Sin[c + d*x])^3) + (b^3*Cos[c + d*x]^3*(a + b*Tan[c + d*x])
^3)/(4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Si...
```

3.39.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sin(c+dx)^5}dx$$

$$\Big\downarrow \text{ 4000}$$

$$\int \left(a^3\csc^5(c+dx)+3a^2b\csc^4(c+dx)\sec(c+dx)+3ab^2\csc^3(c+dx)\sec^2(c+dx)+b^3\csc^2(c+dx)\sec^3(c+dx)\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3a^3\text{arctanh}(\cos(c+dx))}{8d}-\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{8d}+\frac{3a^2b\text{arctanh}(\sin(c+dx))}{d}-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}-\frac{9ab^2\text{arctanh}(\cos(c+dx))}{2d}+\frac{9ab^2\sec(c+dx)}{2d}-\frac{3ab^2\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{3b^3\text{arctanh}(\sin(c+dx))}{2d}-\frac{3b^3\csc(c+dx)}{2d}+\frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d}$$

input
```
Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

output
```
(-3*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (9*a*b^2*ArcTanh[Cos[c + d*x]])/(2*
d) + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (3*b^3*ArcTanh[Sin[c + d*x]])/(2*
d) - (3*a^2*b*Csc[c + d*x])/d - (3*b^3*Csc[c + d*x])/(2*d) - (3*a^3*Cot[c
+ d*x]*Csc[c + d*x])/(8*d) - (a^2*b*Csc[c + d*x]^3)/d - (a^3*Cot[c + d*x]*
Csc[c + d*x]^3)/(4*d) + (9*a*b^2*Sec[c + d*x])/(2*d) - (3*a*b^2*Csc[c + d*
x]^2*Sec[c + d*x])/(2*d) + (b^3*Csc[c + d*x]*Sec[c + d*x]^2)/(2*d)
```

3.39.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.39.4 Maple [A] (verified)

Time = 10.94 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{b^3\left(\frac{1}{2\sin(dx+c)\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a\,b^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc}{}\right.}{}$
default	$\frac{b^3\left(\frac{1}{2\sin(dx+c)\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a\,b^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc}{}\right.}{}$
risch	$-\frac{ie^{i(dx+c)}\left(-5ia^3e^{2i(dx+c)}-30ia^3e^{6i(dx+c)}+24a^2b\,e^{10i(dx+c)}+12b^3e^{10i(dx+c)}+3ia^3+36ia\,b^2-56a^2b\,e^{8i(dx+c)}-28b^3e\right.}{}$

input
```
int(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(1/2/sin(d*x+c)/cos(d*x+c)^2-3/2/sin(d*x+c)+3/2*ln(sec(d*x+c)+tan
(d*x+c)))+3*a*b^2*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(
d*x+c)-cot(d*x+c)))+3*a^2*b*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+
tan(d*x+c)))+a^3*((-1/4*csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(csc
(d*x+c)-cot(d*x+c))))
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(211) = 422$.

Time = 0.36 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.86

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{6\left(a^3+12\,ab^2\right)\cos\left(dx+c\right)^5+48\,ab^2\cos\left(dx+c\right)-10\left(a^3+12\,ab^2\right)\cos\left(dx+c\right)^3-3\left(\left(a^3+12\,ab^2\right)\cos\left(a\right.\right.}{}$$

input
```
integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/16*(6*(a^3 + 12*a*b^2)*cos(d*x + c)^5 + 48*a*b^2*cos(d*x + c) - 10*(a^3
+ 12*a*b^2)*cos(d*x + c)^3 - 3*((a^3 + 12*a*b^2)*cos(d*x + c)^6 - 2*(a^3 +
 12*a*b^2)*cos(d*x + c)^4 + (a^3 + 12*a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d
*x + c) + 1/2) + 3*((a^3 + 12*a*b^2)*cos(d*x + c)^6 - 2*(a^3 + 12*a*b^2)*c
os(d*x + c)^4 + (a^3 + 12*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1
/2) + 12*((2*a^2*b + b^3)*cos(d*x + c)^6 - 2*(2*a^2*b + b^3)*cos(d*x + c)^
4 + (2*a^2*b + b^3)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 12*((2*a^2*b +
 b^3)*cos(d*x + c)^6 - 2*(2*a^2*b + b^3)*cos(d*x + c)^4 + (2*a^2*b + b^3)*
cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 8*(3*(2*a^2*b + b^3)*cos(d*x + c)
^4 + b^3 - 4*(2*a^2*b + b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)
^6 - 2*d*cos(d*x + c)^4 + d*cos(d*x + c)^2)
```

3.39.6 Sympy [F]

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\csc^5(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**5*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**5, x)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.09

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a^3\left(\frac{2\left(3\cos(dx+c)^3-5\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1} - 3\log(\cos(dx+c)+1) + 3\log(\cos(dx+c)-1)\right) + 12ab^2\left(\frac{2\left(3\cos(dx+c)\right.}{\cos(dx+c)^3-\cos\left(\right.}\right.}{}$$

input
```
integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/16*(a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 12*a
*b^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(cos
(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 4*b^3*(2*(3*sin(d*x + c)^2 - 2
)/(sin(d*x + c)^3 - sin(d*x + c)) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*
x + c) - 1)) - 8*a^2*b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(si
n(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.63

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$= \frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 8\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 8\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 24\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 120\,a^2b}{}$$

input
```
integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/64*(a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 8*a^3*
tan(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 120*a^2*b*tan(1
/2*d*x + 1/2*c) - 32*b^3*tan(1/2*d*x + 1/2*c) + 96*(2*a^2*b + b^3)*log(abs
(tan(1/2*d*x + 1/2*c) + 1)) - 96*(2*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + 24*(a^3 + 12*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 64*(b^3*t
an(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 + b^3*tan(1/2*d*x +
 1/2*c) + 6*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (50*a^3*tan(1/2*d*x +
1/2*c)^4 + 600*a*b^2*tan(1/2*d*x + 1/2*c)^4 + 120*a^2*b*tan(1/2*d*x + 1/2*
c)^3 + 32*b^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^3*tan(1/2*d*x + 1/2*c)^2 + 24*a
*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*
d*x + 1/2*c)^4)/d
```

3.39.9 Mupad [B] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 698, normalized size of antiderivative = 3.05

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a^3}{8}+\frac{3\,a\,b^2}{8}\right)}{d} - \frac{\mathrm{atan}\left(\frac{\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a^3}{4}+9\,a\,b^2\right)+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)-6\,a^2\,b-3\,b^3\right)1\mathrm{i}-\left(3\,a^2\,b+\ldots\right.}{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(36\,a^4\,b^2+36\,a^2\,b^4+9\,b^6\right)+27\,a\,b^5+\frac{9\,a^5\,b}{2}-\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a^3}{4}+9\,a\,b^2\right)+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(3\,a^2\,b+\frac{3\,b^3}{2}\right)-\ldots\right.}\right)\ldots}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{3\,a^3}{2}+6\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(2\,a^3+102\,a\,b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{15\,a^3}{4}+108\,a\,b^2\right)+\tan(\ldots}{d\left(16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-32\ldots\right.} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{15\,a^2\,b}{8}+\frac{b^3}{2}\right)}{d}+\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^2+12\,b^2\right)}{8\,d}-\frac{a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{8\,d}$$

input
```
int((a + b*tan(c + d*x))^3/sin(c + d*x)^5,x)
```

output
```
(a^3*tan(c/2 + (d*x)/2)^4)/(64*d) + (tan(c/2 + (d*x)/2)^2*((3*a*b^2)/8 + a
^3/8))/d - (atan(((3*a^2*b + (3*b^3)/2)*(tan(c/2 + (d*x)/2)*(9*a*b^2 + (3*
a^3)/4) + 6*tan(c/2 + (d*x)/2)*(3*a^2*b + (3*b^3)/2) - 6*a^2*b - 3*b^3)*1i
 - (3*a^2*b + (3*b^3)/2)*(6*tan(c/2 + (d*x)/2)*(3*a^2*b + (3*b^3)/2) - tan
(c/2 + (d*x)/2)*(9*a*b^2 + (3*a^3)/4) + 6*a^2*b + 3*b^3)*1i)/(2*tan(c/2 +
(d*x)/2)*(9*b^6 + 36*a^2*b^4 + 36*a^4*b^2) + 27*a*b^5 + (9*a^5*b)/2 - (3*a
^2*b + (3*b^3)/2)*(tan(c/2 + (d*x)/2)*(9*a*b^2 + (3*a^3)/4) + 6*tan(c/2 +
(d*x)/2)*(3*a^2*b + (3*b^3)/2) - 6*a^2*b - 3*b^3) - (3*a^2*b + (3*b^3)/2)*
(6*tan(c/2 + (d*x)/2)*(3*a^2*b + (3*b^3)/2) - tan(c/2 + (d*x)/2)*(9*a*b^2
+ (3*a^3)/4) + 6*a^2*b + 3*b^3) + (225*a^3*b^3)/4))*(a^2*b*6i + b^3*3i))/d
 - (tan(c/2 + (d*x)/2)^2*(6*a*b^2 + (3*a^3)/2) + tan(c/2 + (d*x)/2)^6*(102
*a*b^2 + 2*a^3) - tan(c/2 + (d*x)/2)^4*(108*a*b^2 + (15*a^3)/4) + tan(c/2
+ (d*x)/2)^3*(26*a^2*b + 8*b^3) + tan(c/2 + (d*x)/2)^7*(30*a^2*b - 8*b^3)
- tan(c/2 + (d*x)/2)^5*(58*a^2*b + 32*b^3) + a^3/4 + 2*a^2*b*tan(c/2 + (d*
x)/2))/(d*(16*tan(c/2 + (d*x)/2)^4 - 32*tan(c/2 + (d*x)/2)^6 + 16*tan(c/2
+ (d*x)/2)^8)) - (tan(c/2 + (d*x)/2)*((15*a^2*b)/8 + b^3/2))/d + (3*a*log(
tan(c/2 + (d*x)/2))*(a^2 + 12*b^2))/(8*d) - (a^2*b*tan(c/2 + (d*x)/2)^3)/(
8*d)
```

3.40 $\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx$

3.40.1 Optimal result

Integrand size = 21, antiderivative size = 167

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{a(a^2+6b^2)\cot(c+dx)}{d} - \frac{b(6a^2+b^2)\cot^2(c+dx)}{2d} - \frac{a(2a^2+3b^2)\cot^3(c+dx)}{3d} - \frac{3a^2b\cot^4(c+dx)}{4d} - \frac{a^3\cot^5(c+dx)}{5d} + \frac{b(3a^2+2b^2)\log(\tan(c+dx))}{d} + \frac{3ab^2\tan(c+dx)}{d} + \frac{b^3\tan^2(c+dx)}{2d}$$

output

```
-a*(a^2+6*b^2)*cot(d*x+c)/d-1/2*b*(6*a^2+b^2)*cot(d*x+c)^2/d-1/3*a*(2*a^2+
3*b^2)*cot(d*x+c)^3/d-3/4*a^2*b*cot(d*x+c)^4/d-1/5*a^3*cot(d*x+c)^5/d+b*(3
*a^2+2*b^2)*ln(tan(d*x+c))/d+3*a*b^2*tan(d*x+c)/d+1/2*b^3*tan(d*x+c)^2/d
```

3.40.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 515 vs. $2(167) = 334$.

Time = 3.55 (sec) , antiderivative size = 515, normalized size of antiderivative = 3.08

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{\csc^5(c+dx)\sec^2(c+dx)\left(40a(5a^2+3b^2)\cos(c+dx)+8(a^3+15ab^2)\cos(3(c+dx))-24a^3\cos(5(c+\right.}{}$$

input
```
Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

output
```
-1/960*(Csc[c + d*x]^5*Sec[c + d*x]^2*(40*a*(5*a^2 + 3*b^2)*Cos[c + d*x] +
 8*(a^3 + 15*a*b^2)*Cos[3*(c + d*x)] - 24*a^3*Cos[5*(c + d*x)] - 360*a*b^2
*Cos[5*(c + d*x)] + 8*a^3*Cos[7*(c + d*x)] + 120*a*b^2*Cos[7*(c + d*x)] +
360*a^2*b*Sin[c + d*x] - 240*b^3*Sin[c + d*x] + 225*a^2*b*Log[Cos[c + d*x]
]*Sin[c + d*x] + 150*b^3*Log[Cos[c + d*x]]*Sin[c + d*x] - 225*a^2*b*Log[Si
n[c + d*x]]*Sin[c + d*x] - 150*b^3*Log[Sin[c + d*x]]*Sin[c + d*x] + 270*a^
2*b*Sin[3*(c + d*x)] + 180*b^3*Sin[3*(c + d*x)] + 45*a^2*b*Log[Cos[c + d*x
]]*Sin[3*(c + d*x)] + 30*b^3*Log[Cos[c + d*x]]*Sin[3*(c + d*x)] - 45*a^2*b
*Log[Sin[c + d*x]]*Sin[3*(c + d*x)] - 30*b^3*Log[Sin[c + d*x]]*Sin[3*(c +
d*x)] - 90*a^2*b*Sin[5*(c + d*x)] - 60*b^3*Sin[5*(c + d*x)] - 135*a^2*b*Lo
g[Cos[c + d*x]]*Sin[5*(c + d*x)] - 90*b^3*Log[Cos[c + d*x]]*Sin[5*(c + d*x
)] + 135*a^2*b*Log[Sin[c + d*x]]*Sin[5*(c + d*x)] + 90*b^3*Log[Sin[c + d*x
]]*Sin[5*(c + d*x)] + 45*a^2*b*Log[Cos[c + d*x]]*Sin[7*(c + d*x)] + 30*b^3
*Log[Cos[c + d*x]]*Sin[7*(c + d*x)] - 45*a^2*b*Log[Sin[c + d*x]]*Sin[7*(c
+ d*x)] - 30*b^3*Log[Sin[c + d*x]]*Sin[7*(c + d*x)]))/d
```

3.40.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3999, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sin(c+dx)^6}dx$$

$$\Big\downarrow \text{ 3999}$$

$$\frac{b\int \frac{\cot^6(c+dx)(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)^2}{b^6}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 522}$$

$$\frac{b\int \left(\frac{a^3\cot^6(c+dx)}{b^2}+\frac{3a^2\cot^5(c+dx)}{b}+\frac{(3ab^4+2a^3b^2)\cot^4(c+dx)}{b^4}+\frac{(b^4+6a^2b^2)\cot^3(c+dx)}{b^3}+\frac{(a^3+6b^2a)\cot^2(c+dx)}{b^2}+\frac{(3a^2+2b^2)\cot(\cdots}{b}\cdots\right.}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\left(-\frac{a^3\cot^5(c+dx)}{5b}-\frac{a(2a^2+3b^2)\cot^3(c+dx)}{3b}-\frac{1}{2}(6a^2+b^2)\cot^2(c+dx)-\frac{a(a^2+6b^2)\cot(c+dx)}{b}+(3a^2+2b^2)\log(b\tan(c+\cdots\right.}{d}$$

input
```
Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*(-((a*(a^2 + 6*b^2)*Cot[c + d*x])/b) - ((6*a^2 + b^2)*Cot[c + d*x]^2)/2
 - (a*(2*a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*b) - (3*a^2*Cot[c + d*x]^4)/4 - (
a^3*Cot[c + d*x]^5)/(5*b) + (3*a^2 + 2*b^2)*Log[b*Tan[c + d*x]] + 3*a*b*Ta
n[c + d*x] + (b^2*Tan[c + d*x]^2)/2))/d
```

3.40.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.40.4 Maple [A] (verified)

Time = 19.52 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{b^3\left(\frac{1}{2\sin(dx+c)^2\cos(dx+c)^2}-\frac{1}{\sin(dx+c)^2}+2\ln(\tan(dx+c))\right)+3a\,b^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(\cdots}{3}\cdots\right.}{d}$
default	$\frac{b^3\left(\frac{1}{2\sin(dx+c)^2\cos(dx+c)^2}-\frac{1}{\sin(dx+c)^2}+2\ln(\tan(dx+c))\right)+3a\,b^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(\cdots}{3}\cdots\right.}{d}$
risch	$\frac{6a^2b\,e^{12i(dx+c)}+4b^3e^{12i(dx+c)}-18a^2b\,e^{10i(dx+c)}-12b^3e^{10i(dx+c)}+\frac{16ia^3e^{2i(dx+c)}}{5}+32ia\,b^2e^{8i(dx+c)}-24a^2b\,e^{8i(dx+c)}+\cdots}{\cdots}$

input
```
int(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(1/2/sin(d*x+c)^2/cos(d*x+c)^2-1/sin(d*x+c)^2+2*ln(tan(d*x+c)))+3
*a*b^2*(-1/3/sin(d*x+c)^3/cos(d*x+c)+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x
+c))+3*a^2*b*(-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a^3*(-8/1
5-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c))
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(157) = 314.

Time = 0.28 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.05

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{32\,(a^3+15\,ab^2)\cos{(dx+c)}^7-80\,(a^3+15\,ab^2)\cos{(dx+c)}^5-180\,ab^2\cos{(dx+c)}+60\,(a^3+15\,ab^2)\,\cdots}{\cdots}$$

input
```
integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(32*(a^3 + 15*a*b^2)*cos(d*x + c)^7 - 80*(a^3 + 15*a*b^2)*cos(d*x +
c)^5 - 180*a*b^2*cos(d*x + c) + 60*(a^3 + 15*a*b^2)*cos(d*x + c)^3 + 30*((
3*a^2*b + 2*b^3)*cos(d*x + c)^6 - 2*(3*a^2*b + 2*b^3)*cos(d*x + c)^4 + (3*
a^2*b + 2*b^3)*cos(d*x + c)^2)*log(cos(d*x + c)^2)*sin(d*x + c) - 30*((3*a
^2*b + 2*b^3)*cos(d*x + c)^6 - 2*(3*a^2*b + 2*b^3)*cos(d*x + c)^4 + (3*a^2
*b + 2*b^3)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) -
15*(2*(3*a^2*b + 2*b^3)*cos(d*x + c)^4 + 2*b^3 - 3*(3*a^2*b + 2*b^3)*cos(d
*x + c)^2)*sin(d*x + c))/((d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^4 + d*cos(d
*x + c)^2)*sin(d*x + c))
```

3.40.6 Sympy [F]

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\csc^6(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**6*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*csc(c + d*x)**6, x)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.85

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{30\,b^3\tan(dx+c)^2 + 180\,ab^2\tan(dx+c) + 60\,(3\,a^2b+2\,b^3)\log(\tan(dx+c)) - \frac{60\,(a^3+6\,ab^2)\tan(dx+c)^4+45\,a^2b}{}}{60\,d}$$

input
```
integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/60*(30*b^3*tan(d*x + c)^2 + 180*a*b^2*tan(d*x + c) + 60*(3*a^2*b + 2*b^3
)*log(tan(d*x + c)) - (60*(a^3 + 6*a*b^2)*tan(d*x + c)^4 + 45*a^2*b*tan(d*
x + c) + 30*(6*a^2*b + b^3)*tan(d*x + c)^3 + 12*a^3 + 20*(2*a^3 + 3*a*b^2)
*tan(d*x + c)^2)/tan(d*x + c)^5)/d
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.13

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{30\,b^3\tan(dx+c)^2 + 180\,ab^2\tan(dx+c) + 60\,(3\,a^2b+2\,b^3)\log(|\tan(dx+c)|) - \frac{411\,a^2b\tan(dx+c)^5+274\,b^3\tan(\cdots}{\cdots}}{\cdots}$$

input
```
integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(30*b^3*tan(d*x + c)^2 + 180*a*b^2*tan(d*x + c) + 60*(3*a^2*b + 2*b^3
)*log(abs(tan(d*x + c))) - (411*a^2*b*tan(d*x + c)^5 + 274*b^3*tan(d*x + c
)^5 + 60*a^3*tan(d*x + c)^4 + 360*a*b^2*tan(d*x + c)^4 + 180*a^2*b*tan(d*x
 + c)^3 + 30*b^3*tan(d*x + c)^3 + 40*a^3*tan(d*x + c)^2 + 60*a*b^2*tan(d*x
 + c)^2 + 45*a^2*b*tan(d*x + c) + 12*a^3)/tan(d*x + c)^5)/d
```

3.40.9 Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.87

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\ln(\tan(c+d\,x))\,\left(3\,a^2\,b+2\,b^3\right)}{d} - \frac{\cot(c+d\,x)^5\left(\tan(c+d\,x)^2\left(\frac{2\,a^3}{3}+a\,b^2\right)+\tan(c+d\,x)^3\left(3\,a^2\,b+\frac{b^3}{2}\right)+\frac{a^3}{5}+\tan(c+d\,x)^4\left(a^3+6\,a\,b^2\cdots\right.\right.}{d} + \frac{b^3\tan(c+d\,x)^2}{2\,d} + \frac{3\,a\,b^2\tan(c+d\,x)}{d}$$

input
```
int((a + b*tan(c + d*x))^3/sin(c + d*x)^6,x)
```

output
```
(log(tan(c + d*x))*(3*a^2*b + 2*b^3))/d - (cot(c + d*x)^5*(tan(c + d*x)^2*
(a*b^2 + (2*a^3)/3) + tan(c + d*x)^3*(3*a^2*b + b^3/2) + a^3/5 + tan(c + d
*x)^4*(6*a*b^2 + a^3) + (3*a^2*b*tan(c + d*x))/4))/d + (b^3*tan(c + d*x)^2
)/(2*d) + (3*a*b^2*tan(c + d*x))/d
```

3.41 $\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx$

3.41.1 Optimal result

Integrand size = 21, antiderivative size = 275

$$\begin{aligned}\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx &= \frac{4a^3b\operatorname{arctanh}(\sin(c+dx))}{d} \\ &- \frac{10ab^3\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{a^4\cos(c+dx)}{d} \\ &+ \frac{12a^2b^2\cos(c+dx)}{d} - \frac{3b^4\cos(c+dx)}{d} \\ &+ \frac{a^4\cos^3(c+dx)}{3d} - \frac{2a^2b^2\cos^3(c+dx)}{d} \\ &+ \frac{b^4\cos^3(c+dx)}{3d} + \frac{6a^2b^2\sec(c+dx)}{d} \\ &- \frac{3b^4\sec(c+dx)}{d} + \frac{b^4\sec^3(c+dx)}{3d} \\ &- \frac{4a^3b\sin(c+dx)}{d} + \frac{10ab^3\sin(c+dx)}{d} \\ &- \frac{4a^3b\sin^3(c+dx)}{3d} + \frac{10ab^3\sin^3(c+dx)}{3d} \\ &+ \frac{2ab^3\sin^3(c+dx)\tan^2(c+dx)}{d}\end{aligned}$$

output
```
4*a^3*b*arctanh(sin(d*x+c))/d-10*a*b^3*arctanh(sin(d*x+c))/d-a^4*cos(d*x+c
)/d+12*a^2*b^2*cos(d*x+c)/d-3*b^4*cos(d*x+c)/d+1/3*a^4*cos(d*x+c)^3/d-2*a^
2*b^2*cos(d*x+c)^3/d+1/3*b^4*cos(d*x+c)^3/d+6*a^2*b^2*sec(d*x+c)/d-3*b^4*s
ec(d*x+c)/d+1/3*b^4*sec(d*x+c)^3/d-4*a^3*b*sin(d*x+c)/d+10*a*b^3*sin(d*x+c
)/d-4/3*a^3*b*sin(d*x+c)^3/d+10/3*a*b^3*sin(d*x+c)^3/d+2*a*b^3*sin(d*x+c)^
3*tan(d*x+c)^2/d
```

3.41.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1017 vs. 2(275) = 550.

Time = 7.87 (sec) , antiderivative size = 1017, normalized size of antiderivative = 3.70

$$\begin{aligned}
\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx &= -\frac{b^2(-36a^2+17b^2)\cos^4(c+dx)(a+b\tan(c+dx))^4}{6d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&- \frac{(3a^4-42a^2b^2+11b^4)\cos^5(c+dx)(a+b\tan(c+dx))^4}{4d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{(a^4-6a^2b^2+b^4)\cos^4(c+dx)\cos(3(c+dx))(a+b\tan(c+dx))^4}{12d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&- \frac{2(2a^3b-5ab^3)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{2(2a^3b-5ab^3)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{(12ab^3+b^4)\cos^4(c+dx)(a+b\tan(c+dx))^4}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{b^4\cos^4(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a\cos(c+dx)+b\sin(c+dx))^4} \\
&- \frac{b^4\cos^4(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{(-12ab^3+b^4)\cos^4(c+dx)(a+b\tan(c+dx))^4}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{\cos^4(c+dx)\left(36a^2b^2\sin\left(\frac{1}{2}(c+dx)\right)-17b^4\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{\cos^4(c+dx)\left(-36a^2b^2\sin\left(\frac{1}{2}(c+dx)\right)+17b^4\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^4} \\
&- \frac{ab(5a^2-9b^2)\cos^4(c+dx)\sin(c+dx)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{ab(a^2-b^2)\cos^4(c+dx)\sin(3(c+dx))(a+b\tan(c+dx))^4}{3d(a\cos(c+dx)+b\sin(c+dx))^4}
\end{aligned}$$

input
```
Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

output
```
-1/6*(b^2*(-36*a^2 + 17*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) - ((3*a^4 - 42*a^2*b^2 + 11*b^4)*Cos[c +
 d*x]^5*(a + b*Tan[c + d*x])^4)/(4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
+ ((a^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x]^4*Cos[3*(c + d*x)]*(a + b*Tan[c +
d*x])^4)/(12*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*(2*a^3*b - 5*a*b^
3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c +
d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b - 5*a*b^3)*
Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x
])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((12*a*b^3 + b^4)*Cos[c +
d*x]^4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*Sin[(c + d*x
)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*
(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*Sin[(c + d*x)/2
]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-12*a*b^3 + b^4)*Cos[c + d*x]^4*(a +
 b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(36*a^2*b^2*Sin[(c + d*x)/2
] - 17*b^4*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]
^4*(-36*a^2*b^2*Sin[(c + d*x)/2] + 17*b^4*Sin[(c + d*x)/2])*(a + b*Tan[...
```

3.41.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(c+dx)^3(a+b\tan(c+dx))^4dx$$

$$\Big\downarrow \; 4000$$

$$\int \left(a^4\sin^3(c+dx)+4a^3b\sin^3(c+dx)\tan(c+dx)+6a^2b^2\sin^3(c+dx)\tan^2(c+dx)+4ab^3\sin^3(c+dx)\tan^3(c+\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{a^4\cos^3(c+dx)}{3d}-\frac{a^4\cos(c+dx)}{d}+\frac{4a^3b\text{arctanh}(\sin(c+dx))}{d}-\frac{4a^3b\sin^3(c+dx)}{3d}-\frac{4a^3b\sin(c+dx)}{d}-\frac{2a^2b^2\cos^3(c+dx)}{d}+\frac{12a^2b^2\cos(c+dx)}{d}+\frac{6a^2b^2\sec(c+dx)}{d}-\frac{10ab^3\text{arctanh}(\sin(c+dx))}{d}+\frac{10ab^3\sin^3(c+dx)}{3d}+\frac{10ab^3\sin(c+dx)}{d}+\frac{2ab^3\sin^3(c+dx)\tan^2(c+dx)}{d}+\frac{b^4\cos^3(c+dx)}{3d}-\frac{3b^4\cos(c+dx)}{d}+\frac{b^4\sec^3(c+dx)}{3d}-\frac{3b^4\sec(c+dx)}{d}$$

input
```
Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

output
```
(4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (10*a*b^3*ArcTanh[Sin[c + d*x]])/d - (
a^4*Cos[c + d*x])/d + (12*a^2*b^2*Cos[c + d*x])/d - (3*b^4*Cos[c + d*x])/d
 + (a^4*Cos[c + d*x]^3)/(3*d) - (2*a^2*b^2*Cos[c + d*x]^3)/d + (b^4*Cos[c
+ d*x]^3)/(3*d) + (6*a^2*b^2*Sec[c + d*x])/d - (3*b^4*Sec[c + d*x])/d + (b
^4*Sec[c + d*x]^3)/(3*d) - (4*a^3*b*Sin[c + d*x])/d + (10*a*b^3*Sin[c + d*
x])/d - (4*a^3*b*Sin[c + d*x]^3)/(3*d) + (10*a*b^3*Sin[c + d*x]^3)/(3*d) +
 (2*a*b^3*Sin[c + d*x]^3*Tan[c + d*x]^2)/d
```

3.41.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.41.4 Maple [A] (verified)

Time = 4.71 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{-\frac{a^4\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+4a^3b\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+6a^2b^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}\right.\right.}{}$
default	$\frac{-\frac{a^4\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+4a^3b\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+6a^2b^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}\right.\right.}{}$
risch	$-\frac{3\,e^{i(dx+c)}a^4}{8d}-\frac{11\,e^{i(dx+c)}b^4}{8d}+\frac{e^{-3i(dx+c)}a^4}{24d}+\frac{e^{-3i(dx+c)}b^4}{24d}-\frac{3\,e^{-i(dx+c)}a^4}{8d}-\frac{11\,e^{-i(dx+c)}b^4}{8d}+\frac{e^{3i(dx+c)}}{24d}$

input
```
int(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a^4*(2+sin(d*x+c)^2)*cos(d*x+c)+4*a^3*b*(-1/3*sin(d*x+c)^3-sin(d
*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+6*a^2*b^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+s
in(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+4*a*b^3*(1/2*sin(d*x+c)^7/cos(d*
x+c)^2+1/2*sin(d*x+c)^5+5/6*sin(d*x+c)^3+5/2*sin(d*x+c)-5/2*ln(sec(d*x+c)+
tan(d*x+c)))+b^4*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c
)-5/3*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c)))
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.81

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$=\frac{(a^4-6\,a^2b^2+b^4)\cos{(dx+c)}^6-3\,(a^4-12\,a^2b^2+3\,b^4)\cos{(dx+c)}^4+3\,(2\,a^3b-5\,ab^3)\cos{(dx+c)}^3\log}{}$$

input
```
integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/3*((a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^6 - 3*(a^4 - 12*a^2*b^2 + 3*b^4)
*cos(d*x + c)^4 + 3*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) +
1) - 3*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + b^4 + 9
*(2*a^2*b^2 - b^4)*cos(d*x + c)^2 + 2*(2*(a^3*b - a*b^3)*cos(d*x + c)^5 +
3*a*b^3*cos(d*x + c) - 2*(4*a^3*b - 7*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))
/(d*cos(d*x + c)^3)
```

3.41.6 Sympy [F]

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx = \int (a+b\tan(c+dx))^4\sin^3(c+dx)\,dx$$

input
```
integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**4,x)
```

output
```
Integral((a + b*tan(c + d*x))**4*sin(c + d*x)**3, x)
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.79

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{(\cos(dx+c)^3 - 3\cos(dx+c))a^4 - 2(2\sin(dx+c)^3 - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1}{}$$

input
```
integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/3*((cos(d*x + c)^3 - 3*cos(d*x + c))*a^4 - 2*(2*sin(d*x + c)^3 - 3*log(s
in(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^3*b - 6*(co
s(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^2*b^2 + (4*sin(d*x + c)^
3 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*lo
g(sin(d*x + c) - 1) + 24*sin(d*x + c))*a*b^3 + (cos(d*x + c)^3 - (9*cos(d*
x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x + c))*b^4)/d
```

3.41.8 Giac [F(-1)]

Timed out.

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
Timed out
```

3.41.9 Mupad [B] (verification not implemented)

Time = 8.57 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.16

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^4\,dx =$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(8a^4-96a^2b^2+32b^4)+4a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(20ab^3-8a^3b)-\frac{4a^4}{3}-\frac{32b}{3}}{}$$
$$-\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(20ab^3-8a^3b)}{d}$$

input
```
int(sin(c + d*x)^3*(a + b*tan(c + d*x))^4,x)
```

output
```
- (tan(c/2 + (d*x)/2)^4*(8*a^4 + 32*b^4 - 96*a^2*b^2) + 4*a^4*tan(c/2 + (d
*x)/2)^8 + tan(c/2 + (d*x)/2)*(20*a*b^3 - 8*a^3*b) - (4*a^4)/3 - (32*b^4)/
3 + 32*a^2*b^2 - tan(c/2 + (d*x)/2)^6*((32*a^4)/3 - 64*a^2*b^2) + tan(c/2
+ (d*x)/2)^3*((20*a*b^3)/3 - (8*a^3*b)/3) - tan(c/2 + (d*x)/2)^11*(20*a*b^
3 - 8*a^3*b) - tan(c/2 + (d*x)/2)^9*((20*a*b^3)/3 - (8*a^3*b)/3) - tan(c/2
 + (d*x)/2)^5*(56*a*b^3 - 48*a^3*b) + tan(c/2 + (d*x)/2)^7*(56*a*b^3 - 48*
a^3*b))/(d*(3*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d
*x)/2)^12 - 1)) - (atanh(tan(c/2 + (d*x)/2))*(20*a*b^3 - 8*a^3*b))/d
```

3.42 $\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx$

3.42.1 Optimal result

Integrand size = 21, antiderivative size = 139

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{1}{2}\left(a^4-18a^2b^2+5b^4\right)x - \frac{4ab(a^2-2b^2)\log(\cos(c+dx))}{d} + \frac{b^2(18a^2-5b^2)\tan(c+dx)}{2d} + \frac{4ab^3\tan^2(c+dx)}{d} + \frac{5b^4\tan^3(c+dx)}{6d} - \frac{\cos(c+dx)\sin(c+dx)(a+b\tan(c+dx))^4}{2d}$$

output
```
1/2*(a^4-18*a^2*b^2+5*b^4)*x-4*a*b*(a^2-2*b^2)*ln(cos(d*x+c))/d+1/2*b^2*(1
8*a^2-5*b^2)*tan(d*x+c)/d+4*a*b^3*tan(d*x+c)^2/d+5/6*b^4*tan(d*x+c)^3/d-1/
2*cos(d*x+c)*sin(d*x+c)*(a+b*tan(d*x+c))^4/d
```

3.42.2 Mathematica [A] (verified)

Time = 6.31 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.89

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{b\left(-\frac{(a^4-6a^2b^2+b^4)\arctan(\tan(c+dx))}{2b}+2a(a-b)(a+b)\cos^2(c+dx)+\frac{1}{2}\left(4a^3-8ab^2+\frac{a^4-12a^2b^2+3b^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b\right.}\right.}{}$$

input
```
Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

output
```
(b*(-1/2*((a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/b + 2*a*(a - b)*(a
 + b)*Cos[c + d*x]^2 + ((4*a^3 - 8*a*b^2 + (a^4 - 12*a^2*b^2 + 3*b^4)/Sqrt
[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((4*a^3 - 8*a*b^2 - (a^4 - 1
2*a^2*b^2 + 3*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/2 - ((a^4
 - 6*a^2*b^2 + b^4)*Cos[c + d*x]*Sin[c + d*x])/(2*b) + 2*b*(3*a^2 - b^2)*T
an[c + d*x] + 2*a*b^2*Tan[c + d*x]^2 + (b^3*Tan[c + d*x]^3)/3))/d
```

3.42.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3999, 531, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \sin(c+dx)^2(a+b\tan(c+dx))^4dx$$
$$\Big\downarrow \; 3999$$
$$\frac{b\int \frac{b^2\tan^2(c+dx)(a+b\tan(c+dx))^4}{(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{d}$$
$$\Big\downarrow \; 531$$

$$\frac{b\left(-\frac{\int -\frac{b^2(a+b\tan(c+dx))^3(a+5b\tan(c+dx))}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2}-\frac{b\tan(c+dx)(a+b\tan(c+dx))^4}{2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{b\left(\frac{\int \frac{b^2(a+b\tan(c+dx))^3(a+5b\tan(c+dx))}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2}-\frac{b\tan(c+dx)(a+b\tan(c+dx))^4}{2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{b\left(\frac{1}{2}\int \frac{(a+b\tan(c+dx))^3(a+5b\tan(c+dx))}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))-\frac{b\tan(c+dx)(a+b\tan(c+dx))^4}{2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \ 657$$

$$\frac{b\left(\frac{1}{2}\int \left(18a^2+16b\tan(c+dx)a-5b^2+5b^2\tan^2(c+dx)+\frac{a^4-18b^2a^2+8b(a^2-2b^2)\tan(c+dx)a+5b^4}{\tan^2(c+dx)b^2+b^2}\right)d(b\tan(c+dx))-\right.}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{b\left(\frac{1}{2}\left(b\left(18a^2-5b^2\right)\tan(c+dx)+4a\left(a^2-2b^2\right)\log\left(b^2\tan^2(c+dx)+b^2\right)+\frac{\left(a^4-18a^2b^2+5b^4\right)\arctan(\tan(c+dx))}{b}+8ab^2\right.\right.}{d}$$

input
```
Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

output
```
(b*(-1/2*(b*Tan[c + d*x]*(a + b*Tan[c + d*x])^4)/(b^2 + b^2*Tan[c + d*x]^2
) + (((a^4 - 18*a^2*b^2 + 5*b^4)*ArcTan[Tan[c + d*x]])/b + 4*a*(a^2 - 2*b^
2)*Log[b^2 + b^2*Tan[c + d*x]^2] + b*(18*a^2 - 5*b^2)*Tan[c + d*x] + 8*a*b
^2*Tan[c + d*x]^2 + (5*b^3*Tan[c + d*x]^3)/3)/2))/d
```

3.42.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 531
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomi
alRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
 + b*x^2, x], x, 1]}, Simp[(c + d*x)^n*(a*f - b*e*x)*((a + b*x^2)^(p + 1)/(
2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b
*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*(c + d*x)*Qx - a*d*f*n + b*c*e*(2*p
 + 3) + b*d*e*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGt
Q[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && GtQ[n, 1] && IntegerQ[2*p]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.42.4 Maple [A] (verified)

Time = 3.53 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.80

method	result
derivativedivides	$a^4\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^3b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+6a^2b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx\cdots}{2}\right.\right.$
default	$a^4\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^3b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+6a^2b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx\cdots}{2}\right.\right.$
risch	$\frac{i\mathrm{e}^{2i(dx+c)}b^4}{8d}-\frac{i\mathrm{e}^{-2i(dx+c)}b^4}{8d}+\frac{x\,a^4}{2}-9x\,a^2b^2+\frac{5x\,b^4}{2}+\frac{\mathrm{e}^{2i(dx+c)}a^3b}{2d}-\frac{\mathrm{e}^{2i(dx+c)}a\,b^3}{2d}-8ixa\,b^3+4i\cdots$

input
```
int(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+4*a^3*b*(-1/2*sin(d*x+
c)^2-ln(cos(d*x+c)))+6*a^2*b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*
sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+4*a*b^3*(1/2*sin(d*x+c)^6/cos(d*x+c)
^2+1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(cos(d*x+c)))+b^4*(1/3*sin(d*x+c)^7/c
os(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*x+c)^5+5/4*sin(d*x+c)^3
+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c))
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.34

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{12\,(a^3b-ab^3)\cos{(dx+c)}^5+12\,ab^3\cos{(dx+c)}-24\,(a^3b-2\,ab^3)\cos{(dx+c)}^3\log{(-\cos{(dx+c)})}-3\,(\ldots}{\ldots}$$

input
```
integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/6*(12*(a^3*b - a*b^3)*cos(d*x + c)^5 + 12*a*b^3*cos(d*x + c) - 24*(a^3*b
 - 2*a*b^3)*cos(d*x + c)^3*log(-cos(d*x + c)) - 3*(2*a^3*b - 2*a*b^3 - (a^
4 - 18*a^2*b^2 + 5*b^4)*d*x)*cos(d*x + c)^3 - (3*(a^4 - 6*a^2*b^2 + b^4)*c
os(d*x + c)^4 - 2*b^4 - 2*(18*a^2*b^2 - 7*b^4)*cos(d*x + c)^2)*sin(d*x + c
))/(d*cos(d*x + c)^3)
```

3.42.6 Sympy [F]

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx = \int (a+b\tan{(c+dx)})^4\sin^2{(c+dx)}\,dx$$

input
```
integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**4,x)
```

output
```
Integral((a + b*tan(c + d*x))**4*sin(c + d*x)**2, x)
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.11

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{2\,b^4\tan\left(dx+c\right)^3+12\,ab^3\tan\left(dx+c\right)^2+3\,(a^4-18\,a^2b^2+5\,b^4)(dx+c)+12\,(a^3b-2\,ab^3)\log\left(\tan\left(dx+\right.\right.}{6\,d}$$

input
```
integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/6*(2*b^4*tan(d*x + c)^3 + 12*a*b^3*tan(d*x + c)^2 + 3*(a^4 - 18*a^2*b^2
+ 5*b^4)*(d*x + c) + 12*(a^3*b - 2*a*b^3)*log(tan(d*x + c)^2 + 1) + 12*(3*
a^2*b^2 - b^4)*tan(d*x + c) + 3*(4*a^3*b - 4*a*b^3 - (a^4 - 6*a^2*b^2 + b^
4)*tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

3.42.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3651 vs. 2(131) = 262.

Time = 2.57 (sec) , antiderivative size = 3651, normalized size of antiderivative = 26.27

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/6*(3*a^4*d*x*tan(d*x)^5*tan(c)^5 - 54*a^2*b^2*d*x*tan(d*x)^5*tan(c)^5 +
15*b^4*d*x*tan(d*x)^5*tan(c)^5 - 12*a^3*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*t
an(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan
(d*x)^5*tan(c)^5 + 24*a*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c)
 + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^5*tan(c)
^5 + 3*a^4*d*x*tan(d*x)^5*tan(c)^3 - 54*a^2*b^2*d*x*tan(d*x)^5*tan(c)^3 +
15*b^4*d*x*tan(d*x)^5*tan(c)^3 - 9*a^4*d*x*tan(d*x)^4*tan(c)^4 + 162*a^2*b
^2*d*x*tan(d*x)^4*tan(c)^4 - 45*b^4*d*x*tan(d*x)^4*tan(c)^4 + 3*a^4*d*x*ta
n(d*x)^3*tan(c)^5 - 54*a^2*b^2*d*x*tan(d*x)^3*tan(c)^5 + 15*b^4*d*x*tan(d*
x)^3*tan(c)^5 + 6*a^3*b*tan(d*x)^5*tan(c)^5 + 6*a*b^3*tan(d*x)^5*tan(c)^5
- 12*a^3*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2
*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^5*tan(c)^3 + 24*a*b^3*log
(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + ta
n(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^5*tan(c)^3 + 36*a^3*b*log(4*(tan(d*x)^2
*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan
(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 72*a*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*
tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*ta
n(d*x)^4*tan(c)^4 + 3*a^4*tan(d*x)^5*tan(c)^4 - 54*a^2*b^2*tan(d*x)^5*tan(
c)^4 + 15*b^4*tan(d*x)^5*tan(c)^4 - 12*a^3*b*log(4*(tan(d*x)^2*tan(c)^2 -
2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1...
```

3.42.9 Mupad [B] (verification not implemented)

Time = 4.98 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.16

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= x\left(\frac{a^4}{2}-9a^2b^2+\frac{5b^4}{2}\right)-\frac{\ln\left(\tan(c+dx)^2+1\right)\left(4ab^3-2a^3b\right)}{d}$$
$$-\frac{\cos\left(c+dx\right)^2\left(\tan(c+dx)\left(\frac{a^4}{2}-3a^2b^2+\frac{b^4}{2}\right)+2ab^3-2a^3b\right)}{d}$$
$$+\frac{b^4\tan(c+dx)^3}{3d}-\frac{\tan(c+dx)\left(2b^4-6a^2b^2\right)}{d}+\frac{2ab^3\tan(c+dx)^2}{d}$$

input
```
int(sin(c + d*x)^2*(a + b*tan(c + d*x))^4,x)
```

output
```
x*(a^4/2 + (5*b^4)/2 - 9*a^2*b^2) - (log(tan(c + d*x)^2 + 1)*(4*a*b^3 - 2*
a^3*b))/d - (cos(c + d*x)^2*(tan(c + d*x)*(a^4/2 + b^4/2 - 3*a^2*b^2) + 2*
a*b^3 - 2*a^3*b))/d + (b^4*tan(c + d*x)^3)/(3*d) - (tan(c + d*x)*(2*b^4 -
6*a^2*b^2))/d + (2*a*b^3*tan(c + d*x)^2)/d
```

3.43 $\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx$

3.43.1 Optimal result

Integrand size = 19, antiderivative size = 180

$$\begin{aligned}\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx &= \frac{4a^3b\,\text{arctanh}(\sin(c+dx))}{d} \\ &- \frac{6ab^3\,\text{arctanh}(\sin(c+dx))}{d} - \frac{a^4\cos(c+dx)}{d} \\ &+ \frac{6a^2b^2\cos(c+dx)}{d} - \frac{b^4\cos(c+dx)}{d} \\ &+ \frac{6a^2b^2\sec(c+dx)}{d} - \frac{2b^4\sec(c+dx)}{d} \\ &+ \frac{b^4\sec^3(c+dx)}{3d} - \frac{4a^3b\sin(c+dx)}{d} \\ &+ \frac{6ab^3\sin(c+dx)}{d} + \frac{2ab^3\sin(c+dx)\tan^2(c+dx)}{d}\end{aligned}$$

output
```
4*a^3*b*arctanh(sin(d*x+c))/d-6*a*b^3*arctanh(sin(d*x+c))/d-a^4*cos(d*x+c)
/d+6*a^2*b^2*cos(d*x+c)/d-b^4*cos(d*x+c)/d+6*a^2*b^2*sec(d*x+c)/d-2*b^4*se
c(d*x+c)/d+1/3*b^4*sec(d*x+c)^3/d-4*a^3*b*sin(d*x+c)/d+6*a*b^3*sin(d*x+c)/
d+2*a*b^3*sin(d*x+c)*tan(d*x+c)^2/d
```

3.43.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 383 vs. 2(180) = 360.

Time = 6.41 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.13

$$\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{72a^2b^2-22b^4-12(a^4-6a^2b^2+b^4)\cos(c+dx)-24ab(2a^2-3b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{}$$

input
```
Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

output
```
(72*a^2*b^2 - 22*b^4 - 12*(a^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x] - 24*a*b*(2
*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 24*a*b*(2*a^2 - 3
*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b^3*(12*a + b))/(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^4*Sin[(c + d*x)/2])/(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])^3 + (2*b^2*(36*a^2 - 11*b^2)*Sin[(c + d*x)/2])/(Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b^4*Sin[(c + d*x)/2])/(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])^3 + (b^3*(-12*a + b))/(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2 + (2*b^2*(-36*a^2 + 11*b^2)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]) - 48*a*b*(a^2 - b^2)*Sin[c + d*x])/(12*d)
```

3.43.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sin(c+dx)(a+b\tan(c+dx))^4dx$$
$$\Big\downarrow \text{ 4000}$$

$$\int \left(a^4 \sin(c+dx) + 4a^3 b \sin(c+dx) \tan(c+dx) + 6a^2 b^2 \sin(c+dx) \tan^2(c+dx) + 4ab^3 \sin(c+dx) \tan^3(c+dx)\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4 \cos(c+dx)}{d} + \frac{4a^3 b \operatorname{arctanh}(\sin(c+dx))}{d} - \frac{4a^3 b \sin(c+dx)}{d} + \frac{6a^2 b^2 \cos(c+dx)}{d} + \frac{6a^2 b^2 \sec(c+dx)}{d} - \frac{6ab^3 \operatorname{arctanh}(\sin(c+dx))}{d} + \frac{6ab^3 \sin(c+dx)}{d} + \frac{2ab^3 \sin(c+dx) \tan^2(c+dx)}{d} - \frac{b^4 \cos(c+dx)}{d} + \frac{b^4 \sec^3(c+dx)}{3d} - \frac{2b^4 \sec(c+dx)}{d}$$

input
```
Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

output
```
(4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (6*a*b^3*ArcTanh[Sin[c + d*x]])/d - (a
^4*Cos[c + d*x])/d + (6*a^2*b^2*Cos[c + d*x])/d - (b^4*Cos[c + d*x])/d + (
6*a^2*b^2*Sec[c + d*x])/d - (2*b^4*Sec[c + d*x])/d + (b^4*Sec[c + d*x]^3)/
(3*d) - (4*a^3*b*Sin[c + d*x])/d + (6*a*b^3*Sin[c + d*x])/d + (2*a*b^3*Sin
[c + d*x]*Tan[c + d*x]^2)/d
```

3.43.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.43.4 Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-a^4\cos(dx+c)+4a^3b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+6a^2b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+4a}{}$
default	$\frac{-a^4\cos(dx+c)+4a^3b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+6a^2b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+4a}{}$
risch	$\frac{2ie^{i(dx+c)}a^3b}{d}-\frac{2ie^{i(dx+c)}a\,b^3}{d}-\frac{e^{i(dx+c)}a^4}{2d}+\frac{3\,e^{i(dx+c)}a^2b^2}{d}-\frac{e^{i(dx+c)}b^4}{2d}-\frac{2ie^{-i(dx+c)}a^3b}{d}+\frac{2ie^{-i(dx+c)}c}{d}$

input
```
int(sin(d*x+c)*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^4*cos(d*x+c)+4*a^3*b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+6*a^2
*b^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+4*a*b^3*(1/2*si
n(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(sec(d*x+c)+
tan(d*x+c)))+b^4*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8
/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.98

$$\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx =$$
$$-\frac{3\left(a^4-6\,a^2b^2+b^4\right)\cos\left(dx+c\right)^4-3\left(2\,a^3b-3\,ab^3\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)+3\left(2\,a^3b-3\,ab\right.}{}$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^4 - 3*(2*a^3*b - 3*a*b^3)*cos
(d*x + c)^3*log(sin(d*x + c) + 1) + 3*(2*a^3*b - 3*a*b^3)*cos(d*x + c)^3*l
og(-sin(d*x + c) + 1) - b^4 - 6*(3*a^2*b^2 - b^4)*cos(d*x + c)^2 - 6*(a*b^
3*cos(d*x + c) - 2*(a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*
x + c)^3)
```

3.43.6 Sympy [F]

$$\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx = \int (a+b\tan{(c+dx)})^4 \sin{(c+dx)}\,dx$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))**4,x)
```

output
```
Integral((a + b*tan(c + d*x))**4*sin(c + d*x), x)
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92

$$\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx =$$
$$-\frac{3\,ab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} + 3\,\log\left(\sin\left(dx+c\right)+1\right) - 3\,\log\left(\sin\left(dx+c\right)-1\right) - 4\,\sin\left(dx+c\right)\right) - 18\,a^2b^2\left(\frac{1}{\cos(d\ldots}\right.}{}$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/3*(3*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) +
1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 18*a^2*b^2*(1/cos(d*x + c
) + cos(d*x + c)) + b^4*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x
 + c)) - 6*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*
x + c)) + 3*a^4*cos(d*x + c))/d
```

3.43.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19074 vs. $2(178) = 356$.

Time = 11.22 (sec) , antiderivative size = 19074, normalized size of antiderivative = 105.97

$$\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(6*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/
2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 9*a*b^3*log(2*(tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1
/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) +
1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(
1/2*d*x)^8*tan(1/2*c)^8 - 6*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t
an(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 9
*a*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) -
2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*
x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 3*a^4*tan(1/2*d*x)^8*tan(1/2
*c)^8 - 36*a^2*b^2*tan(1/2*d*x)^8*tan(1/2*c)^8 + 8*b^4*tan(1/2*d*x)^8*tan(
1/2*c)^8 - 12*a^3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*
tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 -
 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 + 18*a*b^3*log...
```

3.43.9 Mupad [B] (verification not implemented)

Time = 8.15 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.49

$$\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx =$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(6\,a^4-48\,a^2\,b^2+\frac{32\,b^4}{3}\right)+2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(12\,a\,b^3-8\,a^3\,b\right)-2\,a^4-\frac{16}{3}}{d\,\left(\tan\right.}$$

$$-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(12\,a\,b^3-8\,a^3\,b\right)}{d}$$

input
```
int(sin(c + d*x)*(a + b*tan(c + d*x))^4,x)
```

output
```
- (tan(c/2 + (d*x)/2)^2*(6*a^4 + (32*b^4)/3 - 48*a^2*b^2) + 2*a^4*tan(c/2
+ (d*x)/2)^6 + tan(c/2 + (d*x)/2)*(12*a*b^3 - 8*a^3*b) - 2*a^4 - (16*b^4)/
3 + 24*a^2*b^2 - tan(c/2 + (d*x)/2)^4*(6*a^4 - 24*a^2*b^2) - tan(c/2 + (d*
x)/2)^7*(12*a*b^3 - 8*a^3*b) - tan(c/2 + (d*x)/2)^3*(20*a*b^3 - 24*a^3*b)
+ tan(c/2 + (d*x)/2)^5*(20*a*b^3 - 24*a^3*b))/(d*(2*tan(c/2 + (d*x)/2)^2 -
 2*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 - 1)) - (atanh(tan(c/2 + (d
*x)/2))*(12*a*b^3 - 8*a^3*b))/d
```

3.43. $\int \sin(c+dx)(a+b\tan(c+dx))^4\,dx$

3.44 $\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx$

3.44.1 Optimal result

Integrand size = 19, antiderivative size = 118

$$\begin{aligned}\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx = &-\frac{a^4\operatorname{arctanh}(\cos(c+dx))}{d}\\&+\frac{4a^3b\operatorname{arctanh}(\sin(c+dx))}{d}\\&-\frac{2ab^3\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{6a^2b^2\sec(c+dx)}{d}\\&-\frac{b^4\sec(c+dx)}{d}+\frac{b^4\sec^3(c+dx)}{3d}\\&+\frac{2ab^3\sec(c+dx)\tan(c+dx)}{d}\end{aligned}$$

output
```
-a^4*arctanh(cos(d*x+c))/d+4*a^3*b*arctanh(sin(d*x+c))/d-2*a*b^3*arctanh(s
in(d*x+c))/d+6*a^2*b^2*sec(d*x+c)/d-b^4*sec(d*x+c)/d+1/3*b^4*sec(d*x+c)^3/
d+2*a*b^3*sec(d*x+c)*tan(d*x+c)/d
```

3.44.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 352 vs. 2(118) = 236.

Time = 6.85 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.98

$$\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{72a^2b^2-10b^4-12a^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-48a^3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+24ab^3\log\left(c\right.}{}$$

input
```
Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

output
```
(72*a^2*b^2 - 10*b^4 - 12*a^4*Log[Cos[(c + d*x)/2]] - 48*a^3*b*Log[Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]] + 24*a*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + 12*a^4*Log[Sin[(c + d*x)/2]] + 48*a^3*b*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]] - 24*a*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (12
*a*b^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + b^4/(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])^2 + 2*b^2*(36*a^2 - b^2 + 2*b^2*Cos[c + d*x] + (36*a^2 -
 5*b^2)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*Sin[(c + d*x)/2]^2 - (12*a*b^3)/(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + b^4/(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^2)/(12*d)
```

3.44.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+b\tan(c+dx))^4}{\sin(c+dx)}dx$$
$$\Big\downarrow \text{4000}$$

$$\int \left(a^4 \csc(c+dx) + 4a^3 b \sec(c+dx) + 6a^2 b^2 \tan(c+dx)\sec(c+dx) + 4ab^3 \tan^2(c+dx)\sec(c+dx) + b^4 \tan^3(c+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4 \text{arctanh}(\cos(c+dx))}{d} + \frac{4a^3 b \text{arctanh}(\sin(c+dx))}{d} + \frac{6a^2 b^2 \sec(c+dx)}{d} - \frac{2ab^3 \text{arctanh}(\sin(c+dx))}{d} + \frac{2ab^3 \tan(c+dx)\sec(c+dx)}{d} + \frac{b^4 \sec^3(c+dx)}{3d} - \frac{b^4 \sec(c+dx)}{d}$$

input
```
Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^4,x]
```

output
```
-((a^4*ArcTanh[Cos[c + d*x]])/d) + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (2*
a*b^3*ArcTanh[Sin[c + d*x]])/d + (6*a^2*b^2*Sec[c + d*x])/d - (b^4*Sec[c +
 d*x])/d + (b^4*Sec[c + d*x]^3)/(3*d) + (2*a*b^3*Sec[c + d*x]*Tan[c + d*x]
)/d
```

3.44.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.44.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{b^4\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+4a\,b^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+}{d}$
default	$\frac{b^4\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+4a\,b^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+}{d}$
risch	$-\frac{2b^2e^{i(dx+c)}\left(-18a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}+6iab\,e^{4i(dx+c)}-36a^2e^{2i(dx+c)}+2b^2e^{2i(dx+c)}-18a^2+3b^2-6iab\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{4a^3}{}$

input
```
int(csc(d*x+c)*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^4*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2
+sin(d*x+c)^2)*cos(d*x+c))+4*a*b^3*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(
d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+6*a^2*b^2/cos(d*x+c)+4*a^3*b*ln(sec(
d*x+c)+tan(d*x+c))+a^4*ln(csc(d*x+c)-cot(d*x+c)))
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.48

$$\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx =$$

$$-\frac{3\,a^4\cos\left(dx+c\right)^3\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-3\,a^4\cos\left(dx+c\right)^3\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-12\,ab^3\cos\left(d\right.}{}$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/6*(3*a^4*cos(d*x + c)^3*log(1/2*cos(d*x + c) + 1/2) - 3*a^4*cos(d*x + c
)^3*log(-1/2*cos(d*x + c) + 1/2) - 12*a*b^3*cos(d*x + c)*sin(d*x + c) - 6*
(2*a^3*b - a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 6*(2*a^3*b - a*b^
3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*b^4 - 6*(6*a^2*b^2 - b^4)*cos
(d*x + c)^2)/(d*cos(d*x + c)^3)
```

3.44.6 Sympy [F]

$$\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx=\int (a+b\tan(c+dx))^4\csc(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c))**4,x)
```

output
```
Integral((a + b*tan(c + d*x))**4*csc(c + d*x), x)
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.18

$$\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx=$$
$$-\frac{3\,ab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)-6\,a^3b(\log(\sin(dx+c)+1)-\log}{3\,d}$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/3*(3*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) - 6*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x +
 c) - 1)) + 3*a^4*log(cot(d*x + c) + csc(d*x + c)) - 18*a^2*b^2/cos(d*x +
c) + (3*cos(d*x + c)^2 - 1)*b^4/cos(d*x + c)^3)/d
```

3.44.8 Giac [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.64

$$\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$=\frac{3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+6\,(2\,a^3b-ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,(2\,a^3b-ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx\right.\right.\right.}{}$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 6*(2*a^3*b - a*b^3)*log(abs(ta
n(1/2*d*x + 1/2*c) + 1)) - 6*(2*a^3*b - a*b^3)*log(abs(tan(1/2*d*x + 1/2*c
) - 1)) + 4*(3*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b^2*tan(1/2*d*x + 1/2*
c)^4 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*b^4*tan(1/2*d*x + 1/2*c)^2 -
3*a*b^3*tan(1/2*d*x + 1/2*c) - 9*a^2*b^2 + b^4)/(tan(1/2*d*x + 1/2*c)^2 -
1)^3)/d
```

3.44.9 Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 496, normalized size of antiderivative = 4.20

$$\int \csc(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$
$$-\frac{12\,a^2\,b^2-\frac{4\,b^4}{3}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(4\,b^4-24\,a^2\,b^2\right)+12\,a^2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4\,a\,b^3\tan}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$
$$-\frac{a\,b\,\mathrm{atan}\left(\frac{a\,b\,\left(2\,a^2-b^2\right)\,\left(4\,a\,b^3-8\,a^3\,b+2\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-b^2\right)\right)\,2\mathrm{i}+a\,b\,\left(2\,a^2-b^2\right)\,\left(\right.}{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(64\,a^6\,b^2-64\,a^4\,b^4+16\,a^2\,b^6\right)+16\,a^7\,b-8\,a^5\,b^3+2\,a\,b\,\left(2\,a^2-b^2\right)\,\left(4\,a\,b^3-8\,a^3\,b+2\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12\,a\,b\tan\left(\frac{c}{2}+\right.\right.}\right.}{d}$$

input
```
int((a + b*tan(c + d*x))^4/sin(c + d*x),x)
```

output
```
(a^4*log(tan(c/2 + (d*x)/2)))/d - (12*a^2*b^2 - (4*b^4)/3 + tan(c/2 + (d*x
)/2)^2*(4*b^4 - 24*a^2*b^2) + 12*a^2*b^2*tan(c/2 + (d*x)/2)^4 + 4*a*b^3*ta
n(c/2 + (d*x)/2) - 4*a*b^3*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^
2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (a*b*atan((a*b*(
2*a^2 - b^2)*(4*a*b^3 - 8*a^3*b + 2*a^4*tan(c/2 + (d*x)/2) - 12*a*b*tan(c/
2 + (d*x)/2)*(2*a^2 - b^2))*2i + a*b*(2*a^2 - b^2)*(4*a*b^3 - 8*a^3*b + 2*
a^4*tan(c/2 + (d*x)/2) + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2))*2i)/(2*t
an(c/2 + (d*x)/2)*(16*a^2*b^6 - 64*a^4*b^4 + 64*a^6*b^2) + 16*a^7*b - 8*a^
5*b^3 + 2*a*b*(2*a^2 - b^2)*(4*a*b^3 - 8*a^3*b + 2*a^4*tan(c/2 + (d*x)/2)
- 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2)) - 2*a*b*(2*a^2 - b^2)*(4*a*b^3
- 8*a^3*b + 2*a^4*tan(c/2 + (d*x)/2) + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 -
b^2))))*(2*a^2 - b^2)*4i)/d
```

3.45 $\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx$

3.45.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{a^4\cot(c+dx)}{d}+\frac{4a^3b\log(\tan(c+dx))}{d}+\frac{6a^2b^2\tan(c+dx)}{d}+\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}$$

output
```
-a^4*cot(d*x+c)/d+4*a^3*b*ln(tan(d*x+c))/d+6*a^2*b^2*tan(d*x+c)/d+2*a*b^3*
tan(d*x+c)^2/d+1/3*b^4*tan(d*x+c)^3/d
```

3.45.2 Mathematica [A] (verified)

Time = 3.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.95

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{\csc(c+dx)\sec^3(c+dx)\left(4(3a^4+b^4)\cos(2(c+dx))+(3a^4+18a^2b^2-b^4)\cos(4(c+dx))+3(3a^4-6a\right.}{}$$

input
```
Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

output
```
-1/24*(Csc[c + d*x]*Sec[c + d*x]^3*(4*(3*a^4 + b^4)*Cos[2*(c + d*x)] + (3*
a^4 + 18*a^2*b^2 - b^4)*Cos[4*(c + d*x)] + 3*(3*a^4 - 6*a^2*b^2 - b^4 + 8*
a*b*(-b^2 + a^2*Log[Cos[c + d*x]] - a^2*Log[Sin[c + d*x]])*Sin[2*(c + d*x)
] + 4*a^3*b*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])*Sin[4*(c + d*x)])))/d
```

3.45.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3999, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^4}{\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3999}$$

$$\frac{b\int \frac{\cot^2(c+dx)(a+b\tan(c+dx))^4}{b^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{49}$$

$$\frac{b\int \left(\frac{\cot^2(c+dx)a^4}{b^2}+\frac{4\cot(c+dx)a^3}{b}+6a^2+4b\tan(c+dx)a+b^2\tan^2(c+dx)\right)d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{a^4\cot(c+dx)}{b}+4a^3\log(b\tan(c+dx))+6a^2b\tan(c+dx)+2ab^2\tan^2(c+dx)+\frac{1}{3}b^3\tan^3(c+dx)\right)}{d}$$

input
```
Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^4,x]
```

output
```
(b*(-((a^4*Cot[c + d*x])/b) + 4*a^3*Log[b*Tan[c + d*x]] + 6*a^2*b*Tan[c +
d*x] + 2*a*b^2*Tan[c + d*x]^2 + (b^3*Tan[c + d*x]^3)/3))/d
```

3.45.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.45.4 Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{b^4\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{2a\,b^3}{\cos(dx+c)^2}+6a^2b^2\tan(dx+c)+4a^3b\ln(\tan(dx+c))-a^4\cot(dx+c)}{d}$
default	$\frac{\frac{b^4\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{2a\,b^3}{\cos(dx+c)^2}+6a^2b^2\tan(dx+c)+4a^3b\ln(\tan(dx+c))-a^4\cot(dx+c)}{d}$
risch	$-\frac{2i\left(12ia\,b^3e^{6i(dx+c)}+3a^4e^{6i(dx+c)}-18a^2b^2e^{6i(dx+c)}+3b^4e^{6i(dx+c)}+9a^4e^{4i(dx+c)}-18a^2b^2e^{4i(dx+c)}-3b^4e^{4i(dx+c)}-1\right.}{3d\left(e^{2i(dx+c)}+1\right)^3\left(e^{2i(dx+c)}-1\right)}$

input
```
int(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*b^4*sin(d*x+c)^3/cos(d*x+c)^3+2*a*b^3/cos(d*x+c)^2+6*a^2*b^2*tan(
d*x+c)+4*a^3*b*ln(tan(d*x+c))-a^4*cot(d*x+c))
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.92

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{6\,a^3b\cos(dx+c)^3\log\left(\cos(dx+c)^2\right)\sin(dx+c) - 6\,a^3b\cos(dx+c)^3\log\left(-\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\right)\sin(\text{[...]}}{3\,d\cos(dx\text{[...]}}$$

input
```
integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/3*(6*a^3*b*cos(d*x + c)^3*log(cos(d*x + c)^2)*sin(d*x + c) - 6*a^3*b*co
s(d*x + c)^3*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 6*a*b^3*cos(d*x
 + c)*sin(d*x + c) + (3*a^4 + 18*a^2*b^2 - b^4)*cos(d*x + c)^4 - b^4 - 2*(
9*a^2*b^2 - b^4)*cos(d*x + c)^2)/(d*cos(d*x + c)^3*sin(d*x + c))
```

3.45.6 Sympy [F]

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx = \int (a+b\tan(c+dx))^4\csc^2(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**4,x)
```

output
```
Integral((a + b*tan(c + d*x))**4*csc(c + d*x)**2, x)
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{b^4\tan(dx+c)^3 + 6\,ab^3\tan(dx+c)^2 + 12\,a^3b\log(\tan(dx+c)) + 18\,a^2b^2\tan(dx+c) - \frac{3\,a^4}{\tan(dx+c)}}{3\,d}$$

input
```
integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 12*a^3*b*log(tan(d*x +
c)) + 18*a^2*b^2*tan(d*x + c) - 3*a^4/tan(d*x + c))/d
```

3.45.8 Giac [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{b^4\tan{(dx+c)}^3+6\,ab^3\tan{(dx+c)}^2+12\,a^3b\log{(|\tan{(dx+c)}|)}+18\,a^2b^2\tan{(dx+c)}-\frac{3\,(4\,a^3b\tan(dx+c)+}{\tan(dx+c)}}{3\,d}$$

input
```
integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 12*a^3*b*log(abs(tan(d*
x + c))) + 18*a^2*b^2*tan(d*x + c) - 3*(4*a^3*b*tan(d*x + c) + a^4)/tan(d*
x + c))/d
```

3.45.9 Mupad [B] (verification not implemented)

Time = 4.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{b^4\tan(c+d\,x)^3}{3\,d}-\frac{a^4\cot(c+d\,x)}{d}+\frac{6\,a^2\,b^2\tan(c+d\,x)}{d}+\frac{2\,a\,b^3\tan(c+d\,x)^2}{d}+\frac{4\,a^3\,b\,\ln{(\tan(c+d\,x))}}{d}$$

input
```
int((a + b*tan(c + d*x))^4/sin(c + d*x)^2,x)
```

output
```
(b^4*tan(c + d*x)^3)/(3*d) - (a^4*cot(c + d*x))/d + (6*a^2*b^2*tan(c + d*x
))/d + (2*a*b^3*tan(c + d*x)^2)/d + (4*a^3*b*log(tan(c + d*x)))/d
```

3.46 $\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx$

3.46.1 Optimal result

Integrand size = 21, antiderivative size = 161

$$\begin{aligned}\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx = & -\frac{a^4\text{arctanh}(\cos(c+dx))}{2d} \\ & -\frac{6a^2b^2\text{arctanh}(\cos(c+dx))}{d} \\ & +\frac{4a^3b\text{arctanh}(\sin(c+dx))}{d} \\ & +\frac{2ab^3\text{arctanh}(\sin(c+dx))}{d}-\frac{4a^3b\csc(c+dx)}{d} \\ & -\frac{a^4\cot(c+dx)\csc(c+dx)}{2d}+\frac{6a^2b^2\sec(c+dx)}{d} \\ & +\frac{b^4\sec^3(c+dx)}{3d}+\frac{2ab^3\sec(c+dx)\tan(c+dx)}{d}\end{aligned}$$

output
```
-1/2*a^4*arctanh(cos(d*x+c))/d-6*a^2*b^2*arctanh(cos(d*x+c))/d+4*a^3*b*arc
tanh(sin(d*x+c))/d+2*a*b^3*arctanh(sin(d*x+c))/d-4*a^3*b*csc(d*x+c)/d-1/2*
a^4*cot(d*x+c)*csc(d*x+c)/d+6*a^2*b^2*sec(d*x+c)/d+1/3*b^4*sec(d*x+c)^3/d+
2*a*b^3*sec(d*x+c)*tan(d*x+c)/d
```

3.46.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1128 vs. $2(161) = 322$.

Time = 8.04 (sec) , antiderivative size = 1128, normalized size of antiderivative = 7.01

$$\begin{aligned}
&\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx \\
&= \frac{b^2(36a^2+b^2)\cos^4(c+dx)(a+b\tan(c+dx))^4}{6d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&- \frac{2a^3b\cos^4(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&- \frac{a^4\cos^4(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{8d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{(-a^4-12a^2b^2)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{2d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&- \frac{2(2a^3b+ab^3)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{(a^4+12a^2b^2)\cos^4(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{2d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{2(2a^3b+ab^3)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{a^4\cos^4(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{8d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{(12ab^3+b^4)\cos^4(c+dx)(a+b\tan(c+dx))^4}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{b^4\cos^4(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a\cos(c+dx)+b\sin(c+dx))^4} \\
&- \frac{b^4\cos^4(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{(-12ab^3+b^4)\cos^4(c+dx)(a+b\tan(c+dx))^4}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{\cos^4(c+dx)\left(-36a^2b^2\sin\left(\frac{1}{2}(c+dx)\right)-b^4\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{\cos^4(c+dx)\left(36a^2b^2\sin\left(\frac{1}{2}(c+dx)\right)+b^4\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^4} \\
&- \frac{2a^3b\cos^4(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4}
\end{aligned}$$

input
```
Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

output
```
(b^2*(36*a^2 + b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(6*d*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^4) - (2*a^3*b*Cos[c + d*x]^4*Cot[(c + d*x)/2]*(a +
 b*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a^4*Cos[c +
 d*x]^4*Csc[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] +
b*Sin[c + d*x])^4) + ((-a^4 - 12*a^2*b^2)*Cos[c + d*x]^4*Log[Cos[(c + d*x)
/2]]*(a + b*Tan[c + d*x])^4)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (
2*(2*a^3*b + a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
]*(a + b*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((a^4
+ 12*a^2*b^2)*Cos[c + d*x]^4*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)
/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b + a*b^3)*Cos[c +
d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d
*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (a^4*Cos[c + d*x]^4*Sec[(c + d*x)/
2]^2*(a + b*Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (
(12*a*b^3 + b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Co
s[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c
 + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(6*d*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-12*a*b^3 +
 b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(12*d*(Cos[(c + d*x)/2] + ...
```

3.46.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^4}{\sin(c+dx)^3}dx$$

$$\Big\downarrow \text{4000}$$

$$\int \left(a^4 \csc^3(c+dx) + 4a^3 b \csc^2(c+dx) \sec(c+dx) + 6a^2 b^2 \csc(c+dx) \sec^2(c+dx) + 4ab^3 \sec^3(c+dx) + b^4 \tan(c$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^4 \text{arctanh}(\cos(c+dx))}{2d} - \frac{a^4 \cot(c+dx)\csc(c+dx)}{2d} + \frac{4a^3 b \text{arctanh}(\sin(c+dx))}{d} - \frac{4a^3 b \csc(c+dx)}{d} - \frac{6a^2 b^2 \text{arctanh}(\cos(c+dx))}{d} + \frac{6a^2 b^2 \sec(c+dx)}{d} + \frac{2ab^3 \text{arctanh}(\sin(c+dx))}{d} + \frac{2ab^3 \tan(c+dx)\sec(c+dx)}{d} + \frac{b^4 \sec^3(c+dx)}{3d}$$

input
```
Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^4,x]
```

output
```
-1/2*(a^4*ArcTanh[Cos[c + d*x]])/d - (6*a^2*b^2*ArcTanh[Cos[c + d*x]])/d +
 (4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (2*a*b^3*ArcTanh[Sin[c + d*x]])/d - (
4*a^3*b*Csc[c + d*x])/d - (a^4*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (6*a^2*b
^2*Sec[c + d*x])/d + (b^4*Sec[c + d*x]^3)/(3*d) + (2*a*b^3*Sec[c + d*x]*Ta
n[c + d*x])/d
```

3.46.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.46.4 Maple [A] (verified)

Time = 6.39 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{\frac{b^4}{3\cos(dx+c)^3}+4a\,b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+6a^2b^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+4a}{d}$
default	$\frac{\frac{b^4}{3\cos(dx+c)^3}+4a\,b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+6a^2b^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+4a}{d}$
risch	$-\frac{ie^{i(dx+c)}\left(8ib^4\mathrm{e}^{6i(dx+c)}+8ib^4\mathrm{e}^{2i(dx+c)}+24a^3b\,\mathrm{e}^{8i(dx+c)}+12a\,b^3\mathrm{e}^{8i(dx+c)}+36ia^2b^2-16ib^4\mathrm{e}^{4i(dx+c)}+48a^3b\,\mathrm{e}^{6i(dx+c)}\right.}{}$

input
```
int(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*b^4/cos(d*x+c)^3+4*a*b^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*
x+c)+tan(d*x+c)))+6*a^2*b^2*(1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+4*a^3
*b*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^4*(-1/2*csc(d*x+c)*cot(d*x+
c)+1/2*ln(csc(d*x+c)-cot(d*x+c))))
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(155) = 310.

Time = 0.33 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.15

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx$$

$$=\frac{6\left(a^4+12\,a^2b^2\right)\cos\left(dx+c\right)^4-4\,b^4-4\left(18\,a^2b^2-b^4\right)\cos\left(dx+c\right)^2-3\left(\left(a^4+12\,a^2b^2\right)\cos\left(dx+c\right)^5-\left(a^4\right.\right.}{}$$

input
```
integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/12*(6*(a^4 + 12*a^2*b^2)*cos(d*x + c)^4 - 4*b^4 - 4*(18*a^2*b^2 - b^4)*c
os(d*x + c)^2 - 3*((a^4 + 12*a^2*b^2)*cos(d*x + c)^5 - (a^4 + 12*a^2*b^2)*
cos(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) + 3*((a^4 + 12*a^2*b^2)*cos(d*
x + c)^5 - (a^4 + 12*a^2*b^2)*cos(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2)
 + 12*((2*a^3*b + a*b^3)*cos(d*x + c)^5 - (2*a^3*b + a*b^3)*cos(d*x + c)^3
)*log(sin(d*x + c) + 1) - 12*((2*a^3*b + a*b^3)*cos(d*x + c)^5 - (2*a^3*b
+ a*b^3)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 24*(a*b^3*cos(d*x + c) -
 (2*a^3*b + a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5 - d*cos
(d*x + c)^3)
```

3.46.6 Sympy [F]

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx = \int (a+b\tan(c+dx))^4\csc^3(c+dx)\,dx$$

input
```
integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**4,x)
```

output
```
Integral((a + b*tan(c + d*x))**4*csc(c + d*x)**3, x)
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.17

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{3a^4\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-12ab^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c\right.}{}$$

input
```
integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/12*(3*a^4*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) +
 log(cos(d*x + c) - 1)) - 12*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) -
log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 36*a^2*b^2*(2/cos(d*x + c
) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 24*a^3*b*(2/sin(d*x +
 c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*b^4/cos(d*x + c)^
3)/d
```

3.46.8 Giac [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.86

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{3a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-48a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+48(2a^3b+ab^3)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-48(2a^3b}{}$$

input
```
integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/24*(3*a^4*tan(1/2*d*x + 1/2*c)^2 - 48*a^3*b*tan(1/2*d*x + 1/2*c) + 48*(2
*a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 48*(2*a^3*b + a*b^3)*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 12*(a^4 + 12*a^2*b^2)*log(abs(tan(1/2
*d*x + 1/2*c))) - 3*(6*a^4*tan(1/2*d*x + 1/2*c)^2 + 72*a^2*b^2*tan(1/2*d*x
 + 1/2*c)^2 + 16*a^3*b*tan(1/2*d*x + 1/2*c) + a^4)/tan(1/2*d*x + 1/2*c)^2
+ 16*(6*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 -
 3*b^4*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*b^
3*tan(1/2*d*x + 1/2*c) - 18*a^2*b^2 - b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)
/d
```

3.46.9 Mupad [B] (verification not implemented)

Time = 5.10 (sec) , antiderivative size = 670, normalized size of antiderivative = 4.16

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}$$
$$-\frac{\frac{a^4}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{3\,a^4}{2}+48\,a^2\,b^2+\frac{8\,b^4}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{a^4}{2}+48\,a^2\,b^2+8\,b^4\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{3\,a^4}{2}\right.}{d\,\left(-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+12\tan\right.}$$
$$+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{a^4}{2}+6\,a^2\,b^2\right)}{d}-\frac{2\,a^3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$
$$-\frac{a\,b\,\mathrm{atan}\left(\frac{a\,b\,\left(2\,a^2+b^2\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^4+12\,a^2\,b^2\right)-4\,a\,b^3-8\,a^3\,b+12\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2+b^2\right)\right)\,2\mathrm{i}-}{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(64\,a^6\,b^2+64\,a^4\,b^4+16\,a^2\,b^6\right)+8\,a^7\,b+48\,a^3\,b^5+100\,a^5\,b^3-2\,a\,b\,\left(2\,a^2+b^2\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^4+12\,a^2\,b^2\right)-4\,a\,b^3-8\,a\right.}\right.}{}$$

input
```
int((a + b*tan(c + d*x))^4/sin(c + d*x)^3,x)
```

output
```
(a^4*tan(c/2 + (d*x)/2)^2)/(8*d) - (a^4/2 - tan(c/2 + (d*x)/2)^2*((3*a^4)/
2 + (8*b^4)/3 + 48*a^2*b^2) - tan(c/2 + (d*x)/2)^6*(a^4/2 + 8*b^4 + 48*a^2
*b^2) + tan(c/2 + (d*x)/2)^4*((3*a^4)/2 + 96*a^2*b^2) + tan(c/2 + (d*x)/2)
^7*(16*a*b^3 - 8*a^3*b) - tan(c/2 + (d*x)/2)^3*(16*a*b^3 + 24*a^3*b) + 8*a
^3*b*tan(c/2 + (d*x)/2) + 24*a^3*b*tan(c/2 + (d*x)/2)^5)/(d*(4*tan(c/2 + (
d*x)/2)^2 - 12*tan(c/2 + (d*x)/2)^4 + 12*tan(c/2 + (d*x)/2)^6 - 4*tan(c/2
+ (d*x)/2)^8)) + (log(tan(c/2 + (d*x)/2))*(a^4/2 + 6*a^2*b^2))/d - (2*a^3*
b*tan(c/2 + (d*x)/2))/d - (a*b*atan((a*b*(2*a^2 + b^2)*(tan(c/2 + (d*x)/2)
*(a^4 + 12*a^2*b^2) - 4*a*b^3 - 8*a^3*b + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2
 + b^2))*2i - a*b*(2*a^2 + b^2)*(4*a*b^3 - tan(c/2 + (d*x)/2)*(a^4 + 12*a^
2*b^2) + 8*a^3*b + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 + b^2))*2i)/(2*tan(c/2
 + (d*x)/2)*(16*a^2*b^6 + 64*a^4*b^4 + 64*a^6*b^2) + 8*a^7*b + 48*a^3*b^5
+ 100*a^5*b^3 - 2*a*b*(2*a^2 + b^2)*(tan(c/2 + (d*x)/2)*(a^4 + 12*a^2*b^2)
 - 4*a*b^3 - 8*a^3*b + 12*a*b*tan(c/2 + (d*x)/2)*(2*a^2 + b^2)) - 2*a*b*(2
*a^2 + b^2)*(4*a*b^3 - tan(c/2 + (d*x)/2)*(a^4 + 12*a^2*b^2) + 8*a^3*b + 1
2*a*b*tan(c/2 + (d*x)/2)*(2*a^2 + b^2))))*(2*a^2 + b^2)*4i)/d
```

3.46. $\int \csc^3(c+dx)(a+b\tan(c+dx))^4\,dx$

3.47 $\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx$

3.47.1 Optimal result

Integrand size = 21, antiderivative size = 137

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{a^2(a^2+6b^2)\cot(c+dx)}{d}-\frac{2a^3b\cot^2(c+dx)}{d} -\frac{a^4\cot^3(c+dx)}{3d}+\frac{4ab(a^2+b^2)\log(\tan(c+dx))}{d} +\frac{b^2(6a^2+b^2)\tan(c+dx)}{d} +\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}$$

output
```
-a^2*(a^2+6*b^2)*cot(d*x+c)/d-2*a^3*b*cot(d*x+c)^2/d-1/3*a^4*cot(d*x+c)^3/
d+4*a*b*(a^2+b^2)*ln(tan(d*x+c))/d+b^2*(6*a^2+b^2)*tan(d*x+c)/d+2*a*b^3*ta
n(d*x+c)^2/d+1/3*b^4*tan(d*x+c)^3/d
```

3.47.2 Mathematica [A] (verified)

Time = 5.98 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.37

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{(b+a\cot(c+dx))^4\sin(c+dx)\left(\cos(c+dx)\left(-6ab^3+6a^3b\cot^2(c+dx)+a^4\cot^3(c+dx)\right)+2a\cos^3(c\right.}{}$$

input
```
Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^4,x]
```

output
```
-1/3*((b + a*Cot[c + d*x])^4*Sin[c + d*x]*(Cos[c + d*x]*(-6*a*b^3 + 6*a^3*
b*Cot[c + d*x]^2 + a^4*Cot[c + d*x]^3) + 2*a*Cos[c + d*x]^3*(a*(a^2 + 9*b^
2)*Cot[c + d*x] + 6*b*(a^2 + b^2)*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]]))
 - b^4*Sin[c + d*x] - 2*b^2*(9*a^2 + b^2)*Cos[c + d*x]^2*Sin[c + d*x])*Tan
[c + d*x]^3)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

3.47.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3999, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^4}{\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{3999}$$

$$\frac{b\int \frac{\cot^4(c+dx)(a+b\tan(c+dx))^4(\tan^2(c+dx)b^2+b^2)}{b^4}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{b\int \left(\frac{a^4\cot^4(c+dx)}{b^2}+\frac{4a^3\cot^3(c+dx)}{b}+\frac{(a^4+6b^2a^2)\cot^2(c+dx)}{b^2}+\frac{4a(a^2+b^2)\cot(c+dx)}{b}+b^2\tan^2(c+dx)+6a^2\left(\frac{b^2}{6a^2}+1\right)+4a$$

$$d$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{a^4\cot^3(c+dx)}{3b}-2a^3\cot^2(c+dx)+b(6a^2+b^2)\tan(c+dx)-\frac{a^2(a^2+6b^2)\cot(c+dx)}{b}+4a(a^2+b^2)\log(b\tan(c+da$$

$$d$$

input
```
Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^4,x]
```

output
```
(b*(-((a^2*(a^2 + 6*b^2)*Cot[c + d*x])/b) - 2*a^3*Cot[c + d*x]^2 - (a^4*Co
t[c + d*x]^3)/(3*b) + 4*a*(a^2 + b^2)*Log[b*Tan[c + d*x]] + b*(6*a^2 + b^2
)*Tan[c + d*x] + 2*a*b^2*Tan[c + d*x]^2 + (b^3*Tan[c + d*x]^3)/3))/d
```

3.47.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.47.4 Maple [A] (verified)

Time = 11.77 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{-b^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+4a\,b^3\left(\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+6a^2b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)}{d}$
default	$\frac{-b^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+4a\,b^3\left(\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+6a^2b^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)}{d}$
risch	$8a^3b\,e^{10i(dx+c)}+8a\,b^3e^{10i(dx+c)}-\frac{32ib^4e^{6i(dx+c)}}{3}-\frac{4ib^4}{3}-24ia^2b^2+16a^3b\,e^{8i(dx+c)}-16a\,b^3e^{8i(dx+c)}+8ib^4e^{4i(dx+c)}$

input
```
int(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+4*a*b^3*(1/2/cos(d*x+c)^2+ln(
tan(d*x+c)))+6*a^2*b^2*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+4*a^3*b*(-1/
2/sin(d*x+c)^2+ln(tan(d*x+c)))+a^4*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c))
```

3.47.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(133) = 266$.

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.95

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx =$$
$$-\frac{2\,(a^4+18\,a^2b^2+b^4)\cos{(dx+c)}^6+18\,a^2b^2\cos{(dx+c)}^2-3\,(a^4+18\,a^2b^2+b^4)\cos{(dx+c)}^4+b^4+6}{}$$

input
```
integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/3*(2*(a^4 + 18*a^2*b^2 + b^4)*cos(d*x + c)^6 + 18*a^2*b^2*cos(d*x + c)^
2 - 3*(a^4 + 18*a^2*b^2 + b^4)*cos(d*x + c)^4 + b^4 + 6*((a^3*b + a*b^3)*c
os(d*x + c)^5 - (a^3*b + a*b^3)*cos(d*x + c)^3)*log(cos(d*x + c)^2)*sin(d*
x + c) - 6*((a^3*b + a*b^3)*cos(d*x + c)^5 - (a^3*b + a*b^3)*cos(d*x + c)^
3)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) + 6*(a*b^3*cos(d*x + c) - (
a^3*b + a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/((d*cos(d*x + c)^5 - d*cos(d*
x + c)^3)*sin(d*x + c))
```

3.47.6 Sympy [F]

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx = \int (a+b\tan{(c+dx)})^4\csc^4{(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**4,x)
```

output
```
Integral((a + b*tan(c + d*x))**4*csc(c + d*x)**4, x)
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{b^4\tan\left(dx+c\right)^3 + 6\,ab^3\tan\left(dx+c\right)^2 + 12\left(a^3b+ab^3\right)\log\left(\tan\left(dx+c\right)\right) + 3\left(6\,a^2b^2+b^4\right)\tan\left(dx+c\right) - \frac{6}{}}{3\,d}$$

input
```
integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 12*(a^3*b + a*b^3)*log(
tan(d*x + c)) + 3*(6*a^2*b^2 + b^4)*tan(d*x + c) - (6*a^3*b*tan(d*x + c) +
 a^4 + 3*(a^4 + 6*a^2*b^2)*tan(d*x + c)^2)/tan(d*x + c)^3)/d
```

3.47.8 Giac [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{b^4\tan\left(dx+c\right)^3 + 6\,ab^3\tan\left(dx+c\right)^2 + 18\,a^2b^2\tan\left(dx+c\right) + 3\,b^4\tan\left(dx+c\right) + 12\left(a^3b+ab^3\right)\log\left(|\tan\left(\right.\right.}{3\,d}$$

input
```
integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 18*a^2*b^2*tan(d*x + c)
 + 3*b^4*tan(d*x + c) + 12*(a^3*b + a*b^3)*log(abs(tan(d*x + c))) - (22*a^
3*b*tan(d*x + c)^3 + 22*a*b^3*tan(d*x + c)^3 + 3*a^4*tan(d*x + c)^2 + 18*a
^2*b^2*tan(d*x + c)^2 + 6*a^3*b*tan(d*x + c) + a^4)/tan(d*x + c)^3)/d
```

3.47.9 Mupad [B] (verification not implemented)

Time = 4.78 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$=\frac{\ln(\tan(c+d\,x))\,(4\,a^3\,b+4\,a\,b^3)}{d}-\frac{\cot(c+d\,x)^3\left(\tan(c+d\,x)^2\,(a^4+6\,a^2\,b^2)+\frac{a^4}{3}+2\,a^3\,b\tan(c+d\,x)\right)}{d}+\frac{\tan(c+d\,x)\,(6\,a^2\,b^2+b^4)}{d}+\frac{b^4\tan(c+d\,x)^3}{3\,d}+\frac{2\,a\,b^3\tan(c+d\,x)^2}{d}$$

input
```
int((a + b*tan(c + d*x))^4/sin(c + d*x)^4,x)
```

output
```
(log(tan(c + d*x))*(4*a*b^3 + 4*a^3*b))/d - (cot(c + d*x)^3*(tan(c + d*x)^
2*(a^4 + 6*a^2*b^2) + a^4/3 + 2*a^3*b*tan(c + d*x)))/d + (tan(c + d*x)*(b^
4 + 6*a^2*b^2))/d + (b^4*tan(c + d*x)^3)/(3*d) + (2*a*b^3*tan(c + d*x)^2)/
d
```

3.48 $\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx$

3.48.1 Optimal result

Integrand size = 21, antiderivative size = 274

$$\begin{aligned}\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx = &-\frac{3a^4\text{arctanh}(\cos(c+dx))}{8d} \\ &-\frac{9a^2b^2\text{arctanh}(\cos(c+dx))}{d} \\ &-\frac{b^4\text{arctanh}(\cos(c+dx))}{d} \\ &+\frac{4a^3b\text{arctanh}(\sin(c+dx))}{d} \\ &+\frac{6ab^3\text{arctanh}(\sin(c+dx))}{d}-\frac{4a^3b\csc(c+dx)}{d} \\ &-\frac{6ab^3\csc(c+dx)}{d}-\frac{3a^4\cot(c+dx)\csc(c+dx)}{8d} \\ &-\frac{4a^3b\csc^3(c+dx)}{3d}-\frac{a^4\cot(c+dx)\csc^3(c+dx)}{4d} \\ &+\frac{9a^2b^2\sec(c+dx)}{d}+\frac{b^4\sec(c+dx)}{d} \\ &-\frac{3a^2b^2\csc^2(c+dx)\sec(c+dx)}{d} \\ &+\frac{2ab^3\csc(c+dx)\sec^2(c+dx)}{d}+\frac{b^4\sec^3(c+dx)}{3d}\end{aligned}$$

output
```
-3/8*a^4*arctanh(cos(d*x+c))/d-9*a^2*b^2*arctanh(cos(d*x+c))/d-b^4*arctanh
(cos(d*x+c))/d+4*a^3*b*arctanh(sin(d*x+c))/d+6*a*b^3*arctanh(sin(d*x+c))/d
-4*a^3*b*csc(d*x+c)/d-6*a*b^3*csc(d*x+c)/d-3/8*a^4*cot(d*x+c)*csc(d*x+c)/d
-4/3*a^3*b*csc(d*x+c)^3/d-1/4*a^4*cot(d*x+c)*csc(d*x+c)^3/d+9*a^2*b^2*sec(
d*x+c)/d+b^4*sec(d*x+c)/d-3*a^2*b^2*csc(d*x+c)^2*sec(d*x+c)/d+2*a*b^3*csc(
d*x+c)*sec(d*x+c)^2/d+1/3*b^4*sec(d*x+c)^3/d
```

3.48.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1491 vs. $2(274) = 548$.

Time = 8.07 (sec) , antiderivative size = 1491, normalized size of antiderivative = 5.44

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx = \text{Too large to display}$$

input
```
Integrate[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^4,x]
```

output
```
(b^2*(36*a^2 + 7*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(6*d*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^4) + ((-7*a^3*b*Cos[(c + d*x)/2] - 6*a*b^3*Cos[(
c + d*x)/2])*Cos[c + d*x]^4*Csc[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(3*d*
(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (3*(a^4 + 8*a^2*b^2)*Cos[c + d*x]^4
*Csc[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^4)/(32*d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^4) - (a^3*b*Cos[c + d*x]^4*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(
a + b*Tan[c + d*x])^4)/(6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a^4*Co
s[c + d*x]^4*Csc[(c + d*x)/2]^4*(a + b*Tan[c + d*x])^4)/(64*d*(a*Cos[c + d
*x] + b*Sin[c + d*x])^4) + ((-3*a^4 - 72*a^2*b^2 - 8*b^4)*Cos[c + d*x]^4*L
og[Cos[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^4) - (2*(2*a^3*b + 3*a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^4) + ((3*a^4 + 72*a^2*b^2 + 8*b^4)*Cos[c + d*x]^4*Log[Sin[(c + d*x)
/2]]*(a + b*Tan[c + d*x])^4)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (
2*(2*a^3*b + 3*a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]]*(a + b*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (3*(
a^4 + 8*a^2*b^2)*Cos[c + d*x]^4*Sec[(c + d*x)/2]^2*(a + b*Tan[c + d*x])^4)
/(32*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (a^4*Cos[c + d*x]^4*Sec[(c +
 d*x)/2]^4*(a + b*Tan[c + d*x])^4)/(64*d*(a*Cos[c + d*x] + b*Sin[c + d*x])
^4) + ((12*a*b^3 + b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(12*d*(C...
```

3.48.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^4}{\sin(c+dx)^5}dx$$

$$\Big\downarrow \text{4000}$$

$$\int \left(a^4\csc^5(c+dx)+4a^3b\csc^4(c+dx)\sec(c+dx)+6a^2b^2\csc^3(c+dx)\sec^2(c+dx)+4ab^3\csc^2(c+dx)\sec^3(c+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3a^4\text{arctanh}(\cos(c+dx))}{8d}-\frac{a^4\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^4\cot(c+dx)\csc(c+dx)}{8d}+\frac{4a^3b\text{arctanh}(\sin(c+dx))}{d}-\frac{4a^3b\csc^3(c+dx)}{3d}-\frac{4a^3b\csc(c+dx)}{d}-\frac{9a^2b^2\text{arctanh}(\cos(c+dx))}{d}+\frac{9a^2b^2\sec(c+dx)}{d}-\frac{3a^2b^2\csc^2(c+dx)\sec(c+dx)}{d}+\frac{6ab^3\text{arctanh}(\sin(c+dx))}{d}-\frac{6ab^3\csc(c+dx)}{d}+\frac{2ab^3\csc(c+dx)\sec^2(c+dx)}{d}-\frac{b^4\text{arctanh}(\cos(c+dx))}{d}+\frac{b^4\sec^3(c+dx)}{3d}+\frac{b^4\sec(c+dx)}{d}$$

input
```
Int[Csc[c + d*x]^5*(a + b*Tan[c + d*x])^4,x]
```

output
```
(-3*a^4*ArcTanh[Cos[c + d*x]])/(8*d) - (9*a^2*b^2*ArcTanh[Cos[c + d*x]])/d
 - (b^4*ArcTanh[Cos[c + d*x]])/d + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d + (6*
a*b^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*b*Csc[c + d*x])/d - (6*a*b^3*Csc[c
 + d*x])/d - (3*a^4*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (4*a^3*b*Csc[c + d*
x]^3)/(3*d) - (a^4*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d) + (9*a^2*b^2*Sec[c +
 d*x])/d + (b^4*Sec[c + d*x])/d - (3*a^2*b^2*Csc[c + d*x]^2*Sec[c + d*x])/
d + (2*a*b^3*Csc[c + d*x]*Sec[c + d*x]^2)/d + (b^4*Sec[c + d*x]^3)/(3*d)
```

3.48.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.48.4 Maple [A] (verified)

Time = 17.35 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{b^4\left(\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+4a\,b^3\left(\frac{1}{2\sin(dx+c)\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)}+\frac{3\ln(\sec(dx+c)+\tan(}{2}\right.}{}$
default	$\frac{b^4\left(\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+4a\,b^3\left(\frac{1}{2\sin(dx+c)\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)}+\frac{3\ln(\sec(dx+c)+\tan(}{2}\right.}{}$
risch	$\frac{e^{i(dx+c)}\left(216a^2b^2+9a^4+24b^4+216a^2b^2e^{12i(dx+c)}-144a^2b^2e^{10i(dx+c)}-216a^2b^2e^{8i(dx+c)}+144ia\,b^3+192ia\,b^3e^{10i(dx+c)}\right.}{}$

input
```
int(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^4*(1/3/cos(d*x+c)^3+1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+4*a*b^3
*(1/2/sin(d*x+c)/cos(d*x+c)^2-3/2/sin(d*x+c)+3/2*ln(sec(d*x+c)+tan(d*x+c))
)+6*a^2*b^2*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c)
-cot(d*x+c)))+4*a^3*b*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*
x+c)))+a^4*((-1/4*csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(csc(d*x+c
)-cot(d*x+c))))
```

3.48.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(264) = 528.

Time = 0.40 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.00

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{6\left(3a^4+72a^2b^2+8b^4\right)\cos\left(dx+c\right)^6-10\left(3a^4+72a^2b^2+8b^4\right)\cos\left(dx+c\right)^4+16b^4+16\left(18a^2b^2+b^4\right)\cdots}{}$$

input
```
integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/48*(6*(3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^6 - 10*(3*a^4 + 72*a^2*b
^2 + 8*b^4)*cos(d*x + c)^4 + 16*b^4 + 16*(18*a^2*b^2 + b^4)*cos(d*x + c)^2
 - 3*((3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 - 2*(3*a^4 + 72*a^2*b^2
+ 8*b^4)*cos(d*x + c)^5 + (3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^3)*log
(1/2*cos(d*x + c) + 1/2) + 3*((3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^7
- 2*(3*a^4 + 72*a^2*b^2 + 8*b^4)*cos(d*x + c)^5 + (3*a^4 + 72*a^2*b^2 + 8*
b^4)*cos(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) + 48*((2*a^3*b + 3*a*b^3
)*cos(d*x + c)^7 - 2*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^5 + (2*a^3*b + 3*a*b
^3)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 48*((2*a^3*b + 3*a*b^3)*cos(d*
x + c)^7 - 2*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^5 + (2*a^3*b + 3*a*b^3)*cos(
d*x + c)^3)*log(-sin(d*x + c) + 1) + 32*(3*(2*a^3*b + 3*a*b^3)*cos(d*x + c
)^5 + 3*a*b^3*cos(d*x + c) - 4*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^3)*sin(d*x
 + c))/(d*cos(d*x + c)^7 - 2*d*cos(d*x + c)^5 + d*cos(d*x + c)^3)
```

3.48.6 Sympy [F]

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^4\csc^5\left(c+dx\right)dx$$

input
```
integrate(csc(d*x+c)**5*(a+b*tan(d*x+c))**4,x)
```

output
```
Integral((a + b*tan(c + d*x))**4*csc(c + d*x)**5, x)
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.11

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{3\,a^4\left(\frac{2\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1} - 3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+72\,a^2b^2\left(\frac{2\left(3\,\cos(dx\right.}{\cos(dx+c)^3-}\right.}{}$$

input
```
integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/48*(3*a^4*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos
(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 72
*a^2*b^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log
(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 48*a*b^3*(2*(3*sin(d*x + c
)^2 - 2)/(sin(d*x + c)^3 - sin(d*x + c)) - 3*log(sin(d*x + c) + 1) + 3*log
(sin(d*x + c) - 1)) + 8*b^4*(2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3*l
og(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 32*a^3*b*(2*(3*sin(d*x +
 c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)))/d
```

3.48.8 Giac [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.75

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-32\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+144\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4}{}$$

input
```
integrate(csc(d*x+c)^5*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/192*(3*a^4*tan(1/2*d*x + 1/2*c)^4 - 32*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 24
*a^4*tan(1/2*d*x + 1/2*c)^2 + 144*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 480*a^3
*b*tan(1/2*d*x + 1/2*c) - 384*a*b^3*tan(1/2*d*x + 1/2*c) + 384*(2*a^3*b +
3*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 384*(2*a^3*b + 3*a*b^3)*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) + 24*(3*a^4 + 72*a^2*b^2 + 8*b^4)*log(abs(t
an(1/2*d*x + 1/2*c))) + 256*(3*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b^2*ta
n(1/2*d*x + 1/2*c)^4 - 3*b^4*tan(1/2*d*x + 1/2*c)^4 + 18*a^2*b^2*tan(1/2*d
*x + 1/2*c)^2 + 3*b^4*tan(1/2*d*x + 1/2*c)^2 - 3*a*b^3*tan(1/2*d*x + 1/2*c
) - 9*a^2*b^2 - 2*b^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3 - (150*a^4*tan(1/2*d
*x + 1/2*c)^4 + 3600*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 400*b^4*tan(1/2*d*x
+ 1/2*c)^4 + 480*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 384*a*b^3*tan(1/2*d*x + 1/
2*c)^3 + 24*a^4*tan(1/2*d*x + 1/2*c)^2 + 144*a^2*b^2*tan(1/2*d*x + 1/2*c)^
2 + 32*a^3*b*tan(1/2*d*x + 1/2*c) + 3*a^4)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.48.9 Mupad [B] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 857, normalized size of antiderivative = 3.13

$$\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx = \text{Too large to display}$$

input
```
int((a + b*tan(c + d*x))^4/sin(c + d*x)^5,x)
```

output
```
(a^4*tan(c/2 + (d*x)/2)^4)/(64*d) - (atan(-((6*a*b^3 + 4*a^3*b)*(12*a*b^3
+ 8*a^3*b - 6*tan(c/2 + (d*x)/2)*(6*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*
((3*a^4)/4 + 2*b^4 + 18*a^2*b^2))*1i + (6*a*b^3 + 4*a^3*b)*(12*a*b^3 + 8*a
^3*b + 6*tan(c/2 + (d*x)/2)*(6*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((3*a
^4)/4 + 2*b^4 + 18*a^2*b^2))*1i)/(2*tan(c/2 + (d*x)/2)*(144*a^2*b^6 + 192*
a^4*b^4 + 64*a^6*b^2) + (6*a*b^3 + 4*a^3*b)*(12*a*b^3 + 8*a^3*b - 6*tan(c/
2 + (d*x)/2)*(6*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((3*a^4)/4 + 2*b^4 +
 18*a^2*b^2)) - (6*a*b^3 + 4*a^3*b)*(12*a*b^3 + 8*a^3*b + 6*tan(c/2 + (d*x
)/2)*(6*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((3*a^4)/4 + 2*b^4 + 18*a^2*
b^2)) + 24*a*b^7 + 6*a^7*b + 232*a^3*b^5 + 153*a^5*b^3))*(a*b^3*12i + a^3*
b*8i))/d - (tan(c/2 + (d*x)/2)*(2*a^3*b + (a*b*(a^2 + 4*b^2))/2))/d + (log
(tan(c/2 + (d*x)/2))*((3*a^4)/8 + b^4 + 9*a^2*b^2))/d + (tan(c/2 + (d*x)/2
)^2*(a^4/8 + (3*a^2*b^2)/4))/d - (tan(c/2 + (d*x)/2)^6*((23*a^4)/4 + 64*b^
4 + 420*a^2*b^2) - tan(c/2 + (d*x)/2)^4*((21*a^4)/4 + (128*b^4)/3 + 228*a^
2*b^2) - tan(c/2 + (d*x)/2)^8*(2*a^4 + 64*b^4 + 204*a^2*b^2) + a^4/4 + tan
(c/2 + (d*x)/2)^2*((5*a^4)/4 + 12*a^2*b^2) + tan(c/2 + (d*x)/2)^3*(32*a*b^
3 + 32*a^3*b) + tan(c/2 + (d*x)/2)^9*(32*a*b^3 - 40*a^3*b) - tan(c/2 + (d*
x)/2)^5*(160*a*b^3 + 112*a^3*b) + tan(c/2 + (d*x)/2)^7*(96*a*b^3 + (352*a^
3*b)/3) + (8*a^3*b*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*x)/2)^4 - 48
*tan(c/2 + (d*x)/2)^6 + 48*tan(c/2 + (d*x)/2)^8 - 16*tan(c/2 + (d*x)/2)...
```

3.48. $\int \csc^5(c+dx)(a+b\tan(c+dx))^4\,dx$

3.49 $\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx$

3.49.1 Optimal result

Integrand size = 21, antiderivative size = 194

$$\begin{aligned}\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx = &-\frac{(a^4+12a^2b^2+b^4)\cot(c+dx)}{d}\\ &-\frac{2ab(2a^2+b^2)\cot^2(c+dx)}{d}\\ &-\frac{2a^2(a^2+3b^2)\cot^3(c+dx)}{3d}-\frac{a^3b\cot^4(c+dx)}{d}\\ &-\frac{a^4\cot^5(c+dx)}{5d}+\frac{4ab(a^2+2b^2)\log(\tan(c+dx))}{d}\\ &+\frac{2b^2(3a^2+b^2)\tan(c+dx)}{d}\\ &+\frac{2ab^3\tan^2(c+dx)}{d}+\frac{b^4\tan^3(c+dx)}{3d}\end{aligned}$$

output
```
-(a^4+12*a^2*b^2+b^4)*cot(d*x+c)/d-2*a*b*(2*a^2+b^2)*cot(d*x+c)^2/d-2/3*a^
2*(a^2+3*b^2)*cot(d*x+c)^3/d-a^3*b*cot(d*x+c)^4/d-1/5*a^4*cot(d*x+c)^5/d+4
*a*b*(a^2+2*b^2)*ln(tan(d*x+c))/d+2*b^2*(3*a^2+b^2)*tan(d*x+c)/d+2*a*b^3*t
an(d*x+c)^2/d+1/3*b^4*tan(d*x+c)^3/d
```

3.49.2 Mathematica [A] (verified)

Time = 6.10 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.20

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx = -\frac{(15a^3b\cot^4(c+dx)+3a^4\cot^5(c+dx)+2a\cos^2(c+dx)\left(-15b^3+15b(a^2+b^2)\cot^2(c+dx)+a(2a^2+\right.}{}$$

input
```
Integrate[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^4,x]
```

output
```
-1/15*((15*a^3*b*Cot[c + d*x]^4 + 3*a^4*Cot[c + d*x]^5 + 2*a*Cos[c + d*x]^
2*(-15*b^3 + 15*b*(a^2 + b^2)*Cot[c + d*x]^2 + a*(2*a^2 + 15*b^2)*Cot[c +
d*x]^3) + Cos[c + d*x]^4*((8*a^4 + 150*a^2*b^2 + 15*b^4)*Cot[c + d*x] + 60
*a*b*(a^2 + 2*b^2)*(Log[Cos[c + d*x]] - Log[Sin[c + d*x]])) - 5*b^2*(18*a^
2 + 5*b^2)*Cos[c + d*x]^3*Sin[c + d*x] - (5*b^4*Sin[2*(c + d*x)])/2)*(a +
b*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

3.49.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3999, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^4}{\sin(c+dx)^6}dx$$

$$\Big\downarrow \text{3999}$$

$$\frac{b\int \frac{\cot^6(c+dx)(a+b\tan(c+dx))^4\left(\tan^2(c+dx)b^2+b^2\right)^2}{b^6}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{b\int\left(\frac{a^4\cot^6(c+dx)}{b^2}+\frac{4a^3\cot^5(c+dx)}{b}+\frac{2a^2(a^2+3b^2)\cot^4(c+dx)}{b^2}+\frac{4a(2a^2+b^2)\cot^3(c+dx)}{b}+\frac{(a^4+12b^2a^2+b^4)\cot^2(c+dx)}{b^2}+\frac{4(a^3+2b^2}{}\right.}{d}$$

$\Big\downarrow$ 2009

$$\frac{b\left(-\frac{a^4\cot^5(c+dx)}{5b}-a^3\cot^4(c+dx)+2b(3a^2+b^2)\tan(c+dx)-\frac{2a^2(a^2+3b^2)\cot^3(c+dx)}{3b}-2a(2a^2+b^2)\cot^2(c+dx)\right.}{d}$$

input
```
Int[Csc[c + d*x]^6*(a + b*Tan[c + d*x])^4,x]
```

output
```
(b*(-(((a^4 + 12*a^2*b^2 + b^4)*Cot[c + d*x])/b) - 2*a*(2*a^2 + b^2)*Cot[c
 + d*x]^2 - (2*a^2*(a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*b) - a^3*Cot[c + d*x]^
4 - (a^4*Cot[c + d*x]^5)/(5*b) + 4*a*(a^2 + 2*b^2)*Log[b*Tan[c + d*x]] + 2
*b*(3*a^2 + b^2)*Tan[c + d*x] + 2*a*b^2*Tan[c + d*x]^2 + (b^3*Tan[c + d*x]
^3)/3))/d
```

3.49.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.49.4 Maple [A] (verified)

Time = 25.78 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{b^4\left(\frac{1}{3\sin(dx+c)\cos(dx+c)^3}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+4a\,b^3\left(\frac{1}{2\sin(dx+c)^2\cos(dx+c)^2}-\frac{1}{\sin(dx+c)^2}+2\ln(\tan(dx+}{}$
default	$\frac{b^4\left(\frac{1}{3\sin(dx+c)\cos(dx+c)^3}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+4a\,b^3\left(\frac{1}{2\sin(dx+c)^2\cos(dx+c)^2}-\frac{1}{\sin(dx+c)^2}+2\ln(\tan(dx+}{}$
risch	$\frac{64ia^2b^2e^{4i(dx+c)}-32ia^2b^2e^{8i(dx+c)}+64ia^2b^2e^{10i(dx+c)}-128ia^2b^2e^{6i(dx+c)}-56a^3b\,e^{10i(dx+c)}+16a\,b^3e^{10i(dx+c)}+64ia^2}{}$

input
```
int(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^4*(1/3/sin(d*x+c)/cos(d*x+c)^3+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*
x+c))+4*a*b^3*(1/2/sin(d*x+c)^2/cos(d*x+c)^2-1/sin(d*x+c)^2+2*ln(tan(d*x+c
)))+6*a^2*b^2*(-1/3/sin(d*x+c)^3/cos(d*x+c)+4/3/sin(d*x+c)/cos(d*x+c)-8/3*
cot(d*x+c))+4*a^3*b*(-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a^
4*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c))
```

3.49.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(188) = 376$.

Time = 0.29 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.99

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx =$$

$$-\frac{8\left(a^4+30\,a^2b^2+5\,b^4\right)\cos\left(dx+c\right)^8-20\left(a^4+30\,a^2b^2+5\,b^4\right)\cos\left(dx+c\right)^6+15\left(a^4+30\,a^2b^2+5\,b^4\right)\mathrm{co}}{}$$

input
```
integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/15*(8*(a^4 + 30*a^2*b^2 + 5*b^4)*cos(d*x + c)^8 - 20*(a^4 + 30*a^2*b^2
+ 5*b^4)*cos(d*x + c)^6 + 15*(a^4 + 30*a^2*b^2 + 5*b^4)*cos(d*x + c)^4 - 5
*b^4 - 10*(9*a^2*b^2 + b^4)*cos(d*x + c)^2 + 30*((a^3*b + 2*a*b^3)*cos(d*x
 + c)^7 - 2*(a^3*b + 2*a*b^3)*cos(d*x + c)^5 + (a^3*b + 2*a*b^3)*cos(d*x +
 c)^3)*log(cos(d*x + c)^2)*sin(d*x + c) - 30*((a^3*b + 2*a*b^3)*cos(d*x +
c)^7 - 2*(a^3*b + 2*a*b^3)*cos(d*x + c)^5 + (a^3*b + 2*a*b^3)*cos(d*x + c)
^3)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 15*(2*(a^3*b + 2*a*b^3)*
cos(d*x + c)^5 + 2*a*b^3*cos(d*x + c) - 3*(a^3*b + 2*a*b^3)*cos(d*x + c)^3
)*sin(d*x + c))/((d*cos(d*x + c)^7 - 2*d*cos(d*x + c)^5 + d*cos(d*x + c)^3
)*sin(d*x + c))
```

3.49.6 Sympy [F(-1)]

Timed out.

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**6*(a+b*tan(d*x+c))**4,x)
```

output
```
Timed out
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.88

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{5\,b^4\tan(dx+c)^3 + 30\,ab^3\tan(dx+c)^2 + 60\,(a^3b+2\,ab^3)\log(\tan(dx+c)) + 30\,(3\,a^2b^2+b^4)\tan(dx+c}{15\,d}$$

input
```
integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/15*(5*b^4*tan(d*x + c)^3 + 30*a*b^3*tan(d*x + c)^2 + 60*(a^3*b + 2*a*b^3
)*log(tan(d*x + c)) + 30*(3*a^2*b^2 + b^4)*tan(d*x + c) - (15*a^3*b*tan(d*
x + c) + 15*(a^4 + 12*a^2*b^2 + b^4)*tan(d*x + c)^4 + 3*a^4 + 30*(2*a^3*b
+ a*b^3)*tan(d*x + c)^3 + 10*(a^4 + 3*a^2*b^2)*tan(d*x + c)^2)/tan(d*x + c
)^5)/d
```

3.49.8 Giac [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.21

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= \frac{5\,b^4\tan\left(dx+c\right)^3+30\,ab^3\tan\left(dx+c\right)^2+90\,a^2b^2\tan\left(dx+c\right)+30\,b^4\tan\left(dx+c\right)+60\,(a^3b+2\,ab^3)\log\left(\right.}{}$$

input
```
integrate(csc(d*x+c)^6*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/15*(5*b^4*tan(d*x + c)^3 + 30*a*b^3*tan(d*x + c)^2 + 90*a^2*b^2*tan(d*x
+ c) + 30*b^4*tan(d*x + c) + 60*(a^3*b + 2*a*b^3)*log(abs(tan(d*x + c))) -
 (137*a^3*b*tan(d*x + c)^5 + 274*a*b^3*tan(d*x + c)^5 + 15*a^4*tan(d*x + c
)^4 + 180*a^2*b^2*tan(d*x + c)^4 + 15*b^4*tan(d*x + c)^4 + 60*a^3*b*tan(d*
x + c)^3 + 30*a*b^3*tan(d*x + c)^3 + 10*a^4*tan(d*x + c)^2 + 30*a^2*b^2*ta
n(d*x + c)^2 + 15*a^3*b*tan(d*x + c) + 3*a^4)/tan(d*x + c)^5)/d
```

3.49.9 Mupad [B] (verification not implemented)

Time = 4.62 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.93

$$\int \csc^6(c+dx)(a+b\tan(c+dx))^4\,dx = \frac{\ln\left(\tan(c+d\,x)\right)\,\left(4\,a^3\,b+8\,a\,b^3\right)}{d}$$
$$-\frac{\cot(c+d\,x)^5\,\left(\tan(c+d\,x)^2\,\left(\frac{2\,a^4}{3}+2\,a^2\,b^2\right)+\tan(c+d\,x)^3\,\left(4\,a^3\,b+2\,a\,b^3\right)+\frac{a^4}{5}+\tan(c+d\,x)^4\,\left(a^4+\right.\right.}{d}$$
$$+\frac{b^4\tan(c+d\,x)^3}{3\,d}+\frac{\tan(c+d\,x)\,\left(6\,a^2\,b^2+2\,b^4\right)}{d}+\frac{2\,a\,b^3\tan(c+d\,x)^2}{d}$$

input
```
int((a + b*tan(c + d*x))^4/sin(c + d*x)^6,x)
```

output
```
(log(tan(c + d*x))*(8*a*b^3 + 4*a^3*b))/d - (cot(c + d*x)^5*(tan(c + d*x)^
2*((2*a^4)/3 + 2*a^2*b^2) + tan(c + d*x)^3*(2*a*b^3 + 4*a^3*b) + a^4/5 + t
an(c + d*x)^4*(a^4 + b^4 + 12*a^2*b^2) + a^3*b*tan(c + d*x)))/d + (b^4*tan
(c + d*x)^3)/(3*d) + (tan(c + d*x)*(2*b^4 + 6*a^2*b^2))/d + (2*a*b^3*tan(c
 + d*x)^2)/d
```

3.50 $\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx$

3.50.1 Optimal result

Integrand size = 21, antiderivative size = 402

$$\begin{aligned}\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx = &-\frac{5a^4\text{arctanh}(\cos(c+dx))}{16d}\\
&-\frac{45a^2b^2\text{arctanh}(\cos(c+dx))}{4d}\\
&-\frac{5b^4\text{arctanh}(\cos(c+dx))}{2d}\\
&+\frac{4a^3b\text{arctanh}(\sin(c+dx))}{d}\\
&+\frac{10ab^3\text{arctanh}(\sin(c+dx))}{d}-\frac{4a^3b\csc(c+dx)}{d}\\
&-\frac{10ab^3\csc(c+dx)}{d}-\frac{5a^4\cot(c+dx)\csc(c+dx)}{16d}\\
&-\frac{4a^3b\csc^3(c+dx)}{3d}-\frac{10ab^3\csc^3(c+dx)}{3d}\\
&-\frac{5a^4\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{4a^3b\csc^5(c+dx)}{5d}\\
&-\frac{a^4\cot(c+dx)\csc^5(c+dx)}{6d}+\frac{45a^2b^2\sec(c+dx)}{4d}\\
&+\frac{5b^4\sec(c+dx)}{2d}-\frac{15a^2b^2\csc^2(c+dx)\sec(c+dx)}{4d}\\
&-\frac{3a^2b^2\csc^4(c+dx)\sec(c+dx)}{2d}\\
&+\frac{2ab^3\csc^3(c+dx)\sec^2(c+dx)}{d}\\
&+\frac{5b^4\sec^3(c+dx)}{6d}-\frac{b^4\csc^2(c+dx)\sec^3(c+dx)}{2d}\end{aligned}$$

output
```
-5/16*a^4*arctanh(cos(d*x+c))/d-45/4*a^2*b^2*arctanh(cos(d*x+c))/d-5/2*b^4
*arctanh(cos(d*x+c))/d+4*a^3*b*arctanh(sin(d*x+c))/d+10*a*b^3*arctanh(sin(
d*x+c))/d-4*a^3*b*csc(d*x+c)/d-10*a*b^3*csc(d*x+c)/d-5/16*a^4*cot(d*x+c)*c
sc(d*x+c)/d-4/3*a^3*b*csc(d*x+c)^3/d-10/3*a*b^3*csc(d*x+c)^3/d-5/24*a^4*co
t(d*x+c)*csc(d*x+c)^3/d-4/5*a^3*b*csc(d*x+c)^5/d-1/6*a^4*cot(d*x+c)*csc(d*
x+c)^5/d+45/4*a^2*b^2*sec(d*x+c)/d+5/2*b^4*sec(d*x+c)/d-15/4*a^2*b^2*csc(d
*x+c)^2*sec(d*x+c)/d-3/2*a^2*b^2*csc(d*x+c)^4*sec(d*x+c)/d+2*a*b^3*csc(d*x
+c)^3*sec(d*x+c)^2/d+5/6*b^4*sec(d*x+c)^3/d-1/2*b^4*csc(d*x+c)^2*sec(d*x+c
)^3/d
```

3.50.2 Mathematica [A] (verified)

Time = 8.99 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.64

$$\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$= -\frac{5(a^4+36a^2b^2+8b^4)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{16d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$-\frac{2(2a^3b+5ab^3)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{5(a^4+36a^2b^2+8b^4)\cos^4(c+dx)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{16d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{2(2a^3b+5ab^3)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{d(a\cos(c+dx)+b\sin(c+dx))^4}$$
$$+\frac{\cot(c+dx)\csc^5(c+dx)\left(-2545a^4+540a^2b^2+5240b^4-2760a^4\cos(2(c+dx))-7200a^2b^2\cos(2(c+d\right.}{}$$

input
```
Integrate[Csc[c + d*x]^7*(a + b*Tan[c + d*x])^4,x]
```

output
```
(-5*(a^4 + 36*a^2*b^2 + 8*b^4)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2]]*(a + b
*Tan[c + d*x])^4)/(16*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*(2*a^3*b
 + 5*a*b^3)*Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b
*Tan[c + d*x])^4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (5*(a^4 + 36*a
^2*b^2 + 8*b^4)*Cos[c + d*x]^4*Log[Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^
4)/(16*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*(2*a^3*b + 5*a*b^3)*Cos
[c + d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^
4)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cot[c + d*x]*Csc[c + d*x]^5*
(-2545*a^4 + 540*a^2*b^2 + 5240*b^4 - 2760*a^4*Cos[2*(c + d*x)] - 7200*a^2
*b^2*Cos[2*(c + d*x)] - 6720*b^4*Cos[2*(c + d*x)] + 60*a^4*Cos[4*(c + d*x)
] + 2160*a^2*b^2*Cos[4*(c + d*x)] + 480*b^4*Cos[4*(c + d*x)] + 200*a^4*Cos
[6*(c + d*x)] + 7200*a^2*b^2*Cos[6*(c + d*x)] + 1600*b^4*Cos[6*(c + d*x)]
- 75*a^4*Cos[8*(c + d*x)] - 2700*a^2*b^2*Cos[8*(c + d*x)] - 600*b^4*Cos[8*
(c + d*x)] - 15744*a^3*b*Sin[2*(c + d*x)] - 8640*a*b^3*Sin[2*(c + d*x)] -
1152*a^3*b*Sin[4*(c + d*x)] - 2880*a*b^3*Sin[4*(c + d*x)] + 3200*a^3*b*Sin
[6*(c + d*x)] + 8000*a*b^3*Sin[6*(c + d*x)] - 960*a^3*b*Sin[8*(c + d*x)] -
 2400*a*b^3*Sin[8*(c + d*x)])*(a + b*Tan[c + d*x])^4)/(30720*d*(a*Cos[c +
d*x] + b*Sin[c + d*x])^4)
```

3.50.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4000, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^4}{\sin(c+dx)^7}dx$$

$$\Big\downarrow \text{4000}$$

$$\int (a^4\csc^7(c+dx)+4a^3b\csc^6(c+dx)\sec(c+dx)+6a^2b^2\csc^5(c+dx)\sec^2(c+dx)+4ab^3\csc^4(c+dx)\sec^3(c+$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5a^4\text{arctanh}(\cos(c+dx))}{16d}-\frac{a^4\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a^4\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{5a^4\cot(c+dx)\csc(c+dx)}{16d}+\frac{4a^3b\text{arctanh}(\sin(c+dx))}{d}-\frac{4a^3b\csc^5(c+dx)}{5d}-\frac{4a^3b\csc^3(c+dx)}{3d}-\frac{4a^3b\csc(c+dx)}{d}-\frac{45a^2b^2\text{arctanh}(\cos(c+dx))}{4d}+\frac{45a^2b^2\sec(c+dx)}{4d}-\frac{3a^2b^2\csc^4(c+dx)\sec(c+dx)}{2d}-\frac{15a^2b^2\csc^2(c+dx)\sec(c+dx)}{4d}+\frac{10ab^3\text{arctanh}(\sin(c+dx))}{d}-\frac{10ab^3\csc^3(c+dx)}{3d}-\frac{10ab^3\csc(c+dx)}{d}+\frac{2ab^3\csc^3(c+dx)\sec^2(c+dx)}{d}-\frac{5b^4\text{arctanh}(\cos(c+dx))}{2d}+\frac{5b^4\sec^3(c+dx)}{6d}+\frac{5b^4\sec(c+dx)}{2d}-\frac{b^4\csc^2(c+dx)\sec^3(c+dx)}{2d}$$

input
```
Int[Csc[c + d*x]^7*(a + b*Tan[c + d*x])^4,x]
```

output
```
(-5*a^4*ArcTanh[Cos[c + d*x]])/(16*d) - (45*a^2*b^2*ArcTanh[Cos[c + d*x]])
/(4*d) - (5*b^4*ArcTanh[Cos[c + d*x]])/(2*d) + (4*a^3*b*ArcTanh[Sin[c + d*
x]])/d + (10*a*b^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*b*Csc[c + d*x])/d - (
10*a*b^3*Csc[c + d*x])/d - (5*a^4*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (4*a
^3*b*Csc[c + d*x]^3)/(3*d) - (10*a*b^3*Csc[c + d*x]^3)/(3*d) - (5*a^4*Cot[
c + d*x]*Csc[c + d*x]^3)/(24*d) - (4*a^3*b*Csc[c + d*x]^5)/(5*d) - (a^4*Co
t[c + d*x]*Csc[c + d*x]^5)/(6*d) + (45*a^2*b^2*Sec[c + d*x])/(4*d) + (5*b^
4*Sec[c + d*x])/(2*d) - (15*a^2*b^2*Csc[c + d*x]^2*Sec[c + d*x])/(4*d) - (
3*a^2*b^2*Csc[c + d*x]^4*Sec[c + d*x])/(2*d) + (2*a*b^3*Csc[c + d*x]^3*Sec
[c + d*x]^2)/d + (5*b^4*Sec[c + d*x]^3)/(6*d) - (b^4*Csc[c + d*x]^2*Sec[c
+ d*x]^3)/(2*d)
```

3.50.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4000
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Expand[Sin[e + f*x]^m*(a + b*Tan[e + f*x])^n, x], x]
/; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.50.4 Maple [A] (verified)

Time = 50.86 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{b^4\left(\frac{1}{3\sin(dx+c)^2\cos(dx+c)^3}-\frac{5}{6\sin(dx+c)^2\cos(dx+c)}+\frac{5}{2\cos(dx+c)}+\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+4a\,b^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx-}\right.}{}$
default	$\frac{b^4\left(\frac{1}{3\sin(dx+c)^2\cos(dx+c)^3}-\frac{5}{6\sin(dx+c)^2\cos(dx+c)}+\frac{5}{2\cos(dx+c)}+\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+4a\,b^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx-}\right.}{}$
risch	Expression too large to display

input
```
int(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^4*(1/3/sin(d*x+c)^2/cos(d*x+c)^3-5/6/sin(d*x+c)^2/cos(d*x+c)+5/2/co
s(d*x+c)+5/2*ln(csc(d*x+c)-cot(d*x+c)))+4*a*b^3*(-1/3/sin(d*x+c)^3/cos(d*x
+c)^2+5/6/sin(d*x+c)/cos(d*x+c)^2-5/2/sin(d*x+c)+5/2*ln(sec(d*x+c)+tan(d*x
+c)))+6*a^2*b^2*(-1/4/sin(d*x+c)^4/cos(d*x+c)-5/8/sin(d*x+c)^2/cos(d*x+c)+
15/8/cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c)))+4*a^3*b*(-1/5/sin(d*x+c)^5
-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^4*((-1/6*csc(d
*x+c)^5-5/24*csc(d*x+c)^3-5/16*csc(d*x+c))*cot(d*x+c)+5/16*ln(csc(d*x+c)-c
ot(d*x+c))))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.73

$$\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$=\frac{150\left(a^4+36\,a^2b^2+8\,b^4\right)\cos\left(dx+c\right)^8-400\left(a^4+36\,a^2b^2+8\,b^4\right)\cos\left(dx+c\right)^6+330\left(a^4+36\,a^2b^2+8\,b^4\right)}{}$$

input
```
integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/480*(150*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^8 - 400*(a^4 + 36*a^2*b
^2 + 8*b^4)*cos(d*x + c)^6 + 330*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^4
 - 160*b^4 - 480*(6*a^2*b^2 + b^4)*cos(d*x + c)^2 - 75*((a^4 + 36*a^2*b^2
+ 8*b^4)*cos(d*x + c)^9 - 3*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 + 3*
(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^5 - (a^4 + 36*a^2*b^2 + 8*b^4)*cos
(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) + 75*((a^4 + 36*a^2*b^2 + 8*b^4)*
cos(d*x + c)^9 - 3*(a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^7 + 3*(a^4 + 36
*a^2*b^2 + 8*b^4)*cos(d*x + c)^5 - (a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)
^3)*log(-1/2*cos(d*x + c) + 1/2) + 480*((2*a^3*b + 5*a*b^3)*cos(d*x + c)^9
 - 3*(2*a^3*b + 5*a*b^3)*cos(d*x + c)^7 + 3*(2*a^3*b + 5*a*b^3)*cos(d*x +
c)^5 - (2*a^3*b + 5*a*b^3)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 480*((2
*a^3*b + 5*a*b^3)*cos(d*x + c)^9 - 3*(2*a^3*b + 5*a*b^3)*cos(d*x + c)^7 +
3*(2*a^3*b + 5*a*b^3)*cos(d*x + c)^5 - (2*a^3*b + 5*a*b^3)*cos(d*x + c)^3)
*log(-sin(d*x + c) + 1) + 64*(15*(2*a^3*b + 5*a*b^3)*cos(d*x + c)^7 - 35*(
2*a^3*b + 5*a*b^3)*cos(d*x + c)^5 - 15*a*b^3*cos(d*x + c) + 23*(2*a^3*b +
5*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^9 - 3*d*cos(d*x + c
)^7 + 3*d*cos(d*x + c)^5 - d*cos(d*x + c)^3)
```

3.50.6 Sympy [F(-1)]

Timed out.

$$\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx=\text{Timed out}$$

input
```
integrate(csc(d*x+c)**7*(a+b*tan(d*x+c))**4,x)
```

output
```
Timed out
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.96

$$\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx$$
$$=\frac{5\,a^4\left(\frac{2\left(15\,\cos(dx+c)^5-40\,\cos(dx+c)^3+33\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+40}{}$$

input
```
integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/480*(5*a^4*(2*(15*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 33*cos(d*x + c))/
(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 15*log(cos(d*
x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 40*b^4*(2*(15*cos(d*x + c)^4 - 1
0*cos(d*x + c)^2 - 2)/(cos(d*x + c)^5 - cos(d*x + c)^3) - 15*log(cos(d*x +
 c) + 1) + 15*log(cos(d*x + c) - 1)) + 180*a^2*b^2*(2*(15*cos(d*x + c)^4 -
 25*cos(d*x + c)^2 + 8)/(cos(d*x + c)^5 - 2*cos(d*x + c)^3 + cos(d*x + c))
 - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) - 160*a*b^3*(2*(15
*sin(d*x + c)^4 - 10*sin(d*x + c)^2 - 2)/(sin(d*x + c)^5 - sin(d*x + c)^3)
 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 64*a^3*b*(2*(15*
sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 15*log(sin(d*x + c
) + 1) + 15*log(sin(d*x + c) - 1)))/d
```

3.50.8 Giac [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 647, normalized size of antiderivative = 1.61

$$\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx$$

$$= \frac{5\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-48\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+180\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-5}{}$$

input
```
integrate(csc(d*x+c)^7*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/1920*(5*a^4*tan(1/2*d*x + 1/2*c)^6 - 48*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 4
5*a^4*tan(1/2*d*x + 1/2*c)^4 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 560*a^
3*b*tan(1/2*d*x + 1/2*c)^3 - 320*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 225*a^4*ta
n(1/2*d*x + 1/2*c)^2 + 2880*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 240*b^4*tan(1
/2*d*x + 1/2*c)^2 - 5280*a^3*b*tan(1/2*d*x + 1/2*c) - 8640*a*b^3*tan(1/2*d
*x + 1/2*c) + 3840*(2*a^3*b + 5*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- 3840*(2*a^3*b + 5*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 600*(a^4 +
 36*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c))) + 1280*(6*a*b^3*tan(1/
2*d*x + 1/2*c)^5 - 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 9*b^4*tan(1/2*d*x +
 1/2*c)^4 + 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^4*tan(1/2*d*x + 1/2*c
)^2 - 6*a*b^3*tan(1/2*d*x + 1/2*c) - 18*a^2*b^2 - 7*b^4)/(tan(1/2*d*x + 1/
2*c)^2 - 1)^3 - (1470*a^4*tan(1/2*d*x + 1/2*c)^6 + 52920*a^2*b^2*tan(1/2*d
*x + 1/2*c)^6 + 11760*b^4*tan(1/2*d*x + 1/2*c)^6 + 5280*a^3*b*tan(1/2*d*x
+ 1/2*c)^5 + 8640*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 225*a^4*tan(1/2*d*x + 1/2
*c)^4 + 2880*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 240*b^4*tan(1/2*d*x + 1/2*c)
^4 + 560*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 320*a*b^3*tan(1/2*d*x + 1/2*c)^3 +
 45*a^4*tan(1/2*d*x + 1/2*c)^2 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 48*a
^3*b*tan(1/2*d*x + 1/2*c) + 5*a^4)/tan(1/2*d*x + 1/2*c)^6)/d
```

3.50.9 Mupad [B] (verification not implemented)

Time = 4.70 (sec) , antiderivative size = 990, normalized size of antiderivative = 2.46

$$\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx = \text{Too large to display}$$

input
```
int((a + b*tan(c + d*x))^4/sin(c + d*x)^7,x)
```

output
```
(a^4*tan(c/2 + (d*x)/2)^6)/(384*d) - (atan(-((10*a*b^3 + 4*a^3*b)*(20*a*b^
3 + 8*a^3*b - 6*tan(c/2 + (d*x)/2)*(10*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/
2)*((5*a^4)/8 + 5*b^4 + (45*a^2*b^2)/2))*1i + (10*a*b^3 + 4*a^3*b)*(20*a*b
^3 + 8*a^3*b + 6*tan(c/2 + (d*x)/2)*(10*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)
/2)*((5*a^4)/8 + 5*b^4 + (45*a^2*b^2)/2))*1i)/(2*tan(c/2 + (d*x)/2)*(400*a
^2*b^6 + 320*a^4*b^4 + 64*a^6*b^2) + (10*a*b^3 + 4*a^3*b)*(20*a*b^3 + 8*a^
3*b - 6*tan(c/2 + (d*x)/2)*(10*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((5*a
^4)/8 + 5*b^4 + (45*a^2*b^2)/2)) - (10*a*b^3 + 4*a^3*b)*(20*a*b^3 + 8*a^3*
b + 6*tan(c/2 + (d*x)/2)*(10*a*b^3 + 4*a^3*b) - tan(c/2 + (d*x)/2)*((5*a^4
)/8 + 5*b^4 + (45*a^2*b^2)/2)) + 100*a*b^7 + 5*a^7*b + 490*a^3*b^5 + (385*
a^5*b^3)/2))*(a*b^3*20i + a^3*b*8i))/d + (tan(c/2 + (d*x)/2)^4*((a^2*(a^2
+ 12*b^2))/128 + a^4/64))/d + (tan(c/2 + (d*x)/2)^2*((a^2*(a^2 + 12*b^2))/
16 + (7*a^4)/128 + b^4/8 + (3*a^2*b^2)/4))/d - (tan(c/2 + (d*x)/2)*((9*a*b
^3)/2 + (11*a^3*b)/4))/d - (tan(c/2 + (d*x)/2)^4*((7*a^4)/2 + 8*b^4 + 78*a
^2*b^2) - tan(c/2 + (d*x)/2)^10*((15*a^4)/2 + 392*b^4 + 864*a^2*b^2) - tan
(c/2 + (d*x)/2)^6*((109*a^4)/6 + (968*b^4)/3 + 1038*a^2*b^2) + tan(c/2 + (
d*x)/2)^8*(21*a^4 + 536*b^4 + 1818*a^2*b^2) + tan(c/2 + (d*x)/2)^2*(a^4 +
6*a^2*b^2) + a^4/6 - tan(c/2 + (d*x)/2)^11*(32*a*b^3 + 176*a^3*b) + tan(c/
2 + (d*x)/2)^3*((32*a*b^3)/3 + (208*a^3*b)/15) + tan(c/2 + (d*x)/2)^5*(256
*a*b^3 + (624*a^3*b)/5) - tan(c/2 + (d*x)/2)^7*(1088*a*b^3 + (2368*a^3*...
```

3.50. $\int \csc^7(c+dx)(a+b\tan(c+dx))^4\,dx$

3.51 $\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx$

3.51.1 Optimal result

Integrand size = 21, antiderivative size = 274

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{a^5 b \operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2} d}+\frac{a^3b^2\cos(c+dx)}{(a^2+b^2)^3 d}+\frac{ab^2\cos(c+dx)}{(a^2+b^2)^2 d}-\frac{a\cos(c+dx)}{(a^2+b^2) d}-\frac{ab^2\cos^3(c+dx)}{3(a^2+b^2)^2 d}+\frac{2a\cos^3(c+dx)}{3(a^2+b^2) d}-\frac{a\cos^5(c+dx)}{5(a^2+b^2) d}+\frac{a^4b\sin(c+dx)}{(a^2+b^2)^3 d}+\frac{a^2b\sin^3(c+dx)}{3(a^2+b^2)^2 d}+\frac{b\sin^5(c+dx)}{5(a^2+b^2) d}$$

output
```
a^5*b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)
/d+a^3*b^2*cos(d*x+c)/(a^2+b^2)^3/d+a*b^2*cos(d*x+c)/(a^2+b^2)^2/d-a*cos(d
*x+c)/(a^2+b^2)/d-1/3*a*b^2*cos(d*x+c)^3/(a^2+b^2)^2/d+2/3*a*cos(d*x+c)^3/
(a^2+b^2)/d-1/5*a*cos(d*x+c)^5/(a^2+b^2)/d+a^4*b*sin(d*x+c)/(a^2+b^2)^3/d+
1/3*a^2*b*sin(d*x+c)^3/(a^2+b^2)^2/d+1/5*b*sin(d*x+c)^5/(a^2+b^2)/d
```

3.51.2 Mathematica [A] (verified)

Time = 3.80 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.05

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{-480a^5b\,\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sqrt{a^2+b^2}\left(-30a(5a^4-4a^2b^2-b^4)\cos(c+dx)+5a(5a^4+6a^2b^2+b\right.}{}$$

input
```
Integrate[Sin[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

output
```
(-480*a^5*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2
+ b^2]*(-30*a*(5*a^4 - 4*a^2*b^2 - b^4)*Cos[c + d*x] + 5*a*(5*a^4 + 6*a^2*
b^2 + b^4)*Cos[3*(c + d*x)] - 3*a^5*Cos[5*(c + d*x)] - 6*a^3*b^2*Cos[5*(c
+ d*x)] - 3*a*b^4*Cos[5*(c + d*x)] + 330*a^4*b*Sin[c + d*x] + 120*a^2*b^3*
Sin[c + d*x] + 30*b^5*Sin[c + d*x] - 35*a^4*b*Sin[3*(c + d*x)] - 50*a^2*b^
3*Sin[3*(c + d*x)] - 15*b^5*Sin[3*(c + d*x)] + 3*a^4*b*Sin[5*(c + d*x)] +
6*a^2*b^3*Sin[5*(c + d*x)] + 3*b^5*Sin[5*(c + d*x)]))/(240*(a^2 + b^2)^(7/
2)*d)
```

3.51.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.92, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 4001, 3042, 3588, 3042, 3044, 15, 3113, 2009, 3578, 3042, 3113, 2009, 3578, 3042, 3118, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin(c+dx)^5}{a+b\tan(c+dx)}dx$$
$$\Big\downarrow \; 4001$$
$$\int \frac{\sin^5(c+dx)\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^5\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3588}$$

$$\frac{a\int \sin^5(c+dx)dx}{a^2+b^2}+\frac{b\int \cos(c+dx)\sin^4(c+dx)dx}{a^2+b^2}-\frac{ab\int \frac{\sin^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sin(c+dx)^5dx}{a^2+b^2}+\frac{b\int \cos(c+dx)\sin(c+dx)^4dx}{a^2+b^2}-\frac{ab\int \frac{\sin(c+dx)^4}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3044}$$

$$\frac{b\int \sin^4(c+dx)d\sin(c+dx)}{d\left(a^2+b^2\right)}+\frac{a\int \sin(c+dx)^5dx}{a^2+b^2}-\frac{ab\int \frac{\sin(c+dx)^4}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{a\int \sin(c+dx)^5dx}{a^2+b^2}-\frac{ab\int \frac{\sin(c+dx)^4}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\sin^5(c+dx)}{5d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{a\int \left(\cos^4(c+dx)-2\cos^2(c+dx)+1\right)d\cos(c+dx)}{d\left(a^2+b^2\right)}-\frac{ab\int \frac{\sin(c+dx)^4}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\sin^5(c+dx)}{5d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ab\int \frac{\sin(c+dx)^4}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\sin^5(c+dx)}{5d\left(a^2+b^2\right)}-\frac{a\left(\frac{1}{5}\cos^5(c+dx)-\frac{2}{3}\cos^3(c+dx)+\cos(c+dx)\right)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3578}$$

$$-\frac{ab\left(\frac{b\int \sin^3(c+dx)dx}{a^2+b^2}+\frac{a^2\int \frac{\sin^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}-\frac{a\sin^3(c+dx)}{3d(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\sin^5(c+dx)}{5d\left(a^2+b^2\right)}-\frac{a\left(\frac{1}{5}\cos^5(c+dx)-\frac{2}{3}\cos^3(c+dx)+\cos(c+dx)\right)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

3.51. $\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx$

$$-\frac{ab\left(\frac{b\int \sin(c+dx)^3dx}{a^2+b^2}+\frac{a^2\int \frac{\sin(c+dx)^2}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}-\frac{a\sin^3(c+dx)}{3d(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\sin^5(c+dx)}{5d\left(a^2+b^2\right)}-\frac{a\left(\frac{1}{5}\cos^5(c+dx)-\frac{2}{3}\cos^3(c+dx)+\cos(c+dx)\right)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{ab\left(-\frac{b\int \left(1-\cos^2(c+dx)\right)d\cos(c+dx)}{d(a^2+b^2)}+\frac{a^2\int \frac{\sin(c+dx)^2}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}-\frac{a\sin^3(c+dx)}{3d(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\sin^5(c+dx)}{5d\left(a^2+b^2\right)}-\frac{a\left(\frac{1}{5}\cos^5(c+dx)-\frac{2}{3}\cos^3(c+dx)+\cos(c+dx)\right)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ab\left(\frac{a^2\int \frac{\sin(c+dx)^2}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}-\frac{a\sin^3(c+dx)}{3d(a^2+b^2)}-\frac{b\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\sin^5(c+dx)}{5d\left(a^2+b^2\right)}-\frac{a\left(\frac{1}{5}\cos^5(c+dx)-\frac{2}{3}\cos^3(c+dx)+\cos(c+dx)\right)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3578}$$

$$-\frac{ab\left(\frac{a^2\left(\frac{b\int \sin(c+dx)dx}{a^2+b^2}+\frac{a^2\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}-\frac{a\sin(c+dx)}{d\left(a^2+b^2\right)}\right)}{a^2+b^2}-\frac{a\sin^3(c+dx)}{3d(a^2+b^2)}-\frac{b\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\sin^5(c+dx)}{5d\left(a^2+b^2\right)}-\frac{a\left(\frac{1}{5}\cos^5(c+dx)-\frac{2}{3}\cos^3(c+dx)+\cos(c+dx)\right)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\left(\frac{a^2\left(\frac{b\int \sin(c+dx)dx}{a^2+b^2}+\frac{a^2\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}-\frac{a\sin(c+dx)}{d\left(a^2+b^2\right)}\right)}{a^2+b^2}-\frac{a\sin^3(c+dx)}{3d(a^2+b^2)}-\frac{b\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\sin^5(c+dx)}{5d\left(a^2+b^2\right)}-\frac{a\left(\frac{1}{5}\cos^5(c+dx)-\frac{2}{3}\cos^3(c+dx)+\cos(c+dx)\right)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3118}$$

3.51. $\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx$

$$-\frac{ab\left(\frac{a^2\left(\frac{a^2\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}-\frac{a\sin(c+dx)}{d(a^2+b^2)}-\frac{b\cos(c+dx)}{d(a^2+b^2)}\right)}{a^2+b^2}-\frac{a\sin^3(c+dx)}{3d(a^2+b^2)}-\frac{b\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\sin^5(c+dx)}{5d\left(a^2+b^2\right)}-\frac{a\left(\frac{1}{5}\cos^5(c+dx)-\frac{2}{3}\cos^3(c+dx)+\cos(c+dx)\right)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 3553$$

$$-\frac{ab\left(\frac{a^2\left(-\frac{a^2\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{d(a^2+b^2)}-\frac{a\sin(c+dx)}{d(a^2+b^2)}-\frac{b\cos(c+dx)}{d(a^2+b^2)}\right)}{a^2+b^2}-\frac{a\sin^3(c+dx)}{3d(a^2+b^2)}-\frac{b\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\sin^5(c+dx)}{5d\left(a^2+b^2\right)}-\frac{a\left(\frac{1}{5}\cos^5(c+dx)-\frac{2}{3}\cos^3(c+dx)+\cos(c+dx)\right)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 219$$

$$-\frac{ab\left(\frac{a^2\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}-\frac{a\sin(c+dx)}{d(a^2+b^2)}-\frac{b\cos(c+dx)}{d(a^2+b^2)}\right)}{a^2+b^2}-\frac{a\sin^3(c+dx)}{3d(a^2+b^2)}-\frac{b\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\sin^5(c+dx)}{5d\left(a^2+b^2\right)}-\frac{a\left(\frac{1}{5}\cos^5(c+dx)-\frac{2}{3}\cos^3(c+dx)+\cos(c+dx)\right)}{d\left(a^2+b^2\right)}$$

input
```
Int[Sin[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

output
```
-((a*(Cos[c + d*x] - (2*Cos[c + d*x]^3)/3 + Cos[c + d*x]^5/5))/((a^2 + b^2
)*d)) + (b*Sin[c + d*x]^5)/(5*(a^2 + b^2)*d) - (a*b*(-((b*(Cos[c + d*x] -
Cos[c + d*x]^3/3))/((a^2 + b^2)*d)) - (a*Sin[c + d*x]^3)/(3*(a^2 + b^2)*d)
 + (a^2*(-((a^2*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]
)/((a^2 + b^2)^(3/2)*d)) - (b*Cos[c + d*x])/((a^2 + b^2)*d) - (a*Sin[c + d
*x])/((a^2 + b^2)*d)))/(a^2 + b^2)))/(a^2 + b^2)
```

3.51. $\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx$

3.51.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.51. $\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx$

rule 3578
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Simp[a^2/(a^2 + b^2)   Int[Sin[c + d*x]^(m - 2)/(a
*Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Simp[b/(a^2 + b^2)   Int[Sin[c +
d*x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ
[m, 1]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

3.51.4 Maple [A] (verified)

Time = 9.86 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.31

method	result
derivativedivides	$-\frac{64 a^{5} b \,\operatorname{arctanh}\left(\frac{2 a \tan\left(\frac{d x}{2}+\frac{c}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right)}{\left(32 a^{6}+96 a^{4} b^{2}+96 a^{2} b^{4}+32 b^{6}\right) \sqrt{a^{2}+b^{2}}}+\frac{2 a^{4} b \left(\tan^{9}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+2 a^{3} b^{2} \left(\tan^{8}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+2 \left(\frac{16}{3} a^{4} b+\frac{4}{3} a^{2} b^{3}\right) \left(\tan^{7}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+2 \left(6 \cdots\right.}{\cdots}$
default	$-\frac{64 a^{5} b \,\operatorname{arctanh}\left(\frac{2 a \tan\left(\frac{d x}{2}+\frac{c}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right)}{\left(32 a^{6}+96 a^{4} b^{2}+96 a^{2} b^{4}+32 b^{6}\right) \sqrt{a^{2}+b^{2}}}+\frac{2 a^{4} b \left(\tan^{9}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+2 a^{3} b^{2} \left(\tan^{8}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+2 \left(\frac{16}{3} a^{4} b+\frac{4}{3} a^{2} b^{3}\right) \left(\tan^{7}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)+2 \left(6 \cdots\right.}{\cdots}$
risch	$-\frac{i {\mathrm e}^{3 i \left(d x +c \right)} b}{32 \left(-2 i a b +a^{2}-b^{2}\right) d}+\frac{5 \,{\mathrm e}^{3 i \left(d x +c \right)} a}{96 \left(-2 i a b +a^{2}-b^{2}\right) d}+\frac{i {\mathrm e}^{i \left(d x +c \right)} a b}{4 \left(-3 i b \,a^{2}+i b^{3}+a^{3}-3 a \,b^{2}\right) d}-\frac{5 \,{\mathrm e}^{i \left(d x +c \right)} a^{2}}{16 \left(-3 i b \,a^{2}+i b^{3}+a^{3}-3 a \,b^{2}\right) d}+ \cdots$

input
```
int(sin(d*x+c)^5/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-64*a^5*b/(32*a^6+96*a^4*b^2+96*a^2*b^4+32*b^6)/(a^2+b^2)^(1/2)*arcta
nh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+2/(a^4+2*a^2*b^2+b^4)
/(a^2+b^2)*(a^4*b*tan(1/2*d*x+1/2*c)^9+a^3*b^2*tan(1/2*d*x+1/2*c)^8+(16/3*
a^4*b+4/3*a^2*b^3)*tan(1/2*d*x+1/2*c)^7+(6*a^3*b^2+2*a*b^4)*tan(1/2*d*x+1/
2*c)^6+(178/15*a^4*b+136/15*a^2*b^3+16/5*b^5)*tan(1/2*d*x+1/2*c)^5+(-16/3*
a^5-2/3*a*b^4)*tan(1/2*d*x+1/2*c)^4+(16/3*a^4*b+4/3*a^2*b^3)*tan(1/2*d*x+1
/2*c)^3+(2*a^3*b^2-8/3*a^5+2/3*a*b^4)*tan(1/2*d*x+1/2*c)^2+a^4*b*tan(1/2*d
*x+1/2*c)-8/15*a^5+3/5*a^3*b^2+2/15*a*b^4)/(1+tan(1/2*d*x+1/2*c)^2)^5)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.35

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{15\sqrt{a^2+b^2}a^5b\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)-6\,(a^7+3\,a$$

input
```
integrate(sin(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/30*(15*sqrt(a^2 + b^2)*a^5*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2
 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) -
 a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x +
 c)^2 + b^2)) - 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^5 + 1
0*(2*a^7 + 5*a^5*b^2 + 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 30*(a^7 + a^5*b
^2)*cos(d*x + c) + 2*(23*a^6*b + 34*a^4*b^3 + 14*a^2*b^5 + 3*b^7 + 3*(a^6*
b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^4 - (11*a^6*b + 28*a^4*b^3 +
 23*a^2*b^5 + 6*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8 + 4*a^6*b^2 + 6*a
^4*b^4 + 4*a^2*b^6 + b^8)*d)
```

3.51.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**5/(a+b*tan(d*x+c)),x)
```

output
```
Timed out
```

3.51.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(260) = 520$.

Time = 0.31 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.40

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$= \frac{\frac{15\,a^5 b \log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{2\left(8\,a^5-9\,a^3b^2-2\,ab^4-\frac{15\,a^4 b\sin(dx+c)}{\cos(dx+c)+1}-\frac{15\,a^3b^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{15\,a^4 b\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{10\left(4\,a^5-3\,a^3b^2-ab^4\right)s}{(\cos(dx+c)+1)}\right.}{a^6+3\,a^4b^2+3\,a^2b^4+b^6+\frac{5\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\left(a^6+3\,a^4b^2+\right.}{(\cos(}}}{}$$

input
```
integrate(sin(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/15*(15*a^5*b*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2
))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(8*a^5 - 9*a^3*b^2 - 2*a*b^4
 - 15*a^4*b*sin(d*x + c)/(cos(d*x + c) + 1) - 15*a^3*b^2*sin(d*x + c)^8/(c
os(d*x + c) + 1)^8 - 15*a^4*b*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 10*(4*
a^5 - 3*a^3*b^2 - a*b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 20*(4*a^4*b
 + a^2*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*(8*a^5 + a*b^4)*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 - 2*(89*a^4*b + 68*a^2*b^3 + 24*b^5)*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5 - 30*(3*a^3*b^2 + a*b^4)*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 - 20*(4*a^4*b + a^2*b^3)*sin(d*x + c)^7/(cos(d*x + c) + 1
)^7)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*(a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*(a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + (a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10))/d
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.69

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{15\,a^5 b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2\left(15\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+15\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+80\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+20\,a^2b^3\tan\left(\frac{1}{2}\,c\right.}{}}{}$$

input
```
integrate(sin(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/15*(15*a^5*b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))
/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(15*a^4*b*tan(1/2*d*x + 1/2*c)^9
 + 15*a^3*b^2*tan(1/2*d*x + 1/2*c)^8 + 80*a^4*b*tan(1/2*d*x + 1/2*c)^7 + 2
0*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 90*a^3*b^2*tan(1/2*d*x + 1/2*c)^6 + 30*
a*b^4*tan(1/2*d*x + 1/2*c)^6 + 178*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 136*a^2*
b^3*tan(1/2*d*x + 1/2*c)^5 + 48*b^5*tan(1/2*d*x + 1/2*c)^5 - 80*a^5*tan(1/
2*d*x + 1/2*c)^4 - 10*a*b^4*tan(1/2*d*x + 1/2*c)^4 + 80*a^4*b*tan(1/2*d*x
+ 1/2*c)^3 + 20*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 40*a^5*tan(1/2*d*x + 1/2*
c)^2 + 30*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 10*a*b^4*tan(1/2*d*x + 1/2*c)^2
 + 15*a^4*b*tan(1/2*d*x + 1/2*c) - 8*a^5 + 9*a^3*b^2 + 2*a*b^4)/((a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)^5))/d
```

3.51.9 Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 683, normalized size of antiderivative = 2.49

$$\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{2\,(-8\,a^5+9\,a^3\,b^2+2\,a\,b^4)}{15\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)} + \frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(4\,a^4\,b+a^2\,b^3)}{3\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)} + \frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,(3\,a^3\,b^2+a\,b^4)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6} + \frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(-4\,a^5+3\,a^3\,b^2+a\,b^4)}{3\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)} + }{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\tan\left(\frac{c}{2}+\right.\right.}$$
$$+ \frac{2\,a^5\,b\,\mathrm{atanh}\left(\frac{2\,a^6\,b+2\,b^7+6\,a^2\,b^5+6\,a^4\,b^3-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}{2\,(a^2+b^2)^{7/2}}\right)}{d\,(a^2+b^2)^{7/2}}$$

input
```
int(sin(c + d*x)^5/(a + b*tan(c + d*x)),x)
```

output
```
((2*(2*a*b^4 - 8*a^5 + 9*a^3*b^2))/(15*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)
) + (8*tan(c/2 + (d*x)/2)^3*(4*a^4*b + a^2*b^3))/(3*(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2)) + (4*tan(c/2 + (d*x)/2)^6*(a*b^4 + 3*a^3*b^2))/(a^6 + b^6 +
 3*a^2*b^4 + 3*a^4*b^2) + (4*tan(c/2 + (d*x)/2)^2*(a*b^4 - 4*a^5 + 3*a^3*b
^2))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (4*tan(c/2 + (d*x)/2)^5*(89
*a^4*b + 24*b^5 + 68*a^2*b^3))/(15*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) -
(4*tan(c/2 + (d*x)/2)^4*(a*b^4 + 8*a^5))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2)) + (2*a^3*b^2*tan(c/2 + (d*x)/2)^8)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b
^2) + (8*b*tan(c/2 + (d*x)/2)^7*(4*a^4 + a^2*b^2))/(3*(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2)) + (2*a^4*b*tan(c/2 + (d*x)/2))/(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2) + (2*a^4*b*tan(c/2 + (d*x)/2)^9)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 +
(d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (2*a^5
*b*atanh((2*a^6*b + 2*b^7 + 6*a^2*b^5 + 6*a^4*b^3 - 2*a*tan(c/2 + (d*x)/2)
*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))/(2*(a^2 + b^2)^(7/2))))/(d*(a^2 + b^
2)^(7/2))
```

3.51. $\int \frac{\sin^5(c+dx)}{a+b\tan(c+dx)}\,dx$

3.52 $\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx$

3.52.1 Optimal result

Integrand size = 21, antiderivative size = 158

$$\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{a(3a^4-6a^2b^2-b^4)\,x}{8\,(a^2+b^2)^3}+\frac{a^4b\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}+\frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\,(a^2+b^2)\,d}-\frac{\cos^2(c+dx)\,(4b(2a^2+b^2)+a(5a^2+b^2)\tan(c+dx))}{8\,(a^2+b^2)^2\,d}$$

output
```
1/8*a*(3*a^4-6*a^2*b^2-b^4)*x/(a^2+b^2)^3+a^4*b*ln(a*cos(d*x+c)+b*sin(d*x+
c))/(a^2+b^2)^3/d+1/4*cos(d*x+c)^4*(b+a*tan(d*x+c))/(a^2+b^2)/d-1/8*cos(d*
x+c)^2*(4*b*(2*a^2+b^2)+a*(5*a^2+b^2)*tan(d*x+c))/(a^2+b^2)^2/d
```

3.52.2 Mathematica [A] (verified)

Time = 4.17 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.58

$$\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{2ab(5a^4+6a^2b^2+b^4)\arctan(\tan(c+dx))+8b^2(2a^4+3a^2b^2+b^4)\cos^2(c+dx)-4b^2(a^2+b^2)^2\cos^4(c}{}$$

input
```
Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

output
```
-1/16*(2*a*b*(5*a^4 + 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]] + 8*b^2*(2*a^4
 + 3*a^2*b^2 + b^4)*Cos[c + d*x]^2 - 4*b^2*(a^2 + b^2)^2*Cos[c + d*x]^4 +
8*a^4*((b^2 + a*Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*b^2*Log[a
 + b*Tan[c + d*x]] + (b^2 - a*Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]]
) - 4*a*b*(a^2 + b^2)^2*Cos[c + d*x]^3*Sin[c + d*x] + a*(5*a^4*b + 6*a^2*b
^3 + b^5)*Sin[2*(c + d*x)])/(b*(a^2 + b^2)^3*d)
```

3.52.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.53, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3999, 601, 2178, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(c+dx)^4}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \; 3999$$

$$\frac{b\int \frac{b^4\tan^4(c+dx)}{(a+b\tan(c+dx))(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 601$$

$$\frac{b\left(\frac{b^2\left(ab\tan(c+dx)+b^2\right)}{4(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}-\frac{\int \frac{-\frac{3a\tan(c+dx)b^5}{a^2+b^2}-4\tan^2(c+dx)b^4+\frac{a^2b^4}{a^2+b^2}}{(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2}\right)}{d}$$

$$\Big\downarrow \; 2178$$

$$\frac{b\left(\frac{b^2(ab\tan(c+dx)+b^2)}{4(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}-\frac{\frac{b^2\left(ab\left(5a^2+b^2\right)\tan(c+dx)+4b^2\left(2a^2+b^2\right)\right)}{2\left(a^2+b^2\right)^2\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{\int \frac{ab^4\left(a\left(3a^2-b^2\right)-b\left(5a^2+b^2\right)\tan(c+dx)\right)}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2}}{4b^2}\right)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{b^2(ab\tan(c+dx)+b^2)}{4(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}-\frac{\frac{b^2\left(ab\left(5a^2+b^2\right)\tan(c+dx)+4b^2\left(2a^2+b^2\right)\right)}{2\left(a^2+b^2\right)^2\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{ab^2\int \frac{a\left(3a^2-b^2\right)-b\left(5a^2+b^2\right)\tan(c+dx)}{(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2\left(a^2+b^2\right)^2}}{4b^2}\right)}{d}$$

$$\Big\downarrow \; 657$$

$$\frac{b\left(\frac{b^2(ab\tan(c+dx)+b^2)}{4(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}-\frac{\frac{b^2\left(ab\left(5a^2+b^2\right)\tan(c+dx)+4b^2\left(2a^2+b^2\right)\right)}{2\left(a^2+b^2\right)^2\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{ab^2\int\left(\frac{8a^3}{\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{3a^4-8b\tan(c+dx)a^3-6b^2a^2-b^4}{\left(a^2+b^2\right)\left(\tan^2(c+dx)b^2+b^2\right)}\right)d(}{2\left(a^2+b^2\right)^2}}{4b^2}}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\left(\frac{b^2(ab\tan(c+dx)+b^2)}{4(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}-\frac{\frac{b^2\left(ab\left(5a^2+b^2\right)\tan(c+dx)+4b^2\left(2a^2+b^2\right)\right)}{2\left(a^2+b^2\right)^2\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{ab^2\left(\frac{\left(3a^4-6a^2b^2-b^4\right)\arctan(\tan(c+dx))}{b\left(a^2+b^2\right)}-\frac{4a^3\log\left(b^2\tan^2(c+dx)+b^2\right)}{a^2+b^2}\right.}{2\left(a^2+b^2\right)^2}}{4b^2}}{d}$$

input

```
Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

3.52. $\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx$

output
```
(b*((b^2*(b^2 + a*b*Tan[c + d*x]))/(4*(a^2 + b^2)*(b^2 + b^2*Tan[c + d*x]^
2)^2) - (-1/2*(a*b^2*(((3*a^4 - 6*a^2*b^2 - b^4)*ArcTan[Tan[c + d*x]])/(b*
(a^2 + b^2)) + (8*a^3*Log[a + b*Tan[c + d*x]])/(a^2 + b^2) - (4*a^3*Log[b^
2 + b^2*Tan[c + d*x]^2])/(a^2 + b^2)))/(a^2 + b^2)^2 + (b^2*(4*b^2*(2*a^2
+ b^2) + a*b*(5*a^2 + b^2)*Tan[c + d*x]))/(2*(a^2 + b^2)^2*(b^2 + b^2*Tan[
c + d*x]^2)))/(4*b^2)))/d
```

3.52.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.52.4 Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{5}{8}a^5-\frac{3}{4}a^3b^2-\frac{1}{8}ab^4\right)\left(\tan^3(dx+c)\right)+\left(-a^4b-\frac{3}{2}a^2b^3-\frac{1}{2}b^5\right)\left(\tan^2(dx+c)\right)+\left(-\frac{3}{8}a^5-\frac{1}{4}a^3b^2+\frac{1}{8}ab^4\right)\tan(dx+c)-\frac{3a^4b}{4}-a^2b^3-\cdots}{\left(1+\tan^2(dx+c)\right)^2}}{\left(a^2+b^2\right)^3}\cdots}{d}$
default	$\frac{\frac{\frac{\left(-\frac{5}{8}a^5-\frac{3}{4}a^3b^2-\frac{1}{8}ab^4\right)\left(\tan^3(dx+c)\right)+\left(-a^4b-\frac{3}{2}a^2b^3-\frac{1}{2}b^5\right)\left(\tan^2(dx+c)\right)+\left(-\frac{3}{8}a^5-\frac{1}{4}a^3b^2+\frac{1}{8}ab^4\right)\tan(dx+c)-\frac{3a^4b}{4}-a^2b^3-\cdots}{\left(1+\tan^2(dx+c)\right)^2}}{\left(a^2+b^2\right)^3}\cdots}{d}$
risch	$\frac{iaxb}{24ib\,a^2-8ib^3-8a^3+24a\,b^2}-\frac{3a^2x}{8(3ib\,a^2-ib^3-a^3+3a\,b^2)}+\frac{e^{2i(dx+c)}b}{16(-2iab+a^2-b^2)d}+\frac{ie^{2i(dx+c)}a}{8(-2iab+a^2-b^2)d}+\frac{e^{-2i(dx+c)}\cdots}{16(ib+a)^2\cdots}$

input
```
int(sin(d*x+c)^4/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^3*(((-5/8*a^5-3/4*a^3*b^2-1/8*a*b^4)*tan(d*x+c)^3+(-a^4*b
-3/2*a^2*b^3-1/2*b^5)*tan(d*x+c)^2+(-3/8*a^5-1/4*a^3*b^2+1/8*a*b^4)*tan(d*
x+c)-3/4*a^4*b-a^2*b^3-1/4*b^5)/(1+tan(d*x+c)^2)^2+1/8*a*(-4*a^3*b*ln(1+ta
n(d*x+c)^2)+(3*a^4-6*a^2*b^2-b^4)*arctan(tan(d*x+c))))+a^4*b/(a^2+b^2)^3*l
n(a+b*tan(d*x+c)))
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.37

$$\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{4\,a^4b\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4}{}$$

input
```
integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/8*(4*a^4*b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c
)^2 + b^2) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + (3*a^5 - 6*a^3*b
^2 - a*b^4)*d*x - 4*(2*a^4*b + 3*a^2*b^3 + b^5)*cos(d*x + c)^2 + (2*(a^5 +
 2*a^3*b^2 + a*b^4)*cos(d*x + c)^3 - (5*a^5 + 6*a^3*b^2 + a*b^4)*cos(d*x +
 c))*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

3.52.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

output
```
Timed out
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.77

$$\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{8\,a^4b\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{4\,a^4b\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{\left(3\,a^5-6\,a^3b^2-ab^4\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{\left(5\,a^3+ab^2\right)\tan(dx+c)^3+6\,a^2b+2\,b^3+4\left(2\,a^2b+b^3\right)\mathrm{ta}}{\left(a^4+2\,a^2b^2+b^4\right)\tan(dx+c)^4+a^4+2\,a^2b^2+b^4+2}}{8\,d}$$

input
```
integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/8*(8*a^4*b*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
 4*a^4*b*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3*
a^5 - 6*a^3*b^2 - a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (
(5*a^3 + a*b^2)*tan(d*x + c)^3 + 6*a^2*b + 2*b^3 + 4*(2*a^2*b + b^3)*tan(d
*x + c)^2 + (3*a^3 - a*b^2)*tan(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*tan(d*x
 + c)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^2
))/d
```

3.52.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(152) = 304$.

Time = 0.40 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.11

$$\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{8\,a^4b^2\log\left(|b\tan(dx+c)+a|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{4\,a^4b\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{\left(3\,a^5-6\,a^3b^2-ab^4\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{6\,a^4b\tan(dx+c)^4-5\,a^5\tan(dx+c)^3-6\,a^3b^2\tan(dx}{}}{8}$$

input
```
integrate(sin(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/8*(8*a^4*b^2*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5
 + b^7) - 4*a^4*b*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6) + (3*a^5 - 6*a^3*b^2 - a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6) + (6*a^4*b*tan(d*x + c)^4 - 5*a^5*tan(d*x + c)^3 - 6*a^3*b^2*tan(d*x
 + c)^3 - a*b^4*tan(d*x + c)^3 + 4*a^4*b*tan(d*x + c)^2 - 12*a^2*b^3*tan(d
*x + c)^2 - 4*b^5*tan(d*x + c)^2 - 3*a^5*tan(d*x + c) - 2*a^3*b^2*tan(d*x
+ c) + a*b^4*tan(d*x + c) - 8*a^2*b^3 - 2*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)*(tan(d*x + c)^2 + 1)^2))/d
```

3.52.9 Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.98

$$\int \frac{\sin^4(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{a^4\,b\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,\left(a^2+b^2\right)^3} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a\,b-a^2\,3\mathrm{i}\right)}{16\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}$$
$$- \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-3\,a^2+a\,b\,1\mathrm{i}\right)}{16\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}$$
$$- \frac{\frac{3\,a^2\,b+b^3}{4\,(a^4+2\,a^2\,b^2+b^4)} + \frac{\tan(c+d\,x)^3\,(5\,a^3+a\,b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)} + \frac{\tan(c+d\,x)^2\,(2\,a^2\,b+b^3)}{2\,(a^4+2\,a^2\,b^2+b^4)} + \frac{a\tan(c+d\,x)\,(3\,a^2-b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}$$

input
```
int(sin(c + d*x)^4/(a + b*tan(c + d*x)),x)
```

output
```
(a^4*b*log(a + b*tan(c + d*x)))/(d*(a^2 + b^2)^3) - (log(tan(c + d*x) - 1i
)*(a*b - a^2*3i))/(16*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) - (log(tan(c
+ d*x) + 1i)*(a*b*1i - 3*a^2))/(16*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3))
- ((3*a^2*b + b^3)/(4*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)^3*(a*b^2 +
5*a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)^2*(2*a^2*b + b^3))/(2*
(a^4 + b^4 + 2*a^2*b^2)) + (a*tan(c + d*x)*(3*a^2 - b^2))/(8*(a^4 + b^4 +
2*a^2*b^2)))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

3.53 $\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx$

3.53.1 Optimal result

Integrand size = 21, antiderivative size = 168

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{a^3 b \operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2} d} + \frac{ab^2\cos(c+dx)}{\left(a^2+b^2\right)^2 d} - \frac{a\cos(c+dx)}{\left(a^2+b^2\right) d} + \frac{a\cos^3(c+dx)}{3\left(a^2+b^2\right) d} + \frac{a^2 b\sin(c+dx)}{\left(a^2+b^2\right)^2 d} + \frac{b\sin^3(c+dx)}{3\left(a^2+b^2\right) d}$$

output
```
a^3*b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)
/d+a*b^2*cos(d*x+c)/(a^2+b^2)^2/d-a*cos(d*x+c)/(a^2+b^2)/d+1/3*a*cos(d*x+c
)^3/(a^2+b^2)/d+a^2*b*sin(d*x+c)/(a^2+b^2)^2/d+1/3*b*sin(d*x+c)^3/(a^2+b^2
)/d
```

3.53.2 Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.83

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{-24a^3 b \operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right) + \sqrt{a^2+b^2}\left(\left(-9a^3+3ab^2\right)\cos(c+dx)+a\left(a^2+b^2\right)\cos(3(c+dx)) - 2\right.}{12\left(a^2+b^2\right)^{5/2} d}$$

input
```
Integrate[Sin[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

output
```
(-24*a^3*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 +
 b^2]*((-9*a^3 + 3*a*b^2)*Cos[c + d*x] + a*(a^2 + b^2)*Cos[3*(c + d*x)] -
2*b*(-7*a^2 - b^2 + (a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*(a^2
+ b^2)^(5/2)*d)
```

3.53.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4001, 3042, 3588, 3042, 3044, 15, 3113, 2009, 3578, 3042, 3118, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^3}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4001}$$

$$\int \frac{\sin^3(c+dx)\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^3\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3588}$$

$$\frac{a\int \sin^3(c+dx)dx}{a^2+b^2}+\frac{b\int \cos(c+dx)\sin^2(c+dx)dx}{a^2+b^2}-\frac{ab\int \frac{\sin^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sin(c+dx)^3dx}{a^2+b^2}+\frac{b\int \cos(c+dx)\sin(c+dx)^2dx}{a^2+b^2}-\frac{ab\int \frac{\sin(c+dx)^2}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3044}$$

$$\frac{b\int \sin^2(c+dx)d\sin(c+dx)}{d\left(a^2+b^2\right)}+\frac{a\int \sin(c+dx)^3dx}{a^2+b^2}-\frac{ab\int \frac{\sin(c+dx)^2}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{a\int \sin(c+dx)^3dx}{a^2+b^2}-\frac{ab\int \frac{\sin(c+dx)^2}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\sin^3(c+dx)}{3d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{a\int \left(1-\cos^2(c+dx)\right)d\cos(c+dx)}{d\left(a^2+b^2\right)}-\frac{ab\int \frac{\sin(c+dx)^2}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\sin^3(c+dx)}{3d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ab\int \frac{\sin(c+dx)^2}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\sin^3(c+dx)}{3d\left(a^2+b^2\right)}-\frac{a\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3578}$$

$$-\frac{ab\left(\frac{b\int \sin(c+dx)dx}{a^2+b^2}+\frac{a^2\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}-\frac{a\sin(c+dx)}{d(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\sin^3(c+dx)}{3d\left(a^2+b^2\right)}-\frac{a\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\left(\frac{b\int \sin(c+dx)dx}{a^2+b^2}+\frac{a^2\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}-\frac{a\sin(c+dx)}{d(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\sin^3(c+dx)}{3d\left(a^2+b^2\right)}-\frac{a\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{ab\left(\frac{a^2\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}-\frac{a\sin(c+dx)}{d(a^2+b^2)}-\frac{b\cos(c+dx)}{d(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\sin^3(c+dx)}{3d\left(a^2+b^2\right)}-\frac{a\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{ab\left(-\frac{a^2\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{d(a^2+b^2)}-\frac{a\sin(c+dx)}{d(a^2+b^2)}-\frac{b\cos(c+dx)}{d(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\sin^3(c+dx)}{3d\left(a^2+b^2\right)}-\frac{a\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{219}$$

3.53. $\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx$

$$-\frac{ab\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}-\frac{a\sin(c+dx)}{d(a^2+b^2)}-\frac{b\cos(c+dx)}{d(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\sin^3(c+dx)}{3d\,(a^2+b^2)}-\frac{a\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d\,(a^2+b^2)}$$

input
```
Int[Sin[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

output
```
-((a*(Cos[c + d*x] - Cos[c + d*x]^3/3))/((a^2 + b^2)*d)) + (b*Sin[c + d*x]
^3)/(3*(a^2 + b^2)*d) - (a*b*(-((a^2*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d
*x])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d)) - (b*Cos[c + d*x])/((a^2 + b
^2)*d) - (a*Sin[c + d*x])/((a^2 + b^2)*d)))/(a^2 + b^2)
```

3.53.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3578
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Simp[a^2/(a^2 + b^2)   Int[Sin[c + d*x]^(m - 2)/(a
*Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Simp[b/(a^2 + b^2)   Int[Sin[c +
d*x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ
[m, 1]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

3.53. $\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx$

3.53.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{-\frac{16a^3b\ \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(8a^4+16a^2b^2+8b^4\right)\sqrt{a^2+b^2}}+\frac{2a^2b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\,b^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\frac{10}{3}a^2b+\frac{4}{3}b^3\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a^3\left(\tan\right.}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{d}$
default	$\frac{-\frac{16a^3b\ \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(8a^4+16a^2b^2+8b^4\right)\sqrt{a^2+b^2}}+\frac{2a^2b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\,b^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\frac{10}{3}a^2b+\frac{4}{3}b^3\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a^3\left(\tan\right.}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{d}$
risch	$\frac{i e^{i(dx+c)}b}{8(-2iab+a^2-b^2)d}-\frac{3\,e^{i(dx+c)}a}{8(-2iab+a^2-b^2)d}-\frac{ie^{-i(dx+c)}b}{8(ib+a)^2d}-\frac{3\,e^{-i(dx+c)}a}{8(ib+a)^2d}-\frac{ib\,a^3\ln\left(e^{i(dx+c)}-\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)^2d}+\frac{ib\,a}{}$

input
```
int(sin(d*x+c)^3/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-16*a^3*b/(8*a^4+16*a^2*b^2+8*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*t
an(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+2/(a^4+2*a^2*b^2+b^4)*(a^2*b*tan(1
/2*d*x+1/2*c)^5+a*b^2*tan(1/2*d*x+1/2*c)^4+(10/3*a^2*b+4/3*b^3)*tan(1/2*d*
x+1/2*c)^3-2*a^3*tan(1/2*d*x+1/2*c)^2+a^2*b*tan(1/2*d*x+1/2*c)-2/3*a^3+1/3
*a*b^2)/(1+tan(1/2*d*x+1/2*c)^2)^3)
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.55

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$=\frac{3\sqrt{a^2+b^2}a^3b\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)+2\left(a^5+2\,a^3\right.}{6\left(a^6\right.}$$

input
```
integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*sqrt(a^2 + b^2)*a^3*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
 b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a
*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c
)^2 + b^2)) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)^3 - 6*(a^5 + a^3*b^
2)*cos(d*x + c) + 2*(4*a^4*b + 5*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)
*cos(d*x + c)^2)*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

3.53.6 Sympy [F]

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

input
```
integrate(sin(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)**3/(a + b*tan(c + d*x)), x)
```

3.53.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(160) = 320$.

Time = 0.30 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.17

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{3a^3b\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(2a^3-ab^2-\frac{3a^2b\sin(dx+c)}{\cos(dx+c)+1}+\frac{6a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3ab^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3a^2b\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2(5a^2b+2b^3)\sin(dx-}{(\cos(dx+c)+1)^3}\right.}{a^4+2a^2b^2+b^4+\frac{3(a^4+2a^2b^2+b^4)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3(a^4+2a^2b^2+b^4)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(a^4+2a^2b^2+b^4)\sin(dx+c}{(\cos(dx+c)+1)^6}}}{3d}$$

input
```
integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*(3*a^3*b*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))
/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*
b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(2*a^3 - a*b^2 - 3*a^2*b*sin(d*x + c)/(cos
(d*x + c) + 1) + 6*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 3*a*b^2*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 - 3*a^2*b*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 - 2*(5*a^2*b + 2*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^4 + 2*a^2
*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^4 +
2*a^2*b^2 + b^4)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.43

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{3\,a^3 b \log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+10\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3\right.}{(a^4+2\,a^2b^2+b^4)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}}{3\,d}$$

input
```
integrate(sin(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/3*(3*a^3*b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/a
bs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2
+ b^4)*sqrt(a^2 + b^2)) + 2*(3*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 3*a*b^2*tan(
1/2*d*x + 1/2*c)^4 + 10*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 4*b^3*tan(1/2*d*x +
 1/2*c)^3 - 6*a^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b*tan(1/2*d*x + 1/2*c) -
2*a^3 + a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 + 1)^3))/d
```

3.53.9 Mupad [B] (verification not implemented)

Time = 7.21 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.93

$$\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{\frac{2\,a\,b^2}{3}-\frac{4\,a^3}{3}}{a^4+2\,a^2\,b^2+b^4} - \frac{4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{20\,a^2\,b}{3}+\frac{8\,b^3}{3}\right)}{a^4+2\,a^2\,b^2+b^4} + \frac{2\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4+2\,a^2\,b^2+b^4} + \frac{2\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{a^4+2\,a^2\,b^2+b^4} + \frac{2\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^4+2\,a^2\,b^2+b^4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$
$$+ \frac{2\,a^3\,b\,\mathrm{atanh}\left(\frac{a^4\,b+b^5+2\,a^2\,b^3-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}{\left(a^2+b^2\right)^{5/2}}\right)}{d\,\left(a^2+b^2\right)^{5/2}}$$

input
```
int(sin(c + d*x)^3/(a + b*tan(c + d*x)),x)
```

output

```
(((2*a*b^2)/3 - (4*a^3)/3)/(a^4 + b^4 + 2*a^2*b^2) - (4*a^3*tan(c/2 + (d*x
)/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^3*((20*a^2*b)/3 + (8
*b^3)/3))/(a^4 + b^4 + 2*a^2*b^2) + (2*a^2*b*tan(c/2 + (d*x)/2))/(a^4 + b^
4 + 2*a^2*b^2) + (2*a*b^2*tan(c/2 + (d*x)/2)^4)/(a^4 + b^4 + 2*a^2*b^2) +
(2*a^2*b*tan(c/2 + (d*x)/2)^5)/(a^4 + b^4 + 2*a^2*b^2))/(d*(3*tan(c/2 + (d
*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (2*a^3*b*
atanh((a^4*b + b^5 + 2*a^2*b^3 - a*tan(c/2 + (d*x)/2)*(a^4 + b^4 + 2*a^2*b
^2))/(a^2 + b^2)^(5/2)))/(d*(a^2 + b^2)^(5/2))
```

3.53. $\int \frac{\sin^3(c+dx)}{a+b\tan(c+dx)}\,dx$

3.54 $\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx$

3.54.1 Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{a(a^2-b^2)\,x}{2\,(a^2+b^2)^2} + \frac{a^2b\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d} - \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\,(a^2+b^2)\,d}$$

output
```
1/2*a*(a^2-b^2)*x/(a^2+b^2)^2+a^2*b*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2
)^2/d-1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))/(a^2+b^2)/d
```

3.54.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.81

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{2ab(a^2+b^2)\arctan(\tan(c+dx)) + 2b^2(a^2+b^2)\cos^2(c+dx) + a\left(2a\left(\left(b^2+a\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\tan\right.\right.\right.}{4b}$$

input
```
Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

output
```
-1/4*(2*a*b*(a^2 + b^2)*ArcTan[Tan[c + d*x]] + 2*b^2*(a^2 + b^2)*Cos[c + d
*x]^2 + a*(2*a*((b^2 + a*Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*
b^2*Log[a + b*Tan[c + d*x]] + (b^2 - a*Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[
c + d*x]]) + b*(a^2 + b^2)*Sin[2*(c + d*x)]))/(b*(a^2 + b^2)^2*d)
```

3.54.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.55, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3999, 601, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^2}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3999}$$

$$\frac{b\int \frac{b^2\tan^2(c+dx)}{(a+b\tan(c+dx))(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{601}$$

$$\frac{b\left(-\frac{\int -\frac{ab^2(a-b\tan(c+dx))}{\left(a^2+b^2\right)(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2}-\frac{ab\tan(c+dx)+b^2}{2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\int \frac{ab^2(a-b\tan(c+dx))}{\left(a^2+b^2\right)(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2}-\frac{ab\tan(c+dx)+b^2}{2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{a\int \frac{a-b\tan(c+dx)}{(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2(a^2+b^2)}-\frac{ab\tan(c+dx)+b^2}{2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{657}$$

$$\frac{b\left(\frac{a\int\left(\frac{2a}{(a^2+b^2)(a+b\tan(c+dx))}+\frac{a^2-2b\tan(c+dx)a-b^2}{(a^2+b^2)(\tan^2(c+dx)b^2+b^2)}\right)d(b\tan(c+dx))}{2(a^2+b^2)}-\frac{ab\tan(c+dx)+b^2}{2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{a\left(\frac{(a^2-b^2)\arctan(\tan(c+dx))}{b(a^2+b^2)}-\frac{a\log(b^2\tan^2(c+dx)+b^2)}{a^2+b^2}+\frac{2a\log(a+b\tan(c+dx))}{a^2+b^2}\right)}{2(a^2+b^2)}-\frac{ab\tan(c+dx)+b^2}{2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

input
```
Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

output
```
(b*((a*(((a^2 - b^2)*ArcTan[Tan[c + d*x]])/(b*(a^2 + b^2)) + (2*a*Log[a +
b*Tan[c + d*x]])/(a^2 + b^2) - (a*Log[b^2 + b^2*Tan[c + d*x]^2])/(a^2 + b^
2)))/(2*(a^2 + b^2)) - (b^2 + a*b*Tan[c + d*x])/(2*(a^2 + b^2)*(b^2 + b^2*
Tan[c + d*x]^2))))/d
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.54.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{1}{2}a^3-\frac{1}{2}a\,b^2\right)\tan(dx+c)-\frac{a^2b}{2}-\frac{b^3}{2}}{1+\tan^2(dx+c)}+\frac{a\left(-ab\ln\left(1+\tan^2(dx+c)\right)+\left(a^2-b^2\right)\arctan(\tan(dx+c))\right)}{2}}{\left(a^2+b^2\right)^2}+\frac{a^2b\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}}{d}$
default	$\frac{\frac{\frac{\left(-\frac{1}{2}a^3-\frac{1}{2}a\,b^2\right)\tan(dx+c)-\frac{a^2b}{2}-\frac{b^3}{2}}{1+\tan^2(dx+c)}+\frac{a\left(-ab\ln\left(1+\tan^2(dx+c)\right)+\left(a^2-b^2\right)\arctan(\tan(dx+c))\right)}{2}}{\left(a^2+b^2\right)^2}+\frac{a^2b\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}}{d}$
risch	$-\frac{ax}{2(2iab-a^2+b^2)}+\frac{i\mathrm{e}^{2i(dx+c)}}{8(-ib+a)d}-\frac{i\mathrm{e}^{-2i(dx+c)}}{8(ib+a)d}-\frac{2ia^2bx}{a^4+2a^2b^2+b^4}-\frac{2ia^2bc}{d(a^4+2a^2b^2+b^4)}+\frac{a^2b\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{d(a^4+2a^2b^2+b^4)}$

input
```
int(sin(d*x+c)^2/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^2*(((-1/2*a^3-1/2*a*b^2)*tan(d*x+c)-1/2*a^2*b-1/2*b^3)/(1
+tan(d*x+c)^2)+1/2*a*(-a*b*ln(1+tan(d*x+c)^2)+(a^2-b^2)*arctan(tan(d*x+c))
))+a^2*b/(a^2+b^2)^2*ln(a+b*tan(d*x+c)))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{a^2b\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+\left(a^3-ab^2\right)dx-\left(a^2b+b^3\right)\cos\left(dx+\right.}{2\left(a^4+2\,a^2b^2+b^4\right)d}$$

input
```
integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a^2*b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^
2 + b^2) + (a^3 - a*b^2)*d*x - (a^2*b + b^3)*cos(d*x + c)^2 - (a^3 + a*b^2
)*cos(d*x + c)*sin(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d)
```

3.54.6 Sympy [F]

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sin^2\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

input
```
integrate(sin(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.53

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{2\,a^2b\log(b\tan(dx+c)+a)}{a^4+2\,a^2b^2+b^4}-\frac{a^2b\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{(a^3-ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{a\tan(dx+c)+b}{(a^2+b^2)\tan(dx+c)^2+a^2+b^2}}{2\,d}$$

input
```
integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(2*a^2*b*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - a^2*b*log(t
an(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 - a*b^2)*(d*x + c)/(a^4
+ 2*a^2*b^2 + b^4) - (a*tan(d*x + c) + b)/((a^2 + b^2)*tan(d*x + c)^2 + a^
2 + b^2))/d
```

3.54.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(90) = 180$.

Time = 0.39 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.96

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{2\,a^2b^2\log(|b\tan(dx+c)+a|)}{a^4b+2\,a^2b^3+b^5} - \frac{a^2b\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} + \frac{(a^3-ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4} + \frac{a^2b\tan(dx+c)^2-a^3\tan(dx+c)-ab^2\tan(dx+c)-b^3}{(a^4+2\,a^2b^2+b^4)\left(\tan(dx+c)^2+1\right)}}{2\,d}$$

input
```
integrate(sin(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*a^2*b^2*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - a^
2*b*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 - a*b^2)*(d*x +
 c)/(a^4 + 2*a^2*b^2 + b^4) + (a^2*b*tan(d*x + c)^2 - a^3*tan(d*x + c) - a
*b^2*tan(d*x + c) - b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(d*x + c)^2 + 1)))/d
```

3.54.9 Mupad [B] (verification not implemented)

Time = 5.01 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.56

$$\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{a^2\,b\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,\left(a^2+b^2\right)^2} - \frac{a\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)}{4\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)} - \frac{\cos\left(c+d\,x\right)^2\left(\frac{b}{2\,(a^2+b^2)}+\frac{a\tan(c+d\,x)}{2\,(a^2+b^2)}\right)}{d} - \frac{a\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{4\,d\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}$$

input
```
int(sin(c + d*x)^2/(a + b*tan(c + d*x)),x)
```

output
```
(a^2*b*log(a + b*tan(c + d*x)))/(d*(a^2 + b^2)^2) - (a*log(tan(c + d*x) +
1i)*1i)/(4*d*(a*b*2i - a^2 + b^2)) - (a*log(tan(c + d*x) - 1i))/(4*d*(2*a*
b - a^2*1i + b^2*1i)) - (cos(c + d*x)^2*(b/(2*(a^2 + b^2)) + (a*tan(c + d*
x))/(2*(a^2 + b^2))))/d
```

3.54. $\int \frac{\sin^2(c+dx)}{a+b\tan(c+dx)}\,dx$

3.55 $\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx$

3.55.1 Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{ab\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{a\cos(c+dx)}{(a^2+b^2)\,d} + \frac{b\sin(c+dx)}{(a^2+b^2)\,d}$$

output
```
a*b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d
-a*cos(d*x+c)/(a^2+b^2)/d+b*sin(d*x+c)/(a^2+b^2)/d
```

3.55.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{-2ab\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right) + \sqrt{a^2+b^2}(-a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^{3/2}\,d}$$

input
```
Integrate[Sin[c + d*x]/(a + b*Tan[c + d*x]),x]
```

output
```
(-2*a*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^
2]*(-(a*Cos[c + d*x]) + b*Sin[c + d*x]))/((a^2 + b^2)^(3/2)*d)
```

3.55.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 4001, 3042, 3588, 3042, 3117, 3118, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 4001}$$

$$\int \frac{\sin(c+dx)\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(c+dx)\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 3588}$$

$$\frac{a\int \sin(c+dx)dx}{a^2+b^2}+\frac{b\int \cos(c+dx)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\int \sin(c+dx)dx}{a^2+b^2}+\frac{b\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{a\int \sin(c+dx)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\sin(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 3118}$$

$$-\frac{ab\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\sin(c+dx)}{d\left(a^2+b^2\right)}-\frac{a\cos(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 3553}$$

$$\frac{ab\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{b\sin(c+dx)}{d\left(a^2+b^2\right)}-\frac{a\cos(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow 219$$

$$\frac{ab\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}+\frac{b\sin(c+dx)}{d\left(a^2+b^2\right)}-\frac{a\cos(c+dx)}{d\left(a^2+b^2\right)}$$

input
```
Int[Sin[c + d*x]/(a + b*Tan[c + d*x]),x]
```

output
```
(a*b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 + b
^2)^(3/2)*d) - (a*Cos[c + d*x])/((a^2 + b^2)*d) + (b*Sin[c + d*x])/((a^2 +
 b^2)*d)
```

3.55.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

3.55.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2a}{\left(a^2+b^2\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4ab\ \text{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}}{d}$	101
default	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2a}{\left(a^2+b^2\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4ab\ \text{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(2a^2+2b^2\right)\sqrt{a^2+b^2}}}{d}$	101
risch	$-\frac{e^{i(dx+c)}}{2(-ib+a)d}-\frac{e^{-i(dx+c)}}{2(ib+a)d}+\frac{iba\ln\left(e^{i(dx+c)}+\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)d}-\frac{iba\ln\left(e^{i(dx+c)}-\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)d}$	169

input
```
int(sin(d*x+c)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/(a^2+b^2)*(b*tan(1/2*d*x+1/2*c)-a)/(1+tan(1/2*d*x+1/2*c)^2)-4*a*b/(
2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2
+b^2)^(1/2)))
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(86) = 172.

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.06

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\sqrt{a^2+b^2}ab\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)-2\,(a^3+ab^2)\,\mathrm{co}}{2\,(a^4+2\,a^2b^2+b^4)d}$$

input
```
integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(sqrt(a^2 + b^2)*a*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin
(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2)) - 2*(a^3 + a*b^2)*cos(d*x + c) + 2*(a^2*b + b^3)*sin(d*x + c))/((a
^4 + 2*a^2*b^2 + b^4)*d)
```

3.55.6 Sympy [F]

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx$$

input
```
integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)/(a + b*tan(c + d*x)), x)
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.57

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{ab\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(a-\frac{b\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2+b^2+\frac{(a^2+b^2)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

input
```
integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
(a*b*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*
sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*
(a - b*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2 + b^2 + (a^2 + b^2)*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2))/d
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.31

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{ab\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\,\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}{(a^2+b^2)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}}{d}$$

input
```
integrate(sin(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
(a*b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(b*t
an(1/2*d*x + 1/2*c) - a)/((a^2 + b^2)*(tan(1/2*d*x + 1/2*c)^2 + 1)))/d
```

3.55.9 Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.22

$$\int \frac{\sin(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\,a\,b\,\mathrm{atanh}\left(\frac{a^2\,b+b^3-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2+b^2)}{(a^2+b^2)^{3/2}}\right)}{d\,(a^2+b^2)^{3/2}}-\frac{\frac{2\,a}{a^2+b^2}-\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2+b^2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int(sin(c + d*x)/(a + b*tan(c + d*x)),x)
```

output
```
(2*a*b*atanh((a^2*b + b^3 - a*tan(c/2 + (d*x)/2)*(a^2 + b^2))/(a^2 + b^2)^
(3/2)))/(d*(a^2 + b^2)^(3/2)) - ((2*a)/(a^2 + b^2) - (2*b*tan(c/2 + (d*x)/
2))/(a^2 + b^2))/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.56 $\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx$

3.56.1 Optimal result

Integrand size = 19, antiderivative size = 66

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{ad} + \frac{b\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}$$

output
```
-arctanh(cos(d*x+c))/a/d+b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(
1/2))/a/d/(a^2+b^2)^(1/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{-\frac{2b\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

input
```
Integrate[Csc[c + d*x]/(a + b*Tan[c + d*x]),x]
```

output
```
((-2*b*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
 - Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]])/(a*d)
```

3.56.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 4001, 3042, 3589, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(c+dx)(a+b\tan(c+dx))}dx$$
$$\Big\downarrow \text{4001}$$
$$\int \frac{\cot(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(c+dx)}{\sin(c+dx)(a\cos(c+dx)+b\sin(c+dx))}dx$$
$$\Big\downarrow \text{3589}$$
$$\int \left(\frac{\csc(c+dx)}{a}-\frac{b}{a(a\cos(c+dx)+b\sin(c+dx))}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{b\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}}-\frac{\text{arctanh}(\cos(c+dx))}{ad}$$

input
```
Int[Csc[c + d*x]/(a + b*Tan[c + d*x]),x]
```

output
```
-(ArcTanh[Cos[c + d*x]]/(a*d)) + (b*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*
x])/Sqrt[a^2 + b^2]])/(a*Sqrt[a^2 + b^2]*d)
```

3.56.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3589
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

3.56.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}}{d}$	63
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}}{d}$	63
risch	$-\frac{ib\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,da}+\frac{ib\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,da}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{ad}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{ad}$	150

input
```
int(csc(d*x+c)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a*ln(tan(1/2*d*x+1/2*c))-2*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan
(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

3.56.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(62) = 124$.

Time = 0.31 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.77

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\sqrt{a^2+b^2}b\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)-(a^2+b^2)\log\left(\frac{1}{2}\right.}{2\,(a^3+ab^2)d}$$

input
```
integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(sqrt(a^2 + b^2)*b*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*
cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d
*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
b^2)) - (a^2 + b^2)*log(1/2*cos(d*x + c) + 1/2) + (a^2 + b^2)*log(-1/2*cos
(d*x + c) + 1/2))/((a^3 + a*b^2)*d)
```

3.56.6 Sympy [F]

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\csc\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

input
```
integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)/(a + b*tan(c + d*x)), x)
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.62

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{b\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

input
```
integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
(b*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*si
n(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + lo
g(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}}{d}$$

input
```
integrate(csc(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
(b*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(ab
s(tan(1/2*d*x + 1/2*c)))/a)/d
```

3.56.9 Mupad [B] (verification not implemented)

Time = 4.91 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.64

$$\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d} - \frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{a^2+b^2}\,\left(1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+2\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+4\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2\right)}{b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,4\mathrm{i}+a\,b^2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}+a^2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+b^2\right)\,1\mathrm{i}}\right)}{a\,d\,\sqrt{a^2+b^2}}$$

input
```
int(1/(sin(c + d*x)*(a + b*tan(c + d*x))),x)
```

output
```
log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d) - (2*b*atanh(((a^2 + b^2)
^(1/2)*(a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2 + (d*x)/2)*4i + a*b*cos(c/
2 + (d*x)/2)*2i))/(b^3*sin(c/2 + (d*x)/2)*4i + a*b^2*cos(c/2 + (d*x)/2)*1i
 + a^2*b*sin(c/2 + (d*x)/2)*3i + a*cos(c/2 + (d*x)/2)*(a^2 + b^2)*1i)))/(a
*d*(a^2 + b^2)^(1/2))
```

3.56. $\int \frac{\csc(c+dx)}{a+b\tan(c+dx)}\,dx$

3.57 $\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx$

3.57.1 Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx=-\frac{\cot(c+dx)}{ad}-\frac{b\log(\tan(c+dx))}{a^2d}+\frac{b\log(a+b\tan(c+dx))}{a^2d}$$

output
```
-cot(d*x+c)/a/d-b*ln(tan(d*x+c))/a^2/d+b*ln(a+b*tan(d*x+c))/a^2/d
```

3.57.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{-a\cot(c+dx)+b(-\log(\sin(c+dx))+\log(a\cos(c+dx)+b\sin(c+dx)))}{a^2d}$$

input
```
Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

output
```
(-(a*Cot[c + d*x]) + b*(-Log[Sin[c + d*x]] + Log[a*Cos[c + d*x] + b*Sin[c
+ d*x]]))/(a^2*d)
```

3.57.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3999, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(c+dx)^2(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \; 3999$$

$$\frac{b\int \frac{\cot^2(c+dx)}{b^2(a+b\tan(c+dx))}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 54$$

$$\frac{b\int \left(\frac{\cot^2(c+dx)}{ab^2}-\frac{\cot(c+dx)}{a^2b}+\frac{1}{a^2(a+b\tan(c+dx))}\right)d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\left(-\frac{\log(b\tan(c+dx))}{a^2}+\frac{\log(a+b\tan(c+dx))}{a^2}-\frac{\cot(c+dx)}{ab}\right)}{d}$$

input
```
Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

output
```
(b*(-(Cot[c + d*x]/(a*b)) - Log[b*Tan[c + d*x]]/a^2 + Log[a + b*Tan[c + d*
x]]/a^2))/d
```

3.57.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.57.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-\frac{1}{a\tan(dx+c)}-\frac{b\ln(\tan(dx+c))}{a^2}+\frac{b\ln(a+b\tan(dx+c))}{a^2}}{d}$	48
default	$\frac{-\frac{1}{a\tan(dx+c)}-\frac{b\ln(\tan(dx+c))}{a^2}+\frac{b\ln(a+b\tan(dx+c))}{a^2}}{d}$	48
risch	$-\frac{2i}{da\left(e^{2i(dx+c)}-1\right)}-\frac{b\ln\left(e^{2i(dx+c)}-1\right)}{a^2d}+\frac{b\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{a^2d}$	82

input
```
int(csc(d*x+c)^2/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a/tan(d*x+c)-1/a^2*b*ln(tan(d*x+c))+1/a^2*b*ln(a+b*tan(d*x+c)))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.90

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{b\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sin\left(dx+c\right)-b\log\left(-\frac{1}{4}\,\cos\left(dx+c\right)^2+\right.}{2\,a^2d\sin\left(dx+c\right)}$$

input
```
integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
b^2)*sin(d*x + c) - b*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 2*a*co
s(d*x + c))/(a^2*d*sin(d*x + c))
```

3.57.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\csc^2\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

input
```
integrate(csc(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{b\log(b\tan(dx+c)+a)}{a^2}-\frac{b\log(\tan(dx+c))}{a^2}-\frac{1}{a\tan(dx+c)}}{d}$$

input
```
integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
(b*log(b*tan(d*x + c) + a)/a^2 - b*log(tan(d*x + c))/a^2 - 1/(a*tan(d*x +
c)))/d
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{b\log(|b\tan(dx+c)+a|)}{a^2} - \frac{b\log(|\tan(dx+c)|)}{a^2} + \frac{b\tan(dx+c)-a}{a^2\tan(dx+c)}}{d}$$

input
```
integrate(csc(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
(b*log(abs(b*tan(d*x + c) + a))/a^2 - b*log(abs(tan(d*x + c)))/a^2 + (b*ta
n(d*x + c) - a)/(a^2*tan(d*x + c)))/d
```

3.57.9 Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{\csc^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\tan(c+d\,x)}{a}+1\right)}{a^2\,d} - \frac{\cot(c+d\,x)}{a\,d}$$

input
```
int(1/(sin(c + d*x)^2*(a + b*tan(c + d*x))),x)
```

output
```
(2*b*atanh((2*b*tan(c + d*x))/a + 1))/(a^2*d) - cot(c + d*x)/(a*d)
```

3.58 $\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx$

3.58.1 Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\cos(c+dx))}{2ad} - \frac{b^2\operatorname{arctanh}(\cos(c+dx))}{a^3d} + \frac{b\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a^3d} + \frac{b\csc(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

output
```
-1/2*arctanh(cos(d*x+c))/a/d-b^2*arctanh(cos(d*x+c))/a^3/d+b*csc(d*x+c)/a^
2/d-1/2*cot(d*x+c)*csc(d*x+c)/a/d+b*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a
^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/a^3/d
```

3.58.2 Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.47

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{-16b\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right) + 4ab\cot\left(\frac{1}{2}(c+dx)\right) - a^2\csc^2\left(\frac{1}{2}(c+dx)\right) - 4a^2\log\left(\cos\left(\frac{1}{2}(c-\right.\right.}{}$$

input
```
Integrate[Csc[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

output
```
(-16*b*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]
+ 4*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 - 4*a^2*Log[Cos[(c + d*x
)/2]] - 8*b^2*Log[Cos[(c + d*x)/2]] + 4*a^2*Log[Sin[(c + d*x)/2]] + 8*b^2*
Log[Sin[(c + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 + 4*a*b*Tan[(c + d*x)/2])/(
8*a^3*d)
```

3.58.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4001, 3042, 3589, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^3(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{4001}$$

$$\int \frac{\cot(c+dx)\csc^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)}{\sin(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3589}$$

$$\int \left(-\frac{b^3\sec^2(c+dx)}{a^3(a\cos(c+dx)+b\sin(c+dx))}+\frac{b^2\csc(c+dx)\sec^2(c+dx)}{a^3}-\frac{b\csc^2(c+dx)\sec(c+dx)}{a^2}+\frac{\csc^3(c+dx)}{a}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2\text{arctanh}(\cos(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}+\frac{b\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{a^3d}-\frac{\text{arctanh}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

input
```
Int[Csc[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

output
```
-1/2*ArcTanh[Cos[c + d*x]]/(a*d) - (b^2*ArcTanh[Cos[c + d*x]])/(a^3*d) + (
b*Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2
]])/(a^3*d) + (b*Csc[c + d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*
d)
```

3.58.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3589
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

3.58.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{2}+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{2b\sqrt{a^2+b^2}\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^3}-\frac{1}{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(2a^2+4b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}}{d}$
default	$\frac{\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{2}+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{2b\sqrt{a^2+b^2}\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^3}-\frac{1}{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(2a^2+4b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}}{d}$
risch	$\frac{ie^{i(dx+c)}\left(-ia\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-ia-2b\right)}{d\,a^2\left(e^{2i(dx+c)}-1\right)^2}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2ad}+\frac{\ln\left(e^{i(dx+c)}-1\right)b^2}{a^3d}-\frac{i\sqrt{-a^2-b^2}\,b\ln\left(e^{i(dx+c)}-\right.}{d\,a^3}$

input
```
int(csc(d*x+c)^3/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4/a^2*(1/2*tan(1/2*d*x+1/2*c)^2*a+2*b*tan(1/2*d*x+1/2*c))-2*b*(a^2+
b^2)^(1/2)/a^3*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-1
/8/a/tan(1/2*d*x+1/2*c)^2+1/4/a^3*(2*a^2+4*b^2)*ln(tan(1/2*d*x+1/2*c))+1/2
/a^2*b/tan(1/2*d*x+1/2*c))
```

3.58.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(114) = 228$.

Time = 0.31 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.21

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$=\frac{2a^2\cos(dx+c)-4ab\sin(dx+c)+2\left(b\cos(dx+c)^2-b\right)\sqrt{a^2+b^2}\log\left(\frac{2ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx}{2ab\cos(dx+c)\sin(d}\right.}{}$$

input
```
integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*a^2*cos(d*x + c) - 4*a*b*sin(d*x + c) + 2*(b*cos(d*x + c)^2 - b)*sq
rt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x +
 c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))
/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - (
(a^2 + 2*b^2)*cos(d*x + c)^2 - a^2 - 2*b^2)*log(1/2*cos(d*x + c) + 1/2) +
((a^2 + 2*b^2)*cos(d*x + c)^2 - a^2 - 2*b^2)*log(-1/2*cos(d*x + c) + 1/2))
/(a^3*d*cos(d*x + c)^2 - a^3*d)
```

3.58.6 Sympy [F]

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**3/(a + b*tan(c + d*x)), x)
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.76

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{\frac{4\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^2}+\frac{4\,(a^2+2\,b^2)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\left(a-\frac{4\,b\sin(dx+c)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)^2}{a^2\sin(dx+c)^2}+\frac{8\,(a^2b+b^3)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a\cdots}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a\cdots}}\right.}{\sqrt{a^2+b^2}\,a^3}}{8\,d}$$

input
```
integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/8*((4*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2)/a^2 + 4*(a^2 + 2*b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 -
(a - 4*b*sin(d*x + c)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)^2/(a^2*sin(d*
x + c)^2) + 8*(a^2*b + b^3)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + s
qrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*a^3))/d
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.71

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}+\frac{4\,(a^2+2\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{8\,(a^2b+b^3)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^3}-\frac{6\,a^2\tan\cdots}{\cdots}}{8\,d}$$

input
```
integrate(csc(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/8*((a*tan(1/2*d*x + 1/2*c)^2 + 4*b*tan(1/2*d*x + 1/2*c))/a^2 + 4*(a^2 +
2*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 8*(a^2*b + b^3)*log(abs(2*a*ta
n(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c)
 - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) - (6*a^2*tan(1/2*d*x +
1/2*c)^2 + 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 4*a*b*tan(1/2*d*x + 1/2*c) + a^
2)/(a^3*tan(1/2*d*x + 1/2*c)^2))/d
```

3.58.9 Mupad [B] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 764, normalized size of antiderivative = 6.26

$$\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{b^2\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2\,\left(\frac{a^3\,d}{2}-\frac{a^3\,d\,\cos(2\,c+2\,d\,x)}{2}\right)}-\frac{a^2\,\left(\frac{\cos(c+d\,x)}{2}-\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4}+\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos(2\,c+2\,d\,x)}{4}\right)}{\frac{a^3\,d}{2}-\frac{a^3\,d\,\cos(2\,c+2\,d\,x)}{2}}$$
$$+\frac{a\,b\,\sin\left(c+d\,x\right)}{\frac{a^3\,d}{2}-\frac{a^3\,d\,\cos(2\,c+2\,d\,x)}{2}}-\frac{b^2\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos\left(2\,c+2\,d\,x\right)}{2\,\left(\frac{a^3\,d}{2}-\frac{a^3\,d\,\cos(2\,c+2\,d\,x)}{2}\right)}$$
$$+\frac{b\,\mathrm{atan}\left(\frac{a^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}\,1{}\mathrm{i}+b^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}\,8{}\mathrm{i}+a\,b^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}\,4{}\mathrm{i}+a^3\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}\,3{}\mathrm{i}+a^2\,b^2\,\sin\left(\frac{c}{2}+\cdots\right.}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^5+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b+5\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3\,b^2+12\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^3+4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^4+8\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cdots}\cdots\right.}{\frac{a^3\,d}{2}-\frac{a^3\,d\,\cos(2\,c+2\,d\,x)}{2}}$$
$$-\frac{b\,\cos\left(2\,c+2\,d\,x\right)\,\mathrm{atan}\left(\frac{a^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}\,1{}\mathrm{i}+b^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}\,8{}\mathrm{i}+a\,b^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}\,4{}\mathrm{i}+a^3\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a\cdots}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^5+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b+5\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3\,b^2+12\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^3+4\,\cos\left(\frac{c}{2}+\cdots\right.}\cdots\right.}{\frac{a^3\,d}{2}-\frac{a^3\,d\,\cos(2\,c+2\,d\,x)}{2}}$$

input
```
int(1/(sin(c + d*x)^3*(a + b*tan(c + d*x))),x)
```

output
```
(b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(2*((a^3*d)/2 - (a^3*d*co
s(2*c + 2*d*x))/2)) - (a^2*(cos(c + d*x)/2 - log(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2))/4 + (log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*
d*x))/4))/((a^3*d)/2 - (a^3*d*cos(2*c + 2*d*x))/2) + (a*b*sin(c + d*x))/((
a^3*d)/2 - (a^3*d*cos(2*c + 2*d*x))/2) - (b^2*log(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2))*cos(2*c + 2*d*x))/(2*((a^3*d)/2 - (a^3*d*cos(2*c + 2*d*x))/
2)) + (b*atan((a^4*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*1i + b^4*sin(c/2 +
 (d*x)/2)*(a^2 + b^2)^(1/2)*8i + a*b^3*cos(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2
)*4i + a^3*b*cos(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*3i + a^2*b^2*sin(c/2 + (
d*x)/2)*(a^2 + b^2)^(1/2)*8i)/(a^5*cos(c/2 + (d*x)/2) + 8*b^5*sin(c/2 + (d
*x)/2) + 4*a*b^4*cos(c/2 + (d*x)/2) + 4*a^4*b*sin(c/2 + (d*x)/2) + 5*a^3*b
^2*cos(c/2 + (d*x)/2) + 12*a^2*b^3*sin(c/2 + (d*x)/2)))*(a^2 + b^2)^(1/2)*
1i)/((a^3*d)/2 - (a^3*d*cos(2*c + 2*d*x))/2) - (b*cos(2*c + 2*d*x)*atan((a
^4*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*1i + b^4*sin(c/2 + (d*x)/2)*(a^2 +
 b^2)^(1/2)*8i + a*b^3*cos(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*4i + a^3*b*cos
(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)*3i + a^2*b^2*sin(c/2 + (d*x)/2)*(a^2 + b
^2)^(1/2)*8i)/(a^5*cos(c/2 + (d*x)/2) + 8*b^5*sin(c/2 + (d*x)/2) + 4*a*b^4
*cos(c/2 + (d*x)/2) + 4*a^4*b*sin(c/2 + (d*x)/2) + 5*a^3*b^2*cos(c/2 + (d*
x)/2) + 12*a^2*b^3*sin(c/2 + (d*x)/2)))*(a^2 + b^2)^(1/2)*1i)/((a^3*d)/2 -
 (a^3*d*cos(2*c + 2*d*x))/2)
```

3.58. $\int \frac{\csc^3(c+dx)}{a+b\tan(c+dx)}\,dx$

3.59 $\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx$

3.59.1 Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{(a^2+b^2)\cot(c+dx)}{a^3d}+\frac{b\cot^2(c+dx)}{2a^2d}-\frac{\cot^3(c+dx)}{3ad}-\frac{b(a^2+b^2)\log(\tan(c+dx))}{a^4d}+\frac{b(a^2+b^2)\log(a+b\tan(c+dx))}{a^4d}$$

output
```
-(a^2+b^2)*cot(d*x+c)/a^3/d+1/2*b*cot(d*x+c)^2/a^2/d-1/3*cot(d*x+c)^3/a/d-
b*(a^2+b^2)*ln(tan(d*x+c))/a^4/d+b*(a^2+b^2)*ln(a+b*tan(d*x+c))/a^4/d
```

3.59.2 Mathematica [A] (verified)

Time = 2.94 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{3a^2b\csc^2(c+dx)-2\cot(c+dx)\left(2a^3+3ab^2+a^3\csc^2(c+dx)\right)-6b(a^2+b^2)\left(\log(\sin(c+dx))-\log(a\,c\right.}{6a^4d}$$

input
```
Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

output
```
(3*a^2*b*Csc[c + d*x]^2 - 2*Cot[c + d*x]*(2*a^3 + 3*a*b^2 + a^3*Csc[c + d*
x]^2) - 6*b*(a^2 + b^2)*(Log[Sin[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c
+ d*x]]))/(6*a^4*d)
```

3.59.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3999, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(c+dx)^4(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{ 3999}$$

$$\frac{b\int \frac{\cot^4(c+dx)\left(\tan^2(c+dx)b^2+b^2\right)}{b^4(a+b\tan(c+dx))}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 522}$$

$$\frac{b\int \left(\frac{\cot^4(c+dx)}{ab^2}-\frac{\cot^3(c+dx)}{a^2b}+\frac{(a^2+b^2)\cot^2(c+dx)}{a^3b^2}+\frac{(-a^2-b^2)\cot(c+dx)}{a^4b}+\frac{a^2+b^2}{a^4(a+b\tan(c+dx))}\right)d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\left(\frac{\cot^2(c+dx)}{2a^2}-\frac{(a^2+b^2)\log(b\tan(c+dx))}{a^4}+\frac{(a^2+b^2)\log(a+b\tan(c+dx))}{a^4}-\frac{(a^2+b^2)\cot(c+dx)}{a^3b}-\frac{\cot^3(c+dx)}{3ab}\right)}{d}$$

input
```
Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

output
```
(b*(-(((a^2 + b^2)*Cot[c + d*x])/(a^3*b)) + Cot[c + d*x]^2/(2*a^2) - Cot[c
 + d*x]^3/(3*a*b) - ((a^2 + b^2)*Log[b*Tan[c + d*x]])/a^4 + ((a^2 + b^2)*L
og[a + b*Tan[c + d*x]])/a^4))/d
```

3.59.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.59.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{\left(a^{2}+b^{2}\right) b \ln \left(a+b \tan \left(d x+c\right)\right)}{a^{4}}-\frac{1}{3 a \tan \left(d x+c\right)^{3}}-\frac{a^{2}+b^{2}}{a^{3} \tan \left(d x+c\right)}+\frac{b}{2 a^{2} \tan \left(d x+c\right)^{2}}-\frac{\left(a^{2}+b^{2}\right) b \ln \left(\tan \left(d x+c\right)\right)}{a^{4}}}{d}$
default	$\frac{\frac{\left(a^{2}+b^{2}\right) b \ln \left(a+b \tan \left(d x+c\right)\right)}{a^{4}}-\frac{1}{3 a \tan \left(d x+c\right)^{3}}-\frac{a^{2}+b^{2}}{a^{3} \tan \left(d x+c\right)}+\frac{b}{2 a^{2} \tan \left(d x+c\right)^{2}}-\frac{\left(a^{2}+b^{2}\right) b \ln \left(\tan \left(d x+c\right)\right)}{a^{4}}}{d}$
risch	$-\frac{2\left(3 i b^{2} {\mathrm e}^{4 i \left(d x+c\right)}+3 a b \,{\mathrm e}^{4 i \left(d x+c\right)}-6 i a^{2} {\mathrm e}^{2 i \left(d x+c\right)}-6 i b^{2} {\mathrm e}^{2 i \left(d x+c\right)}-3 a b \,{\mathrm e}^{2 i \left(d x+c\right)}+2 i a^{2}+3 i b^{2}\right)}{3 d \,a^{3} \left({\mathrm e}^{2 i \left(d x+c\right)}-1\right)^{3}}+\frac{b \ln \left({\mathrm e}^{2 i \left(d x+c\right)}-\frac{i b+a}{i b-a}\right)}{a^{2} d}$

input
```
int(csc(d*x+c)^4/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*((a^2+b^2)/a^4*b*ln(a+b*tan(d*x+c))-1/3/a/tan(d*x+c)^3-(a^2+b^2)/a^3/t
an(d*x+c)+1/2/a^2*b/tan(d*x+c)^2-(a^2+b^2)/a^4*b*ln(tan(d*x+c)))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.93

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{2\left(2a^3+3ab^2\right)\cos\left(dx+c\right)^3+3a^2b\sin\left(dx+c\right)+3\left(a^2b+b^3-\left(a^2b+b^3\right)\cos\left(dx+c\right)^2\right)\log\left(2ab\cos\left(\right.\right.}{}$$

input
```
integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(2*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3 + 3*a^2*b*sin(d*x + c) + 3*(a^2*b
 + b^3 - (a^2*b + b^3)*cos(d*x + c)^2)*log(2*a*b*cos(d*x + c)*sin(d*x + c)
 + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c) - 3*(a^2*b + b^3 - (a^2*
b + b^3)*cos(d*x + c)^2)*log(-1/4*cos(d*x + c)^2 + 1/4)*sin(d*x + c) - 6*(
a^3 + a*b^2)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))
```

3.59.6 Sympy [F]

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\csc^4\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

input
```
integrate(csc(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**4/(a + b*tan(c + d*x)), x)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{6\left(a^2b+b^3\right)\log(b\tan(dx+c)+a)}{a^4}-\frac{6\left(a^2b+b^3\right)\log(\tan(dx+c))}{a^4}+\frac{3ab\tan(dx+c)-6\left(a^2+b^2\right)\tan(dx+c)^2-2a^2}{a^3\tan(dx+c)^3}}{6d}$$

input
```
integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/6*(6*(a^2*b + b^3)*log(b*tan(d*x + c) + a)/a^4 - 6*(a^2*b + b^3)*log(tan
(d*x + c))/a^4 + (3*a*b*tan(d*x + c) - 6*(a^2 + b^2)*tan(d*x + c)^2 - 2*a^
2)/(a^3*tan(d*x + c)^3))/d
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.33

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{6\,(a^2b+b^3)\log(|\tan(dx+c)|)}{a^4} - \frac{6\,(a^2b^2+b^4)\log(|b\tan(dx+c)+a|)}{a^4b} - \frac{11\,a^2b\tan(dx+c)^3+11\,b^3\tan(dx+c)^3-6\,a^3\tan(dx+c)^2-6\,ab^2\tan(dx}{a^4\tan(dx+c)^3}}{6\,d}$$

input
```
integrate(csc(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(6*(a^2*b + b^3)*log(abs(tan(d*x + c)))/a^4 - 6*(a^2*b^2 + b^4)*log(a
bs(b*tan(d*x + c) + a))/(a^4*b) - (11*a^2*b*tan(d*x + c)^3 + 11*b^3*tan(d*
x + c)^3 - 6*a^3*tan(d*x + c)^2 - 6*a*b^2*tan(d*x + c)^2 + 3*a^2*b*tan(d*x
 + c) - 2*a^3)/(a^4*tan(d*x + c)^3))/d
```

3.59.9 Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{\csc^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{b\,(a^2+b^2)\,(a+2\,b\tan(c+d\,x))}{a\,(a^2\,b+b^3)}\right)\,(a^2+b^2)}{a^4\,d} - \frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,(a^2+b^2)}{a^3}-\frac{b\tan(c+d\,x)}{2\,a^2}}{d\tan\left(c+d\,x\right)^3}$$

input
```
int(1/(sin(c + d*x)^4*(a + b*tan(c + d*x))),x)
```

output
```
(2*b*atanh((b*(a^2 + b^2)*(a + 2*b*tan(c + d*x)))/(a*(a^2*b + b^3)))*(a^2
+ b^2))/(a^4*d) - (1/(3*a) + (tan(c + d*x)^2*(a^2 + b^2))/a^3 - (b*tan(c +
 d*x))/(2*a^2))/(d*tan(c + d*x)^3)
```

3.60 $\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx$

3.60.1 Optimal result

Integrand size = 21, antiderivative size = 169

$$\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{(a^2+b^2)^2\cot(c+dx)}{a^5 d}+\frac{b(2a^2+b^2)\cot^2(c+dx)}{2a^4 d}-\frac{(2a^2+b^2)\cot^3(c+dx)}{3a^3 d}+\frac{b\cot^4(c+dx)}{4a^2 d}-\frac{\cot^5(c+dx)}{5ad}-\frac{b(a^2+b^2)^2\log(\tan(c+dx))}{a^6 d}+\frac{b(a^2+b^2)^2\log(a+b\tan(c+dx))}{a^6 d}$$

output
```
-(a^2+b^2)^2*cot(d*x+c)/a^5/d+1/2*b*(2*a^2+b^2)*cot(d*x+c)^2/a^4/d-1/3*(2*
a^2+b^2)*cot(d*x+c)^3/a^3/d+1/4*b*cot(d*x+c)^4/a^2/d-1/5*cot(d*x+c)^5/a/d-
b*(a^2+b^2)^2*ln(tan(d*x+c))/a^6/d+b*(a^2+b^2)^2*ln(a+b*tan(d*x+c))/a^6/d
```

3.60.2 Mathematica [A] (verified)

Time = 6.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{-4\cot(c+dx)\left(8a^5+25a^3b^2+15ab^4+a^3(4a^2+5b^2)\csc^2(c+dx)+3a^5\csc^4(c+dx)\right)+15b\Big(2a^2(a^2+b^2}{60a^6}$$

input
```
Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

output
```
(-4*Cot[c + d*x]*(8*a^5 + 25*a^3*b^2 + 15*a*b^4 + a^3*(4*a^2 + 5*b^2)*Csc[
c + d*x]^2 + 3*a^5*Csc[c + d*x]^4) + 15*b*(2*a^2*(a^2 + b^2)*Csc[c + d*x]^
2 + a^4*Csc[c + d*x]^4 - 4*(a^2 + b^2)^2*(Log[Sin[c + d*x]] - Log[a*Cos[c
+ d*x] + b*Sin[c + d*x]])))/(60*a^6*d)
```

3.60.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3999, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(c+dx)^6(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \; 3999$$

$$\frac{b\int \frac{\cot^6(c+dx)\left(\tan^2(c+dx)b^2+b^2\right)^2}{b^6(a+b\tan(c+dx))}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 522$$

$$\frac{b\int \left(\frac{\cot^6(c+dx)}{ab^2}-\frac{\cot^5(c+dx)}{a^2b}+\frac{(b^4+2a^2b^2)\cot^4(c+dx)}{a^3b^4}+\frac{(-2a^2-b^2)\cot^3(c+dx)}{a^4b}+\frac{(a^2+b^2)^2\cot^2(c+dx)}{a^5b^2}-\frac{(a^2+b^2)^2\cot(c+dx)}{a^6b}+\right.}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\left(\frac{\cot^4(c+dx)}{4a^2}-\frac{(a^2+b^2)^2\log(b\tan(c+dx))}{a^6}+\frac{(a^2+b^2)^2\log(a+b\tan(c+dx))}{a^6}-\frac{(a^2+b^2)^2\cot(c+dx)}{a^5b}+\frac{(2a^2+b^2)\cot^2(c+dx)}{2a^4}-\frac{(2a^2+b^2)}{3}\right.}{d}$$

input
```
Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

output
```
(b*(-(((a^2 + b^2)^2*Cot[c + d*x])/(a^5*b)) + ((2*a^2 + b^2)*Cot[c + d*x]^
2)/(2*a^4) - ((2*a^2 + b^2)*Cot[c + d*x]^3)/(3*a^3*b) + Cot[c + d*x]^4/(4*
a^2) - Cot[c + d*x]^5/(5*a*b) - ((a^2 + b^2)^2*Log[b*Tan[c + d*x]])/a^6 +
((a^2 + b^2)^2*Log[a + b*Tan[c + d*x]])/a^6))/d
```

3.60.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.60.4 Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{1}{5a\tan(dx+c)^5}-\frac{2a^2+b^2}{3a^3\tan(dx+c)^3}-\frac{a^4+2a^2b^2+b^4}{a^5\tan(dx+c)}+\frac{b}{4a^2\tan(dx+c)^4}+\frac{(2a^2+b^2)b}{2a^4\tan(dx+c)^2}-\frac{(a^4+2a^2b^2+b^4)b\ln(\tan(dx+c))}{a^6}+\frac{(a^4+}{}}{d}$
default	$\frac{-\frac{1}{5a\tan(dx+c)^5}-\frac{2a^2+b^2}{3a^3\tan(dx+c)^3}-\frac{a^4+2a^2b^2+b^4}{a^5\tan(dx+c)}+\frac{b}{4a^2\tan(dx+c)^4}+\frac{(2a^2+b^2)b}{2a^4\tan(dx+c)^2}-\frac{(a^4+2a^2b^2+b^4)b\ln(\tan(dx+c))}{a^6}+\frac{(a^4+}{}}{d}$
risch	$-\frac{2i(15ia^3b\,e^{2i(dx+c)}+15ia\,b^3e^{2i(dx+c)}+15a^2b^2e^{8i(dx+c)}+15b^4e^{8i(dx+c)}-45ia\,b^3e^{4i(dx+c)}-15ia^3b\,e^{8i(dx+c)}-90a^2b^2e}{}$

input
```
int(csc(d*x+c)^6/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/5/a/tan(d*x+c)^5-1/3*(2*a^2+b^2)/a^3/tan(d*x+c)^3-(a^4+2*a^2*b^2+b
^4)/a^5/tan(d*x+c)+1/4/a^2*b/tan(d*x+c)^4+1/2*(2*a^2+b^2)/a^4*b/tan(d*x+c)
^2-(a^4+2*a^2*b^2+b^4)/a^6*b*ln(tan(d*x+c))+(a^4+2*a^2*b^2+b^4)/a^6*b*ln(a
+b*tan(d*x+c)))
```

3.60.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(161) = 322$.

Time = 0.29 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.28

$$\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{4\,(8\,a^5+25\,a^3b^2+15\,ab^4)\cos{(dx+c)}^5-20\,(4\,a^5+11\,a^3b^2+6\,ab^4)\cos{(dx+c)}^3-30\,\left(a^4b+2\,a^2b^3+\right.}{}$$

input
```
integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/60*(4*(8*a^5 + 25*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 20*(4*a^5 + 11*a
^3*b^2 + 6*a*b^4)*cos(d*x + c)^3 - 30*(a^4*b + 2*a^2*b^3 + b^5 + (a^4*b +
2*a^2*b^3 + b^5)*cos(d*x + c)^4 - 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)
^2)*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2
)*sin(d*x + c) + 30*(a^4*b + 2*a^2*b^3 + b^5 + (a^4*b + 2*a^2*b^3 + b^5)*c
os(d*x + c)^4 - 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^2)*log(-1/4*cos(d
*x + c)^2 + 1/4)*sin(d*x + c) + 60*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)
- 15*(3*a^4*b + 2*a^2*b^3 - 2*(a^4*b + a^2*b^3)*cos(d*x + c)^2)*sin(d*x +
c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c))
```

3.60.6 Sympy [F]

$$\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\csc^6{(c+dx)}}{a+b\tan{(c+dx)}}\,dx$$

input
```
integrate(csc(d*x+c)**6/(a+b*tan(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)**6/(a + b*tan(c + d*x)), x)
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.99

$$\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{60\left(a^4b+2\,a^2b^3+b^5\right)\log(b\tan(dx+c)+a)}{a^6} - \frac{60\left(a^4b+2\,a^2b^3+b^5\right)\log(\tan(dx+c))}{a^6} + \frac{15\,a^3b\tan(dx+c)-60\left(a^4+2\,a^2b^2+b^4\right)\tan(dx+c)^4-12\,a^4-}{a^5\tan(}}{60\,d}$$

input
```
integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/60*(60*(a^4*b + 2*a^2*b^3 + b^5)*log(b*tan(d*x + c) + a)/a^6 - 60*(a^4*b
 + 2*a^2*b^3 + b^5)*log(tan(d*x + c))/a^6 + (15*a^3*b*tan(d*x + c) - 60*(a
^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^4 - 12*a^4 + 30*(2*a^3*b + a*b^3)*tan(d
*x + c)^3 - 20*(2*a^4 + a^2*b^2)*tan(d*x + c)^2)/(a^5*tan(d*x + c)^5))/d
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.49

$$\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx =$$
$$-\frac{\frac{60\left(a^4b+2\,a^2b^3+b^5\right)\log(|\tan(dx+c)|)}{a^6} - \frac{60\left(a^4b^2+2\,a^2b^4+b^6\right)\log(|b\tan(dx+c)+a|)}{a^6b} - \frac{137\,a^4b\tan(dx+c)^5+274\,a^2b^3\tan(dx+c)^5+137\,b^5\,\mathrm{t}}{}}{}$$

input
```
integrate(csc(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/60*(60*(a^4*b + 2*a^2*b^3 + b^5)*log(abs(tan(d*x + c)))/a^6 - 60*(a^4*b
^2 + 2*a^2*b^4 + b^6)*log(abs(b*tan(d*x + c) + a))/(a^6*b) - (137*a^4*b*ta
n(d*x + c)^5 + 274*a^2*b^3*tan(d*x + c)^5 + 137*b^5*tan(d*x + c)^5 - 60*a^
5*tan(d*x + c)^4 - 120*a^3*b^2*tan(d*x + c)^4 - 60*a*b^4*tan(d*x + c)^4 +
60*a^4*b*tan(d*x + c)^3 + 30*a^2*b^3*tan(d*x + c)^3 - 40*a^5*tan(d*x + c)^
2 - 20*a^3*b^2*tan(d*x + c)^2 + 15*a^4*b*tan(d*x + c) - 12*a^5)/(a^6*tan(d
*x + c)^5))/d
```

3.60.9 Mupad [B] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

$$\int \frac{\csc^6(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{2\,b\,\mathrm{atanh}\left(\frac{b\,(a^2+b^2)^2\,(a+2\,b\tan(c+d\,x))}{a\,(a^4\,b+2\,a^2\,b^3+b^5)}\right)\,(a^2+b^2)^2}{a^6\,d} - \frac{\frac{1}{5\,a}+\frac{\tan(c+d\,x)^2\,(2\,a^2+b^2)}{3\,a^3}+\frac{\tan(c+d\,x)^4\,(a^4+2\,a^2\,b^2+b^4)}{a^5}-\frac{b\tan(c+d\,x)}{4\,a^2}-\frac{b\tan(c+d\,x)^3\,(2\,a^2+b^2)}{2\,a^4}}{d\tan(c+d\,x)^5}$$

input
```
int(1/(sin(c + d*x)^6*(a + b*tan(c + d*x))),x)
```

output
```
(2*b*atanh((b*(a^2 + b^2)^2*(a + 2*b*tan(c + d*x)))/(a*(a^4*b + b^5 + 2*a^
2*b^3)))*(a^2 + b^2)^2)/(a^6*d) - (1/(5*a) + (tan(c + d*x)^2*(2*a^2 + b^2)
)/(3*a^3) + (tan(c + d*x)^4*(a^4 + b^4 + 2*a^2*b^2))/a^5 - (b*tan(c + d*x)
)/(4*a^2) - (b*tan(c + d*x)^3*(2*a^2 + b^2))/(2*a^4))/(d*tan(c + d*x)^5)
```

3.61 $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.61.1 Optimal result

Integrand size = 21, antiderivative size = 297

$$\begin{aligned}\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{(5a^8-80a^6b^2+50a^4b^4+8a^2b^6+b^8)\,x}{16\,(a^2+b^2)^5} \\ &+ \frac{2a^5b(a^2-3b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^5\,d} \\ &- \frac{a^6b}{(a^2+b^2)^4\,d(a+b\tan(c+dx))} - \frac{\cos^6(c+dx)\,(2ab+(a^2-b^2)\tan(c+dx))}{6\,(a^2+b^2)^2\,d} \\ &+ \frac{\cos^4(c+dx)\,(12ab(3a^2+b^2)+(13a^4-18a^2b^2-7b^4)\tan(c+dx))}{24\,(a^2+b^2)^3\,d} \\ &- \frac{\cos^2(c+dx)\,(48a^5b+(11a^6-43a^4b^2-7a^2b^4-b^6)\tan(c+dx))}{16\,(a^2+b^2)^4\,d}\end{aligned}$$

output
```
1/16*(5*a^8-80*a^6*b^2+50*a^4*b^4+8*a^2*b^6+b^8)*x/(a^2+b^2)^5+2*a^5*b*(a^
2-3*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^5/d-a^6*b/(a^2+b^2)^4/d/(
a+b*tan(d*x+c))-1/6*cos(d*x+c)^6*(2*a*b+(a^2-b^2)*tan(d*x+c))/(a^2+b^2)^2/
d+1/24*cos(d*x+c)^4*(12*a*b*(3*a^2+b^2)+(13*a^4-18*a^2*b^2-7*b^4)*tan(d*x+
c))/(a^2+b^2)^3/d-1/16*cos(d*x+c)^2*(48*a^5*b+(11*a^6-43*a^4*b^2-7*a^2*b^4
-b^6)*tan(d*x+c))/(a^2+b^2)^4/d
```

3.61.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 664 vs. $2(297) = 594$.

Time = 6.72 (sec) , antiderivative size = 664, normalized size of antiderivative = 2.24

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{b\left(-\frac{5(a^2-b^2)\arctan(\tan(c+dx))}{16b(a^2+b^2)^2}+\frac{3(3a^4-3a^2b^2-2b^4)\arctan(\tan(c+dx))}{8b(a^2+b^2)^3}-\frac{(3a^6-6a^4b^2-4a^2b^4-b^6)\arctan(\tan(c+dx))}{2b(a^2+b^2)^4}-\frac{3a^5\cos^2(c+}{(a^2+b^2)}\right.}{}$$

input
```
Integrate[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

output
```
(b*((-5*(a^2 - b^2)*ArcTan[Tan[c + d*x]])/(16*b*(a^2 + b^2)^2) + (3*(3*a^4
 - 3*a^2*b^2 - 2*b^4)*ArcTan[Tan[c + d*x]])/(8*b*(a^2 + b^2)^3) - ((3*a^6
- 6*a^4*b^2 - 4*a^2*b^4 - b^6)*ArcTan[Tan[c + d*x]])/(2*b*(a^2 + b^2)^4) -
 (3*a^5*Cos[c + d*x]^2)/(a^2 + b^2)^4 + (a*(3*a^2 + b^2)*Cos[c + d*x]^4)/(
2*(a^2 + b^2)^3) - (a*Cos[c + d*x]^6)/(3*(a^2 + b^2)^2) - (a^5*(2*a^2 - 6*
b^2 - (a^3 - 7*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(2*(a^
2 + b^2)^5) + (2*a^5*(a^2 - 3*b^2)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^5
- (a^5*(2*a^2 - 6*b^2 + (a^3 - 7*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan
[c + d*x]])/(2*(a^2 + b^2)^5) - (5*(a - b)*(a + b)*Cos[c + d*x]*Sin[c + d*
x])/(16*b*(a^2 + b^2)^2) + (3*(3*a^4 - 3*a^2*b^2 - 2*b^4)*Cos[c + d*x]*Sin
[c + d*x])/(8*b*(a^2 + b^2)^3) - ((3*a^6 - 6*a^4*b^2 - 4*a^2*b^4 - b^6)*Co
s[c + d*x]*Sin[c + d*x])/(2*b*(a^2 + b^2)^4) - (5*(a^2 - b^2)*Cos[c + d*x]
^3*Sin[c + d*x])/(24*b*(a^2 + b^2)^2) + ((3*a^4 - 3*a^2*b^2 - 2*b^4)*Cos[c
 + d*x]^3*Sin[c + d*x])/(4*b*(a^2 + b^2)^3) - ((a^2 - b^2)*Cos[c + d*x]^5*
Sin[c + d*x])/(6*b*(a^2 + b^2)^2) - a^6/((a^2 + b^2)^4*(a + b*Tan[c + d*x]
))))/d
```

3.61.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3999, 601, 25, 2178, 27, 2178, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{\sin(c+dx)^6}{(a+b\tan(c+dx))^2}dx$$

$\Big\downarrow$ 3999

$$\frac{b\int \frac{b^6\tan^6(c+dx)}{(a+b\tan(c+dx))^2(\tan^2(c+dx)b^2+b^2)^4}d(b\tan(c+dx))}{d}$$

$\Big\downarrow$ 601

$$\frac{b\left(-\frac{\int -\frac{-\frac{2a(5a^2+b^2)\tan(c+dx)b^7}{(a^2+b^2)^2}+6\tan^4(c+dx)b^6-\frac{(6a^4+17b^2a^2+b^4)\tan^2(c+dx)b^6}{(a^2+b^2)^2}+\frac{a^2(a^2-b^2)b^6}{(a^2+b^2)^2}}{(a+b\tan(c+dx))^2(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{6b^2}-\frac{b^4(b(a^2-b^2)\tan(c+dx)+2a}{6(a^2+b^2)^2(b^2\tan^2(c+dx)+}\right.}{d}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{\int \frac{-\frac{2a(5a^2+b^2)\tan(c+dx)b^7}{(a^2+b^2)^2}+6\tan^4(c+dx)b^6-\frac{(6a^4+17b^2a^2+b^4)\tan^2(c+dx)b^6}{(a^2+b^2)^2}+\frac{a^2(a^2-b^2)b^6}{(a^2+b^2)^2}}{(a+b\tan(c+dx))^2(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{6b^2}-\frac{b^4(b(a^2-b^2)\tan(c+dx)+2ab^2)}{6(a^2+b^2)^2(b^2\tan^2(c+dx)+b^2)^3}\right.}{d}$$

$\Big\downarrow$ 2178

$$\frac{b\left(\frac{\frac{b^4(12ab^2(3a^2+b^2)+b(13a^4-18a^2b^2-7b^4)\tan(c+dx))}{4(a^2+b^2)^3(b^2\tan^2(c+dx)+b^2)^2}-\frac{\int \frac{3\left(-\frac{2a(13a^4+6b^2a^2+b^4)\tan(c+dx)b^7}{(a^2+b^2)^3}-\frac{(8a^6+37b^2a^4+6b^4a^2+b^6)\tan^2(c+dx)b^6}{(a^2+b^2)^3}+\frac{a^2(3a^4-6b}{(a^2+}\right.}{(a+b\tan(c+dx))^2(\tan^2(c+dx)b^2+b^2)^2}}{4b^2}}{6b^2}\right.}{d}$$

$\Big\downarrow$ 27

3.61. $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

$$\frac{b\left(\frac{\frac{b^4\left(12ab^2\left(3a^2+b^2\right)+b\left(13a^4-18a^2b^2-7b^4\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3\left(b^2\tan^2(c+dx)+b^2\right)^2}-\frac{3\int\frac{-\frac{2a\left(13a^4+6b^2a^2+b^4\right)\tan(c+dx)b^7}{\left(a^2+b^2\right)^3}-\frac{\left(8a^6+37b^2a^4+6b^4a^2+b^6\right)\tan^2(c+dx)b^6}{\left(a^2+b^2\right)^3}+\frac{a^2\left(3a^4-6b^2\cdots\right.}{\left(a^2+b\cdots\right.}}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)^2}}{4b^2}}{6b^2}\cdots\right.}{d}$$

$$\Big\downarrow \; 2178$$

$$\frac{b\left(\frac{\frac{b^4\left(12ab^2\left(3a^2+b^2\right)+b\left(13a^4-18a^2b^2-7b^4\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3\left(b^2\tan^2(c+dx)+b^2\right)^2}-\frac{3\left(\frac{b^4\left(48a^5b^2+b\left(11a^6-43a^4b^2-7a^2b^4-b^6\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^4\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{\int\frac{-\frac{\left(11a^6-43b^2a^4-7b^4a^2-b^6\right)\tan^2(c\cdots}{\left(a^2+b^2\right)^4}\cdots}{\cdots}}{\cdots}\right.}{4b^2\cdots}}{6b^2}\cdots\right.}{d}$$

$$\Big\downarrow \; 2160$$

$$\frac{b\left(\frac{\frac{b^4\left(12ab^2\left(3a^2+b^2\right)+b\left(13a^4-18a^2b^2-7b^4\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3\left(b^2\tan^2(c+dx)+b^2\right)^2}-\frac{3\left(\frac{b^4\left(48a^5b^2+b\left(11a^6-43a^4b^2-7a^2b^4-b^6\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^4\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{\int\left(\frac{32a^5\left(a^2-3b^2\right)b^6}{\left(a^2+b^2\right)^5(a+b\tan(c+dx))}+\frac{\left(5a^8-\cdots\right.}{\cdots}\right)}{\cdots}\right.}{\cdots}}{6b^2}\cdots\right.}{d}$$

$$\Big\downarrow \; 2009$$

3.61. $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

$$b\left(\frac{\frac{b^4\left(12ab^2\left(3a^2+b^2\right)+b\left(13a^4-18a^2b^2-7b^4\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^3\left(b^2\tan^2(c+dx)+b^2\right)^2}-\frac{3\left(\frac{b^4\left(48a^5b^2+b\left(11a^6-43a^4b^2-7a^2b^4-b^6\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^4\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{-\frac{16a^6b^6}{\left(a^2+b^2\right)^4(a+b\tan(c+dx))}-\frac{16a^5b^6(\cdots}{\cdots}}{\cdots}\right.}{\cdots}}{6b^2}\right.$$

input
```
Int[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

output
```
(b*(-1/6*(b^4*(2*a*b^2 + b*(a^2 - b^2)*Tan[c + d*x]))/((a^2 + b^2)^2*(b^2
+ b^2*Tan[c + d*x]^2)^3) + ((b^4*(12*a*b^2*(3*a^2 + b^2) + b*(13*a^4 - 18*
a^2*b^2 - 7*b^4)*Tan[c + d*x]))/(4*(a^2 + b^2)^3*(b^2 + b^2*Tan[c + d*x]^2
)^2) - (3*((b^4*(48*a^5*b^2 + b*(11*a^6 - 43*a^4*b^2 - 7*a^2*b^4 - b^6)*Ta
n[c + d*x]))/(2*(a^2 + b^2)^4*(b^2 + b^2*Tan[c + d*x]^2)) - ((b^5*(5*a^8 -
 80*a^6*b^2 + 50*a^4*b^4 + 8*a^2*b^6 + b^8)*ArcTan[Tan[c + d*x]])/(a^2 + b
^2)^5 + (32*a^5*b^6*(a^2 - 3*b^2)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^5 -
 (16*a^5*b^6*(a^2 - 3*b^2)*Log[b^2 + b^2*Tan[c + d*x]^2])/(a^2 + b^2)^5 -
(16*a^6*b^6)/((a^2 + b^2)^4*(a + b*Tan[c + d*x])))/(2*b^2)))/(4*b^2))/(6*b
^2)))/d
```

3.61.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.61. $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.61.4 Maple [A] (verified)

Time = 33.91 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-\frac{b\,a^{6}}{\left(a^{2}+b^{2}\right)^{4}\left(a+b\tan(dx+c)\right)}+\frac{2b\,a^{5}\left(a^{2}-3b^{2}\right)\ln(a+b\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{5}}+\frac{\left(-\frac{11}{16}a^{8}+2a^{6}b^{2}+\frac{25}{8}a^{4}b^{4}+\frac{1}{2}b^{6}a^{2}+\frac{1}{16}b^{8}\right)\left(\tan^{5}(dx+c)\right)+\left(-3 \ldots\right.}{\ldots}}{\ldots}$
default	$\frac{-\frac{b\,a^{6}}{\left(a^{2}+b^{2}\right)^{4}\left(a+b\tan(dx+c)\right)}+\frac{2b\,a^{5}\left(a^{2}-3b^{2}\right)\ln(a+b\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{5}}+\frac{\left(-\frac{11}{16}a^{8}+2a^{6}b^{2}+\frac{25}{8}a^{4}b^{4}+\frac{1}{2}b^{6}a^{2}+\frac{1}{16}b^{8}\right)\left(\tan^{5}(dx+c)\right)+\left(-3 \ldots\right.}{\ldots}}{\ldots}$
risch	Expression too large to display

input
```
int(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b*a^6/(a^2+b^2)^4/(a+b*tan(d*x+c))+2*b*a^5*(a^2-3*b^2)/(a^2+b^2)^5*l
n(a+b*tan(d*x+c))+1/(a^2+b^2)^5*(((-11/16*a^8+2*a^6*b^2+25/8*a^4*b^4+1/2*b
^6*a^2+1/16*b^8)*tan(d*x+c)^5+(-3*a^7*b-3*a^5*b^3)*tan(d*x+c)^4+(-5/6*a^8+
13/3*a^6*b^2+5*a^4*b^4-1/3*b^6*a^2-1/6*b^8)*tan(d*x+c)^3+(-9/2*b*a^7-5/2*b
^3*a^5+5/2*a^3*b^5+1/2*a*b^7)*tan(d*x+c)^2+(-5/16*a^8+2*a^6*b^2+15/8*a^4*b
^4-1/2*b^6*a^2-1/16*b^8)*tan(d*x+c)-11/6*b*a^7-1/2*b^3*a^5+3/2*a^3*b^5+1/6
*a*b^7)/(1+tan(d*x+c)^2)^3+1/32*(-32*a^7*b+96*a^5*b^3)*ln(1+tan(d*x+c)^2)+
1/16*(5*a^8-80*a^6*b^2+50*a^4*b^4+8*a^2*b^6+b^8)*arctan(tan(d*x+c))))
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. 2(289) = 578.

Time = 0.34 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.08

$$\int \frac{\sin^{6}(c+dx)}{(a+b\tan(c+dx))^{2}}\,dx =$$
$$-\frac{8\left(a^{8}b+4\,a^{6}b^{3}+6\,a^{4}b^{5}+4\,a^{2}b^{7}+b^{9}\right)\cos\left(dx+c\right)^{7}-2\left(19\,a^{8}b+68\,a^{6}b^{3}+90\,a^{4}b^{5}+52\,a^{2}b^{7}+11\,b^{9}\right)\cos \ldots}{\ldots}$$

input
```
integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/48*(8*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^7
- 2*(19*a^8*b + 68*a^6*b^3 + 90*a^4*b^5 + 52*a^2*b^7 + 11*b^9)*cos(d*x + c
)^5 + (85*a^8*b + 224*a^6*b^3 + 210*a^4*b^5 + 88*a^2*b^7 + 17*b^9)*cos(d*x
 + c)^3 - (17*a^8*b + 72*a^6*b^3 + 120*a^4*b^5 + 20*a^2*b^7 + 3*b^9 + 3*(5
*a^9 - 80*a^7*b^2 + 50*a^5*b^4 + 8*a^3*b^6 + a*b^8)*d*x)*cos(d*x + c) - 48
*((a^8*b - 3*a^6*b^3)*cos(d*x + c) + (a^7*b^2 - 3*a^5*b^4)*sin(d*x + c))*l
og(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (
98*a^7*b^2 + 24*a^5*b^4 - 30*a^3*b^6 - 4*a*b^8 - 8*(a^9 + 4*a^7*b^2 + 6*a^
5*b^4 + 4*a^3*b^6 + a*b^8)*cos(d*x + c)^6 + 2*(13*a^9 + 44*a^7*b^2 + 54*a^
5*b^4 + 28*a^3*b^6 + 5*a*b^8)*cos(d*x + c)^4 + 3*(5*a^8*b - 80*a^6*b^3 + 5
0*a^4*b^5 + 8*a^2*b^7 + b^9)*d*x - 3*(11*a^9 + 16*a^7*b^2 - 2*a^5*b^4 - 8*
a^3*b^6 - a*b^8)*cos(d*x + c)^2)*sin(d*x + c))/((a^11 + 5*a^9*b^2 + 10*a^7
*b^4 + 10*a^5*b^6 + 5*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^10*b + 5*a^8*b
^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)*d*sin(d*x + c))
```

3.61.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**6/(a+b*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.61.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(289) = 578$.

Time = 0.32 (sec) , antiderivative size = 799, normalized size of antiderivative = 2.69

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{3\,(5\,a^8-80\,a^6b^2+50\,a^4b^4+8\,a^2b^6+b^8)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} + \frac{96\,(a^7b-3\,a^5b^3)\log(b\tan(dx+c)+a)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - \frac{48\,(a^7b-3\,a^5b^3)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} -}{}$$

input
```
integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/48*(3*(5*a^8 - 80*a^6*b^2 + 50*a^4*b^4 + 8*a^2*b^6 + b^8)*(d*x + c)/(a^1
0 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 96*(a^7*b -
3*a^5*b^3)*log(b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4
*b^6 + 5*a^2*b^8 + b^10) - 48*(a^7*b - 3*a^5*b^3)*log(tan(d*x + c)^2 + 1)/
(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - (136*a^6
*b - 64*a^4*b^3 - 8*a^2*b^5 + 3*(27*a^6*b - 43*a^4*b^3 - 7*a^2*b^5 - b^7)*
tan(d*x + c)^6 + 3*(11*a^7 + 5*a^5*b^2 - 7*a^3*b^4 - a*b^6)*tan(d*x + c)^5
 + 8*(41*a^6*b - 31*a^4*b^3 + a^2*b^5 + b^7)*tan(d*x + c)^4 + 8*(5*a^7 - 4
*a^5*b^2 - 11*a^3*b^4 - 2*a*b^6)*tan(d*x + c)^3 + 3*(125*a^6*b - 69*a^4*b^
3 - a^2*b^5 + b^7)*tan(d*x + c)^2 + (15*a^7 - 23*a^5*b^2 - 43*a^3*b^4 - 5*
a*b^6)*tan(d*x + c))/(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8 + (a
^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*tan(d*x + c)^7 + (a^9 + 4*
a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*tan(d*x + c)^6 + 3*(a^8*b + 4*a^6
*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*tan(d*x + c)^5 + 3*(a^9 + 4*a^7*b^2 +
6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*tan(d*x + c)^4 + 3*(a^8*b + 4*a^6*b^3 + 6*a
^4*b^5 + 4*a^2*b^7 + b^9)*tan(d*x + c)^3 + 3*(a^9 + 4*a^7*b^2 + 6*a^5*b^4
+ 4*a^3*b^6 + a*b^8)*tan(d*x + c)^2 + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a
^2*b^7 + b^9)*tan(d*x + c)))/d
```

3.61.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(289) = 578$.

Time = 0.54 (sec) , antiderivative size = 735, normalized size of antiderivative = 2.47

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{3\,(5\,a^8-80\,a^6b^2+50\,a^4b^4+8\,a^2b^6+b^8)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - \frac{48\,(a^7b-3\,a^5b^3)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} + \frac{96\,(a^7b^2-3\,a^5b^4)\log(|b\tan(dx+c)+a|)}{a^{10}b+5\,a^8b^3+10\,a^6b^5+10\,a^4b^7+5\,a^2b^9+b^{11}} - }{}$$

input
```
integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/48*(3*(5*a^8 - 80*a^6*b^2 + 50*a^4*b^4 + 8*a^2*b^6 + b^8)*(d*x + c)/(a^1
0 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 48*(a^7*b -
3*a^5*b^3)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4
*b^6 + 5*a^2*b^8 + b^10) + 96*(a^7*b^2 - 3*a^5*b^4)*log(abs(b*tan(d*x + c)
 + a))/(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11) -
 48*(2*a^7*b^2*tan(d*x + c) - 6*a^5*b^4*tan(d*x + c) + 3*a^8*b - 5*a^6*b^3
)/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*(b*tan(
d*x + c) + a)) + (88*a^7*b*tan(d*x + c)^6 - 264*a^5*b^3*tan(d*x + c)^6 - 3
3*a^8*tan(d*x + c)^5 + 96*a^6*b^2*tan(d*x + c)^5 + 150*a^4*b^4*tan(d*x + c
)^5 + 24*a^2*b^6*tan(d*x + c)^5 + 3*b^8*tan(d*x + c)^5 + 120*a^7*b*tan(d*x
 + c)^4 - 936*a^5*b^3*tan(d*x + c)^4 - 40*a^8*tan(d*x + c)^3 + 208*a^6*b^2
*tan(d*x + c)^3 + 240*a^4*b^4*tan(d*x + c)^3 - 16*a^2*b^6*tan(d*x + c)^3 -
 8*b^8*tan(d*x + c)^3 + 48*a^7*b*tan(d*x + c)^2 - 912*a^5*b^3*tan(d*x + c)
^2 + 120*a^3*b^5*tan(d*x + c)^2 + 24*a*b^7*tan(d*x + c)^2 - 15*a^8*tan(d*x
 + c) + 96*a^6*b^2*tan(d*x + c) + 90*a^4*b^4*tan(d*x + c) - 24*a^2*b^6*tan
(d*x + c) - 3*b^8*tan(d*x + c) - 288*a^5*b^3 + 72*a^3*b^5 + 8*a*b^7)/((a^1
0 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*(tan(d*x + c)^
2 + 1)^3))/d
```

3.61.9 Mupad [B] (verification not implemented)

Time = 6.11 (sec) , antiderivative size = 757, normalized size of antiderivative = 2.55

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{2\,a\,b}{{\left(a^2+b^2\right)}^2}-\frac{12\,a\,b^3}{{\left(a^2+b^2\right)}^3}+\frac{18\,a\,b^5}{{\left(a^2+b^2\right)}^4}-\frac{8\,a\,b^7}{{\left(a^2+b^2\right)}^5}\right)}{d}$$

$$+ \frac{\frac{\tan(c+d\,x)^3\,\left(-5\,a^5+9\,a^3\,b^2+2\,a\,b^4\right)}{6\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{\tan(c+d\,x)^5\,\left(-11\,a^5+6\,a^3\,b^2+a\,b^4\right)}{16\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{\tan(c+d\,x)^6\,\left(-27\,a^6\,b+43\,a^4\,b^3+7\,a^2\,b^5+b^7\right)}{16\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}+\frac{\tan(c+d\,x)}{48\,(a^6}}{d\,\left(b\tan\left(c+d\,x\right)^7+a\tan\left(c+d\,x\right)^6+3\,b\tan\left(c+d\,x\right)^5+3\,a}$$

$$+ \frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(a^3\,5{}\mathrm{i}-7\,a^2\,b+a\,b^2\,5{}\mathrm{i}+b^3\right)}{32\,d\,\left(a^5-a^4\,b\,5{}\mathrm{i}-10\,a^3\,b^2+a^2\,b^3\,10{}\mathrm{i}+5\,a\,b^4-b^5\,1{}\mathrm{i}\right)}$$

$$- \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a^3\,5{}\mathrm{i}+7\,a^2\,b+a\,b^2\,5{}\mathrm{i}-b^3\right)}{32\,d\,\left(a^5+a^4\,b\,5{}\mathrm{i}-10\,a^3\,b^2-a^2\,b^3\,10{}\mathrm{i}+5\,a\,b^4+b^5\,1{}\mathrm{i}\right)}$$

input
```
int(sin(c + d*x)^6/(a + b*tan(c + d*x))^2,x)
```

output
```
(log(a + b*tan(c + d*x))*((2*a*b)/(a^2 + b^2)^2 - (12*a*b^3)/(a^2 + b^2)^3
 + (18*a*b^5)/(a^2 + b^2)^4 - (8*a*b^7)/(a^2 + b^2)^5))/d + ((tan(c + d*x)
^3*(2*a*b^4 - 5*a^5 + 9*a^3*b^2))/(6*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
+ (tan(c + d*x)^5*(a*b^4 - 11*a^5 + 6*a^3*b^2))/(16*(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2)) + (tan(c + d*x)^6*(b^7 - 27*a^6*b + 7*a^2*b^5 + 43*a^4*b^3)
)/(16*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (tan(c + d*x)*(5*
a*b^4 - 15*a^5 + 38*a^3*b^2))/(48*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (
a*(a*b^5 - 17*a^5*b + 8*a^3*b^3))/(6*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 +
3*a^4*b^2)) - (tan(c + d*x)^4*(41*a^6*b + b^7 + a^2*b^5 - 31*a^4*b^3))/(6*
(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c + d*x)^2*(125*a^
6*b + b^7 - a^2*b^5 - 69*a^4*b^3))/(16*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4
+ 3*a^4*b^2)))/(d*(a + b*tan(c + d*x) + 3*a*tan(c + d*x)^2 + 3*a*tan(c + d
*x)^4 + a*tan(c + d*x)^6 + 3*b*tan(c + d*x)^3 + 3*b*tan(c + d*x)^5 + b*tan
(c + d*x)^7)) + (log(tan(c + d*x) + 1i)*(a*b^2*5i - 7*a^2*b + a^3*5i + b^3
))/(32*d*(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2)) -
 (log(tan(c + d*x) - 1i)*(a*b^2*5i + 7*a^2*b + a^3*5i - b^3))/(32*d*(5*a*b
^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2))
```

3.61. $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.62 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.62.1 Optimal result

Integrand size = 21, antiderivative size = 217

$$\begin{aligned}\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{(3a^6-33a^4b^2+13a^2b^4+b^6)\,x}{8\,(a^2+b^2)^4} \\ &+ \frac{2a^3b(a^2-2b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d} \\ &- \frac{a^4b}{(a^2+b^2)^3\,d(a+b\tan(c+dx))} \\ &+ \frac{\cos^4(c+dx)\,(2ab+(a^2-b^2)\tan(c+dx))}{4\,(a^2+b^2)^2\,d} \\ &- \frac{\cos^2(c+dx)\,(16a^3b+(5a^4-12a^2b^2-b^4)\tan(c+dx))}{8\,(a^2+b^2)^3\,d}\end{aligned}$$

output
```
1/8*(3*a^6-33*a^4*b^2+13*a^2*b^4+b^6)*x/(a^2+b^2)^4+2*a^3*b*(a^2-2*b^2)*ln
(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^4/d-a^4*b/(a^2+b^2)^3/d/(a+b*tan(d*x
+c))+1/4*cos(d*x+c)^4*(2*a*b+(a^2-b^2)*tan(d*x+c))/(a^2+b^2)^2/d-1/8*cos(d
*x+c)^2*(16*a^3*b+(5*a^4-12*a^2*b^2-b^4)*tan(d*x+c))/(a^2+b^2)^3/d
```

3.62.2 Mathematica [A] (verified)

Time = 4.47 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.81

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{b\left(\frac{3(a^2-b^2)(a^2+b^2)^2\arctan(\tan(c+dx))}{b}+\frac{4(a^2+b^2)(-2a^4+3a^2b^2+b^4)\arctan(\tan(c+dx))}{b}-16a^3(a^2+b^2)\cos^2(c+dx)+4a(\right.}{}$$

input
```
Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

output
```
(b*((3*(a^2 - b^2)*(a^2 + b^2)^2*ArcTan[Tan[c + d*x]])/b + (4*(a^2 + b^2)*
(-2*a^4 + 3*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/b - 16*a^3*(a^2 + b^2)*Co
s[c + d*x]^2 + 4*a*(a^2 + b^2)^2*Cos[c + d*x]^4 - 4*a^3*(2*a^2 - 4*b^2 + (
-a^3 + 5*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 16*a^3*(a^2
 - 2*b^2)*Log[a + b*Tan[c + d*x]] - 4*a^3*(2*a^2 - 4*b^2 + (a^3 - 5*a*b^2)
/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (2*(a^2 - b^2)*(a^2 + b^2)
^2*Cos[c + d*x]^3*Sin[c + d*x])/b + (3*(a - b)*(a + b)*(a^2 + b^2)^2*Sin[2
*(c + d*x)])/(2*b) + (2*(a^2 + b^2)*(-2*a^4 + 3*a^2*b^2 + b^4)*Sin[2*(c +
d*x)])/b - (8*a^4*(a^2 + b^2))/(a + b*Tan[c + d*x])))/(8*(a^2 + b^2)^4*d)
```

3.62.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3999, 601, 2178, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(c+dx)^4}{(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3999}$$
$$\frac{b\int \frac{b^4\tan^4(c+dx)}{(a+b\tan(c+dx))^2(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 601$$

$$\frac{b\left(\frac{b^2\left(b\left(a^2-b^2\right)\tan(c+dx)+2ab^2\right)}{4(a^2+b^2)^2(b^2\tan^2(c+dx)+b^2)^2}-\frac{\int \frac{-\frac{2a\left(3a^2+b^2\right)\tan(c+dx)b^5}{\left(a^2+b^2\right)^2}-\frac{\left(4a^4+11b^2a^2+b^4\right)\tan^2(c+dx)b^4}{\left(a^2+b^2\right)^2}+\frac{a^2\left(a^2-b^2\right)b^4}{\left(a^2+b^2\right)^2}}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2}\right)}{d}$$

$$\Big\downarrow \; 2178$$

$$\frac{b\left(\frac{b^2\left(b\left(a^2-b^2\right)\tan(c+dx)+2ab^2\right)}{4(a^2+b^2)^2(b^2\tan^2(c+dx)+b^2)^2}-\frac{\frac{b^2\left(16a^3b^2+b\left(5a^4-12a^2b^2-b^4\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{\int \frac{-\frac{\left(5a^4-12b^2a^2-b^4\right)\tan^2(c+dx)b^6}{\left(a^2+b^2\right)^3}-\frac{2a\left(5a^2-b^2\right)\tan(c+dx)b^5}{\left(a^2+b^2\right)^2}+\cdots}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)}\cdots}{2b^2}}{4b^2}\cdots}{d}$$

$$\Big\downarrow \; 2160$$

$$\frac{b\left(\frac{b^2\left(b\left(a^2-b^2\right)\tan(c+dx)+2ab^2\right)}{4(a^2+b^2)^2(b^2\tan^2(c+dx)+b^2)^2}-\frac{\frac{b^2\left(16a^3b^2+b\left(5a^4-12a^2b^2-b^4\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{\int\left(\frac{16a^3\left(a^2-2b^2\right)b^4}{\left(a^2+b^2\right)^4(a+b\tan(c+dx))}+\frac{\left(3a^6-33b^2a^4-16b\left(a^2-2b^2\right)\tan(c+\cdots\right.}{\left(a^2+b^2\right)^4\left(\tan^2(c+dx)\cdots\right.}\cdots\right.}{2b\cdots}}{4b^2}\cdots}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\left(\frac{b^2\left(b\left(a^2-b^2\right)\tan(c+dx)+2ab^2\right)}{4(a^2+b^2)^2(b^2\tan^2(c+dx)+b^2)^2}-\frac{\frac{b^2\left(16a^3b^2+b\left(5a^4-12a^2b^2-b^4\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{-\frac{8a^4b^4}{\left(a^2+b^2\right)^3(a+b\tan(c+dx))}-\frac{8a^3b^4\left(a^2-2b^2\right)\log\left(b^2\tan^2(c+dx)+b^2\cdots\right.}{\left(a^2+b^2\right)^4}\cdots}{\cdots}}{4b^2}\cdots}{d}$$

input

```
Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

3.62. $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

output
```
(b*((b^2*(2*a*b^2 + b*(a^2 - b^2)*Tan[c + d*x]))/(4*(a^2 + b^2)^2*(b^2 + b
^2*Tan[c + d*x]^2)^2) - ((b^2*(16*a^3*b^2 + b*(5*a^4 - 12*a^2*b^2 - b^4)*T
an[c + d*x]))/(2*(a^2 + b^2)^3*(b^2 + b^2*Tan[c + d*x]^2)) - ((b^3*(3*a^6
- 33*a^4*b^2 + 13*a^2*b^4 + b^6)*ArcTan[Tan[c + d*x]])/(a^2 + b^2)^4 + (16
*a^3*b^4*(a^2 - 2*b^2)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^4 - (8*a^3*b^4
*(a^2 - 2*b^2)*Log[b^2 + b^2*Tan[c + d*x]^2])/(a^2 + b^2)^4 - (8*a^4*b^4)/
((a^2 + b^2)^3*(a + b*Tan[c + d*x])))/(2*b^2))/(4*b^2)))/d
```

3.62.3.1 Defintions of rubi rules used

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.62.4 Maple [A] (verified)

Time = 9.21 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{-\frac{a^4 b}{(a^2+b^2)^3(a+b\tan(dx+c))}+\frac{2a^3b(a^2-2b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^4}+\frac{\left(-\frac{5}{8}a^6+\frac{7}{8}a^4b^2+\frac{13}{8}a^2b^4+\frac{1}{8}b^6\right)\left(\tan^3(dx+c)\right)+\left(-2a^5b-2a^3b^3\right.}{}}{}$
default	$\frac{-\frac{a^4 b}{(a^2+b^2)^3(a+b\tan(dx+c))}+\frac{2a^3b(a^2-2b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^4}+\frac{\left(-\frac{5}{8}a^6+\frac{7}{8}a^4b^2+\frac{13}{8}a^2b^4+\frac{1}{8}b^6\right)\left(\tan^3(dx+c)\right)+\left(-2a^5b-2a^3b^3\right.}{}}{}$
risch	$-\frac{ixab}{2(4ia^3b-4ia\,b^3-a^4+6a^2b^2-b^4)}-\frac{3x\,a^2}{8(4ia^3b-4ia\,b^3-a^4+6a^2b^2-b^4)}-\frac{x\,b^2}{8(4ia^3b-4ia\,b^3-a^4+6a^2b^2-b^4)}-\frac{ie}{64(-2a}$

input
```
int(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^4*b/(a^2+b^2)^3/(a+b*tan(d*x+c))+2*a^3*b*(a^2-2*b^2)/(a^2+b^2)^4*l
n(a+b*tan(d*x+c))+1/(a^2+b^2)^4*(((-5/8*a^6+7/8*a^4*b^2+13/8*a^2*b^4+1/8*b
^6)*tan(d*x+c)^3+(-2*a^5*b-2*a^3*b^3)*tan(d*x+c)^2+(-3/8*a^6+9/8*a^4*b^2+1
1/8*a^2*b^4-1/8*b^6)*tan(d*x+c)-3/2*a^5*b-a^3*b^3+1/2*a*b^5)/(1+tan(d*x+c)
^2)^2+1/16*(-16*a^5*b+32*a^3*b^3)*ln(1+tan(d*x+c)^2)+1/8*(3*a^6-33*a^4*b^2
+13*a^2*b^4+b^6)*arctan(tan(d*x+c))))
```

3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(211) = 422.

Time = 0.31 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.05

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$=\frac{4\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\cos\left(dx+c\right)^5-6\left(3\,a^6b+7\,a^4b^3+5\,a^2b^5+b^7\right)\cos\left(dx+c\right)^3+\left(3\,a^6b+8\,a^4b\right.}{}$$

input
```
integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/16*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^5 - 6*(3*a^6*b
+ 7*a^4*b^3 + 5*a^2*b^5 + b^7)*cos(d*x + c)^3 + (3*a^6*b + 8*a^4*b^3 + 23*
a^2*b^5 + 2*b^7 + 2*(3*a^7 - 33*a^5*b^2 + 13*a^3*b^4 + a*b^6)*d*x)*cos(d*x
 + c) + 16*((a^6*b - 2*a^4*b^3)*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4)*sin(d
*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) + (29*a^5*b^2 + 10*a^3*b^4 - 3*a*b^6 + 4*(a^7 + 3*a^5*b^2 + 3*a^3*b
^4 + a*b^6)*cos(d*x + c)^4 + 2*(3*a^6*b - 33*a^4*b^3 + 13*a^2*b^5 + b^7)*d
*x - 2*(5*a^7 + 9*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)^2)*sin(d*x + c
))/((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^
8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*d*sin(d*x + c))
```

3.62.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.62.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. 2(211) = 422.

Time = 0.66 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.34

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{(3a^6-33a^4b^2+13a^2b^4+b^6)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} + \frac{16(a^5b-2a^3b^3)\log(b\tan(dx+c)+a)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} - \frac{8(a^5b-2a^3b^3)\log\left(\tan(dx+c)^2+1\right)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} - \frac{}{a^7+3a^5b^2+3a^3b^4+ab}}{}$$

input
```
integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/8*((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*(d*x + c)/(a^8 + 4*a^6*b^2 +
6*a^4*b^4 + 4*a^2*b^6 + b^8) + 16*(a^5*b - 2*a^3*b^3)*log(b*tan(d*x + c) +
 a)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 8*(a^5*b - 2*a^3*b^3
)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)
- (20*a^4*b - 4*a^2*b^3 + (13*a^4*b - 12*a^2*b^3 - b^5)*tan(d*x + c)^4 + (
5*a^5 + 4*a^3*b^2 - a*b^4)*tan(d*x + c)^3 + (35*a^4*b - 12*a^2*b^3 + b^5)*
tan(d*x + c)^2 + 3*(a^5 - a*b^4)*tan(d*x + c))/(a^7 + 3*a^5*b^2 + 3*a^3*b^
4 + a*b^6 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)^5 + (a^7 +
3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c)^4 + 2*(a^6*b + 3*a^4*b^3 + 3*a
^2*b^5 + b^7)*tan(d*x + c)^3 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan
(d*x + c)^2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)))/d
```

3.62.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(211) = 422$.

Time = 0.53 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.36

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{(3a^6-33a^4b^2+13a^2b^4+b^6)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} - \frac{8(a^5b-2a^3b^3)\log\left(\tan(dx+c)^2+1\right)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} + \frac{16(a^5b^2-2a^3b^4)\log(|b\tan(dx+c)+a|)}{a^8b+4a^6b^3+6a^4b^5+4a^2b^7+b^9} - \frac{8(2a^5b^2\tan(dx+c)}{(a^8+4a^6b^2+6a^4}}{}$$

input
```
integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/8*((3*a^6 - 33*a^4*b^2 + 13*a^2*b^4 + b^6)*(d*x + c)/(a^8 + 4*a^6*b^2 +
6*a^4*b^4 + 4*a^2*b^6 + b^8) - 8*(a^5*b - 2*a^3*b^3)*log(tan(d*x + c)^2 +
1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 16*(a^5*b^2 - 2*a^3*b
^4)*log(abs(b*tan(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^
7 + b^9) - 8*(2*a^5*b^2*tan(d*x + c) - 4*a^3*b^4*tan(d*x + c) + 3*a^6*b -
3*a^4*b^3)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c
) + a)) + (12*a^5*b*tan(d*x + c)^4 - 24*a^3*b^3*tan(d*x + c)^4 - 5*a^6*tan
(d*x + c)^3 + 7*a^4*b^2*tan(d*x + c)^3 + 13*a^2*b^4*tan(d*x + c)^3 + b^6*t
an(d*x + c)^3 + 8*a^5*b*tan(d*x + c)^2 - 64*a^3*b^3*tan(d*x + c)^2 - 3*a^6
*tan(d*x + c) + 9*a^4*b^2*tan(d*x + c) + 11*a^2*b^4*tan(d*x + c) - b^6*tan
(d*x + c) - 32*a^3*b^3 + 4*a*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8)*(tan(d*x + c)^2 + 1)^2))/d
```

3.62.9 Mupad [B] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.22

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{\tan(c+dx)^3\,(a\,b^2-5\,a^3)}{8\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+dx)^4\,(-13\,a^4\,b+12\,a^2\,b^3+b^5)}{8\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{3\tan(c+dx)\,(a\,b^2-a^3)}{8\,(a^4+2\,a^2\,b^2+b^4)}-\frac{\tan(c+dx)^2\,(35\,a^4\,b-12\,a^2\,b^3+b^5)}{8\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}+\frac{a}{2\,(a^2+}}{d\,\left(b\tan\left(c+d\,x\right)^5+a\tan\left(c+d\,x\right)^4+2\,b\tan\left(c+d\,x\right)^3+2\,a\tan\left(c+d\,x\right)^2+b\tan\left(c+d\,x\right)+\right.}$$
$$+\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{2\,a\,b}{(a^2+b^2)^2}-\frac{8\,a\,b^3}{(a^2+b^2)^3}+\frac{6\,a\,b^5}{(a^2+b^2)^4}\right)}{d}$$
$$+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(3\,a^2-a\,b\,4\mathrm{i}+b^2\right)}{16\,d\,\left(a^4\,1\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6\mathrm{i}+4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$
$$-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(3\,a^2+a\,b\,4\mathrm{i}+b^2\right)}{16\,d\,\left(a^4\,1\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6\mathrm{i}-4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$

input
```
int(sin(c + d*x)^4/(a + b*tan(c + d*x))^2,x)
```

output
```
((tan(c + d*x)^3*(a*b^2 - 5*a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d
*x)^4*(b^5 - 13*a^4*b + 12*a^2*b^3))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2
)) + (3*tan(c + d*x)*(a*b^2 - a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)) - (tan(c +
 d*x)^2*(35*a^4*b + b^5 - 12*a^2*b^3))/(8*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b
^2)) + (a*(a*b^3 - 5*a^3*b))/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)))/(d*(
a + b*tan(c + d*x) + 2*a*tan(c + d*x)^2 + a*tan(c + d*x)^4 + 2*b*tan(c + d
*x)^3 + b*tan(c + d*x)^5)) + (log(a + b*tan(c + d*x))*((2*a*b)/(a^2 + b^2)
^2 - (8*a*b^3)/(a^2 + b^2)^3 + (6*a*b^5)/(a^2 + b^2)^4))/d + (log(tan(c +
d*x) - 1i)*(3*a^2 - a*b*4i + b^2))/(16*d*(4*a*b^3 - 4*a^3*b + a^4*1i + b^4
*1i - a^2*b^2*6i)) - (log(tan(c + d*x) + 1i)*(a*b*4i + 3*a^2 + b^2))/(16*d
*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i))
```

3.63 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2} \, dx$

3.63.1 Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2} \, dx = \frac{(a^4-6a^2b^2+b^4)\,x}{2\,(a^2+b^2)^3} + \frac{2ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d} - \frac{a^2b}{(a^2+b^2)^2\,d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\,(2ab+(a^2-b^2)\tan(c+dx))}{2\,(a^2+b^2)^2\,d}$$

output
```
1/2*(a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^3+2*a*b*(a^2-b^2)*ln(a*cos(d*x+c)+b*si
n(d*x+c))/(a^2+b^2)^3/d-a^2*b/(a^2+b^2)^2/d/(a+b*tan(d*x+c))-1/2*cos(d*x+c
)^2*(2*a*b+(a^2-b^2)*tan(d*x+c))/(a^2+b^2)^2/d
```

3.63.2 Mathematica [A] (verified)

Time = 3.75 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.66

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2} \, dx = -\frac{b\left(\frac{(a^2-b^2)(a^2+b^2)\arctan(\tan(c+dx))}{b} + 2a(a^2+b^2)\cos^2(c+dx) + a\left(2a^2-2b^2+\frac{-a^3+3ab^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan\right.\right.}{}$$

input
```
Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

output
```
-1/2*(b*(((a^2 - b^2)*(a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + 2*a*(a^2 + b^2
)*Cos[c + d*x]^2 + a*(2*a^2 - 2*b^2 + (-a^3 + 3*a*b^2)/Sqrt[-b^2])*Log[Sqr
t[-b^2] - b*Tan[c + d*x]] - 4*a*(a - b)*(a + b)*Log[a + b*Tan[c + d*x]] +
a*(2*a^2 - 2*b^2 + (a^3 - 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c +
d*x]] + ((a - b)*(a + b)*(a^2 + b^2)*Sin[2*(c + d*x)])/(2*b) + (2*a^2*(a^2
 + b^2))/(a + b*Tan[c + d*x])))/((a^2 + b^2)^3*d)
```

3.63.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3999, 601, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^2}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3999}$$

$$\frac{b\int \frac{b^2\tan^2(c+dx)}{(a+b\tan(c+dx))^2(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{601}$$

$$\frac{b\left(-\frac{\int -\frac{-\frac{\left(a^2-b^2\right)\tan^2(c+dx)b^4}{\left(a^2+b^2\right)^2}-\frac{2a\tan(c+dx)b^3}{a^2+b^2}+\frac{a^2\left(a^2-b^2\right)b^2}{\left(a^2+b^2\right)^2}}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2}-\frac{b(a^2-b^2)\tan(c+dx)+2ab^2}{2(a^2+b^2)^2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\int \frac{-\frac{(a^2-b^2)\tan^2(c+dx)b^4}{(a^2+b^2)^2}-\frac{2a\tan(c+dx)b^3}{a^2+b^2}+\frac{a^2(a^2-b^2)b^2}{(a^2+b^2)^2}}{(a+b\tan(c+dx))^2(\tan^2(c+dx)b^2+b^2)}d(b\tan(c+dx))}{2b^2}-\frac{b(a^2-b^2)\tan(c+dx)+2ab^2}{2(a^2+b^2)^2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$\Big\downarrow$ 2160

$$\frac{b\left(\frac{\int \left(\frac{4a(a^2-b^2)b^2}{(a^2+b^2)^3(a+b\tan(c+dx))}+\frac{(a^4-6b^2a^2-4b(a^2-b^2)\tan(c+dx)a+b^4)b^2}{(a^2+b^2)^3(\tan^2(c+dx)b^2+b^2)}+\frac{2a^2b^2}{(a^2+b^2)^2(a+b\tan(c+dx))^2}\right)d(b\tan(c+dx))}{2b^2}-\frac{b(a^2-b^2)\tan(c+dx)}{2(a^2+b^2)^2(b^2\tan^2(c+}\right.}{d}$$

$\Big\downarrow$ 2009

$$\frac{b\left(\frac{-\frac{2a^2b^2}{(a^2+b^2)^2(a+b\tan(c+dx))}-\frac{2ab^2(a^2-b^2)\log(b^2\tan^2(c+dx)+b^2)}{(a^2+b^2)^3}+\frac{4ab^2(a^2-b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^3}+\frac{b(a^4-6a^2b^2+b^4)\arctan(\tan(c+dx))}{(a^2+b^2)^3}}{2b^2}-\frac{b(a^2}{2(a^2+}\right.}{d}$$

input
```
Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

output
```
(b*(-1/2*(2*a*b^2 + b*(a^2 - b^2)*Tan[c + d*x])/((a^2 + b^2)^2*(b^2 + b^2*
Tan[c + d*x]^2)) + ((b*(a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/(a^2
+ b^2)^3 + (4*a*b^2*(a^2 - b^2)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^3 - (
2*a*b^2*(a^2 - b^2)*Log[b^2 + b^2*Tan[c + d*x]^2])/(a^2 + b^2)^3 - (2*a^2*
b^2)/((a^2 + b^2)^2*(a + b*Tan[c + d*x]))))/(2*b^2)))/d
```

3.63.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.63.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\frac{\frac{\left(-\frac{a^4}{2}+\frac{b^4}{2}\right)\tan(dx+c)-a^3b-a\,b^3}{1+\tan^2(dx+c)}+\frac{\left(-4a^3b+4a\,b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{4}+\frac{\left(a^4-6a^2b^2+b^4\right)\arctan(\tan(dx+c))}{2}}{\left(a^2+b^2\right)^3}-\frac{a^2b}{\left(a^2+b^2\right)^2(a+b\tan}}{d}$
default	$\frac{\frac{\frac{\left(-\frac{a^4}{2}+\frac{b^4}{2}\right)\tan(dx+c)-a^3b-a\,b^3}{1+\tan^2(dx+c)}+\frac{\left(-4a^3b+4a\,b^3\right)\ln\left(1+\tan^2(dx+c)\right)}{4}+\frac{\left(a^4-6a^2b^2+b^4\right)\arctan(\tan(dx+c))}{2}}{\left(a^2+b^2\right)^3}-\frac{a^2b}{\left(a^2+b^2\right)^2(a+b\tan}}{d}$
risch	$-\frac{ixb}{2(3ib\,a^2-ib^3-a^3+3a\,b^2)}-\frac{xa}{2(3ib\,a^2-ib^3-a^3+3a\,b^2)}+\frac{ie^{2i(dx+c)}}{8(-2iab+a^2-b^2)d}-\frac{ie^{-2i(dx+c)}}{8(2iab+a^2-b^2)d}-\frac{4ia^3b}{a^6+3a^4b^2+3}$

3.63. $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

input
```
int(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^3*(((-1/2*a^4+1/2*b^4)*tan(d*x+c)-a^3*b-a*b^3)/(1+tan(d*x
+c)^2)+1/4*(-4*a^3*b+4*a*b^3)*ln(1+tan(d*x+c)^2)+1/2*(a^4-6*a^2*b^2+b^4)*a
rctan(tan(d*x+c)))-a^2*b/(a^2+b^2)^2/(a+b*tan(d*x+c))+2*a*b*(a^2-b^2)/(a^2
+b^2)^3*ln(a+b*tan(d*x+c)))
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(144) = 288$.

Time = 0.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.97

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{(a^4b+2\,a^2b^3+b^5)\cos{(dx+c)}^3+(a^2b^3-b^5-(a^5-6\,a^3b^2+ab^4)dx)\cos{(dx+c)}-2\,((a^4b-a^2b^3)\cos\cdots}{2\cdots}$$

input
```
integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 + (a^2*b^3 - b^5 - (a^5 - 6
*a^3*b^2 + a*b^4)*d*x)*cos(d*x + c) - 2*((a^4*b - a^2*b^3)*cos(d*x + c) +
(a^3*b^2 - a*b^4)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2
 - b^2)*cos(d*x + c)^2 + b^2) - (3*a^3*b^2 + a*b^4 + (a^4*b - 6*a^2*b^3 +
b^5)*d*x - (a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 +
 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^
2*b^5 + b^7)*d*sin(d*x + c))
```

3.63.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.63.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(144) = 288$.

Time = 0.53 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.98

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{(a^4-6a^2b^2+b^4)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{4(a^3b-ab^3)\log(b\tan(dx+c)+a)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{2(a^3b-ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{4a^2b+(3a^2b}{a^5+2a^3b^2+ab^4+(a^4b+2a^2b^3+b^5)\tan(d}}{2d}$$

input
```
integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*((a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
 + 4*(a^3*b - a*b^3)*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6) - 2*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6) - (4*a^2*b + (3*a^2*b - b^3)*tan(d*x + c)^2 + (a^3 + a*b^2)*t
an(d*x + c))/(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x
+ c)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*tan(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b
^5)*tan(d*x + c)))/d
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.78

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{(a^4-6a^2b^2+b^4)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{2(a^3b-ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{4(a^3b^2-ab^4)\log(|b\tan(dx+c)+a|)}{a^6b+3a^4b^3+3a^2b^5+b^7} - \frac{3a^2b\tan(dx+c)^2-b^3\tan(dx+c)^2+a^3}{(a^4+2a^2b^2+b^4)\left(b\tan(dx+c)^3+a\right.}}{2d}$$

input
```
integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*((a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
 - 2*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6) + 4*(a^3*b^2 - a*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b
^3 + 3*a^2*b^5 + b^7) - (3*a^2*b*tan(d*x + c)^2 - b^3*tan(d*x + c)^2 + a^3
*tan(d*x + c) + a*b^2*tan(d*x + c) + 4*a^2*b)/((a^4 + 2*a^2*b^2 + b^4)*(b*
tan(d*x + c)^3 + a*tan(d*x + c)^2 + b*tan(d*x + c) + a)))/d
```

3.63.9 Mupad [B] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.72

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{2\,a\,b}{\left(a^2+b^2\right)^2}-\frac{4\,a\,b^3}{\left(a^2+b^2\right)^3}\right)}{d} - \frac{\frac{\tan(c+d\,x)^2\,\left(3\,a^2\,b-b^3\right)}{2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{a\tan(c+d\,x)}{2\,\left(a^2+b^2\right)}+\frac{2\,a^2\,b}{\left(a^2+b^2\right)^2}}{d\,\left(b\tan\left(c+d\,x\right)^3+a\tan\left(c+d\,x\right)^2+b\tan\left(c+d\,x\right)+a\right)} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(a+b\,1\mathrm{i}\right)}{4\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)}{4\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}$$

input
```
int(sin(c + d*x)^2/(a + b*tan(c + d*x))^2,x)
```

output
```
(log(a + b*tan(c + d*x))*((2*a*b)/(a^2 + b^2)^2 - (4*a*b^3)/(a^2 + b^2)^3)
)/d - ((tan(c + d*x)^2*(3*a^2*b - b^3))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (a*t
an(c + d*x))/(2*(a^2 + b^2)) + (2*a^2*b)/(a^2 + b^2)^2)/(d*(a + b*tan(c +
d*x) + a*tan(c + d*x)^2 + b*tan(c + d*x)^3)) + (log(tan(c + d*x) + 1i)*(a
+ b*1i))/(4*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) + (log(tan(c + d*x) - 1
i)*(a*1i + b))/(4*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i))
```

3.64 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.64.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx=-\frac{\cot(c+dx)}{a^2d}-\frac{2b\log(\tan(c+dx))}{a^3d}+\frac{2b\log(a+b\tan(c+dx))}{a^3d}-\frac{b}{a^2d(a+b\tan(c+dx))}$$

output
```
-cot(d*x+c)/a^2/d-2*b*ln(tan(d*x+c))/a^3/d+2*b*ln(a+b*tan(d*x+c))/a^3/d-b/
a^2/d/(a+b*tan(d*x+c))
```

3.64.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.51

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx =\frac{-a^2\cot^2(c+dx)-ab\cot(c+dx)(1+2\log(\sin(c+dx))-2\log(a\cos(c+dx)+b\sin(c+dx)))+b^2(1-}{a^3d(b+a\cot(c+dx))}$$

input
```
Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

output
```
(-(a^2*Cot[c + d*x]^2) - a*b*Cot[c + d*x]*(1 + 2*Log[Sin[c + d*x]] - 2*Log
[a*Cos[c + d*x] + b*Sin[c + d*x]]) + b^2*(1 - 2*Log[Sin[c + d*x]] + 2*Log[
a*Cos[c + d*x] + b*Sin[c + d*x]]))/(a^3*d*(b + a*Cot[c + d*x]))
```

3.64.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3999, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(c+dx)^2(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \; 3999$$

$$\frac{b\int \frac{\cot^2(c+dx)}{b^2(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 54$$

$$\frac{b\int \left(\frac{\cot^2(c+dx)}{a^2b^2}-\frac{2\cot(c+dx)}{a^3b}+\frac{2}{a^3(a+b\tan(c+dx))}+\frac{1}{a^2(a+b\tan(c+dx))^2}\right)d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\left(-\frac{2\log(b\tan(c+dx))}{a^3}+\frac{2\log(a+b\tan(c+dx))}{a^3}-\frac{1}{a^2(a+b\tan(c+dx))}-\frac{\cot(c+dx)}{a^2b}\right)}{d}$$

input
```
Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

output
```
(b*(-(Cot[c + d*x]/(a^2*b)) - (2*Log[b*Tan[c + d*x]])/a^3 + (2*Log[a + b*T
an[c + d*x]])/a^3 - 1/(a^2*(a + b*Tan[c + d*x]))))/d
```

3.64.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.64.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

method	result	siz
derivativedivides	$\frac{-\frac{b}{a^{2}(a+b\tan(dx+c))}+\frac{2b\ln(a+b\tan(dx+c))}{a^{3}}-\frac{1}{a^{2}\tan(dx+c)}-\frac{2b\ln(\tan(dx+c))}{a^{3}}}{d}$	67
default	$\frac{-\frac{b}{a^{2}(a+b\tan(dx+c))}+\frac{2b\ln(a+b\tan(dx+c))}{a^{3}}-\frac{1}{a^{2}\tan(dx+c)}-\frac{2b\ln(\tan(dx+c))}{a^{3}}}{d}$	67
risch	$-\frac{2i\left(2iab\,e^{2i(dx+c)}-a^{2}e^{2i(dx+c)}+2b^{2}e^{2i(dx+c)}-a^{2}-2b^{2}\right)}{\left(e^{2i(dx+c)}-1\right)(ia+b)\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)a^{2}d}-\frac{2b\ln\left(e^{2i(dx+c)}-1\right)}{a^{3}d}+\frac{2b\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{a^{3}d}$	17

input
```
int(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^2*b/(a+b*tan(d*x+c))+2/a^3*b*ln(a+b*tan(d*x+c))-1/a^2/tan(d*x+c)
-2/a^3*b*ln(tan(d*x+c)))
```

3.64.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(72) = 144$.

Time = 0.28 (sec) , antiderivative size = 293, normalized size of antiderivative = 4.07

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{a^2b^2 - (a^4 + 2\,a^2b^2)\cos{(dx+c)}^2 - (a^3b + 2\,ab^3)\cos{(dx+c)}\sin{(dx+c)} + \left(a^2b^2 + b^4 - (a^2b^2 + b^4)\,\mathrm{co}\right.}{}$$

input
```
integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(a^2*b^2 - (a^4 + 2*a^2*b^2)*cos(d*x + c)^2 - (a^3*b + 2*a*b^3)*cos(d*x +
 c)*sin(d*x + c) + (a^2*b^2 + b^4 - (a^2*b^2 + b^4)*cos(d*x + c)^2 + (a^3*
b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c)
+ (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*b^2 + b^4 - (a^2*b^2 + b^4)*cos
(d*x + c)^2 + (a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(-1/4*cos(d*x
+ c)^2 + 1/4))/((a^5*b + a^3*b^3)*d*cos(d*x + c)^2 - (a^6 + a^4*b^2)*d*cos
(d*x + c)*sin(d*x + c) - (a^5*b + a^3*b^3)*d)
```

3.64.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\csc^2{(c+dx)}}{(a+b\tan{(c+dx)})^2}\,dx$$

input
```
integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(csc(c + d*x)**2/(a + b*tan(c + d*x))**2, x)
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{2\,b\tan(dx+c)+a}{a^2b\tan(dx+c)^2+a^3\tan(dx+c)} - \frac{2\,b\log(b\tan(dx+c)+a)}{a^3} + \frac{2\,b\log(\tan(dx+c))}{a^3}}{d}$$

input
```
integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-((2*b*tan(d*x + c) + a)/(a^2*b*tan(d*x + c)^2 + a^3*tan(d*x + c)) - 2*b*l
og(b*tan(d*x + c) + a)/a^3 + 2*b*log(tan(d*x + c))/a^3)/d
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{2\,b\log(|b\tan(dx+c)+a|)}{a^3} - \frac{2\,b\log(|\tan(dx+c)|)}{a^3} - \frac{2\,b\tan(dx+c)+a}{\left(b\tan(dx+c)^2+a\tan(dx+c)\right)a^2}}{d}$$

input
```
integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
(2*b*log(abs(b*tan(d*x + c) + a))/a^3 - 2*b*log(abs(tan(d*x + c)))/a^3 - (
2*b*tan(d*x + c) + a)/((b*tan(d*x + c)^2 + a*tan(d*x + c))*a^2))/d
```

3.64.9 Mupad [B] (verification not implemented)

Time = 5.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{2\,b\,\ln\left(\frac{a+b\tan(c+d\,x)}{\tan(c+d\,x)}\right)}{a^3\,d} - \frac{2\,b}{a^2\,d\,\left(a+b\tan\left(c+d\,x\right)\right)} - \frac{1}{a\,d\tan\left(c+d\,x\right)\,\left(a+b\tan\left(c+d\,x\right)\right)}$$

input
```
int(1/(sin(c + d*x)^2*(a + b*tan(c + d*x))^2),x)
```

output
```
(2*b*log((a + b*tan(c + d*x))/tan(c + d*x)))/(a^3*d) - (2*b)/(a^2*d*(a + b
*tan(c + d*x))) - 1/(a*d*tan(c + d*x)*(a + b*tan(c + d*x)))
```

3.65 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.65.1 Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{(a^2+3b^2)\cot(c+dx)}{a^4d} + \frac{b\cot^2(c+dx)}{a^3d} - \frac{\cot^3(c+dx)}{3a^2d} - \frac{2b(a^2+2b^2)\log(\tan(c+dx))}{a^5d} + \frac{2b(a^2+2b^2)\log(a+b\tan(c+dx))}{a^5d} - \frac{b(a^2+b^2)}{a^4d(a+b\tan(c+dx))}$$

output
```
-(a^2+3*b^2)*cot(d*x+c)/a^4/d+b*cot(d*x+c)^2/a^3/d-1/3*cot(d*x+c)^3/a^2/d-
2*b*(a^2+2*b^2)*ln(tan(d*x+c))/a^5/d+2*b*(a^2+2*b^2)*ln(a+b*tan(d*x+c))/a^
5/d-b*(a^2+b^2)/a^4/d/(a+b*tan(d*x+c))
```

3.65.2 Mathematica [A] (verified)

Time = 4.10 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.74

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{-\cot^2(c+dx)\left(2a^4+9a^2b^2+a^4\csc^2(c+dx)\right)+3b^2(a^2+b^2+a^2\csc^2(c+dx)-2(a^2+2b^2)\log(\sin(c+\ldots}{}$$

input
```
Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

output
```
(-(Cot[c + d*x]^2*(2*a^4 + 9*a^2*b^2 + a^4*Csc[c + d*x]^2)) + 3*b^2*(a^2 +
 b^2 + a^2*Csc[c + d*x]^2 - 2*(a^2 + 2*b^2)*Log[Sin[c + d*x]] + 2*a^2*Log[
a*Cos[c + d*x] + b*Sin[c + d*x]] + 4*b^2*Log[a*Cos[c + d*x] + b*Sin[c + d*
x]]) + a*b*Cot[c + d*x]*(-2*a^2 - 9*b^2 + 2*a^2*Csc[c + d*x]^2 - 6*(a^2 +
2*b^2)*Log[Sin[c + d*x]] + 6*a^2*Log[a*Cos[c + d*x] + b*Sin[c + d*x]] + 12
*b^2*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]))/(3*a^5*d*(b + a*Cot[c + d*x]))
```

3.65.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3999, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^4(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3999}$$

$$\frac{b\int \frac{\cot^4(c+dx)\left(\tan^2(c+dx)b^2+b^2\right)}{b^4(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{b\int \left(\frac{\cot^4(c+dx)}{a^2b^2}-\frac{2\cot^3(c+dx)}{a^3b}+\frac{(a^2+3b^2)\cot^2(c+dx)}{a^4b^2}-\frac{2(a^2+2b^2)\cot(c+dx)}{a^5b}+\frac{2(a^2+2b^2)}{a^5(a+b\tan(c+dx))}+\frac{a^2+b^2}{a^4(a+b\tan(c+dx))^2}\right)d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{\cot^2(c+dx)}{a^3}-\frac{\cot^3(c+dx)}{3a^2b}-\frac{2(a^2+2b^2)\log(b\tan(c+dx))}{a^5}+\frac{2(a^2+2b^2)\log(a+b\tan(c+dx))}{a^5}-\frac{a^2+b^2}{a^4(a+b\tan(c+dx))}-\frac{(a^2+3b^2)\cot(c+dx)}{a^4b}\right)}{d}$$

input
```
Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

output
```
(b*(-(((a^2 + 3*b^2)*Cot[c + d*x])/(a^4*b)) + Cot[c + d*x]^2/a^3 - Cot[c +
 d*x]^3/(3*a^2*b) - (2*(a^2 + 2*b^2)*Log[b*Tan[c + d*x]])/a^5 + (2*(a^2 +
2*b^2)*Log[a + b*Tan[c + d*x]])/a^5 - (a^2 + b^2)/(a^4*(a + b*Tan[c + d*x]
))))/d
```

3.65.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.65.4 Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{\left(a^2+b^2\right)b}{a^4(a+b\tan(dx+c))}+\frac{2b\left(a^2+2b^2\right)\ln(a+b\tan(dx+c))}{a^5}-\frac{1}{3a^2\tan(dx+c)^3}-\frac{a^2+3b^2}{a^4\tan(dx+c)}+\frac{b}{a^3\tan(dx+c)^2}-\frac{2b\left(a^2+2b^2\right)\ln(\tan(dx}{a^5}}{d}$
default	$\frac{-\frac{\left(a^2+b^2\right)b}{a^4(a+b\tan(dx+c))}+\frac{2b\left(a^2+2b^2\right)\ln(a+b\tan(dx+c))}{a^5}-\frac{1}{3a^2\tan(dx+c)^3}-\frac{a^2+3b^2}{a^4\tan(dx+c)}+\frac{b}{a^3\tan(dx+c)^2}-\frac{2b\left(a^2+2b^2\right)\ln(\tan(dx}{a^5}}{d}$
risch	$-\frac{4i\left(-12ia\,b^2\mathrm{e}^{2i(dx+c)}-3a^2b\,\mathrm{e}^{4i(dx+c)}+a^2b\,\mathrm{e}^{2i(dx+c)}-3ia^3\mathrm{e}^{4i(dx+c)}+6b^3\mathrm{e}^{6i(dx+c)}-18b^3\mathrm{e}^{4i(dx+c)}+3a^2b\,\mathrm{e}^{6i(dx+c)}+i\right)}{3\left(\mathrm{e}^{2i(dx+c)}-1\right)^3\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)}$

input
```
int(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-(a^2+b^2)*b/a^4/(a+b*tan(d*x+c))+2*b*(a^2+2*b^2)/a^5*ln(a+b*tan(d*x+
c))-1/3/a^2/tan(d*x+c)^3-(a^2+3*b^2)/a^4/tan(d*x+c)+1/a^3*b/tan(d*x+c)^2-2
*b*(a^2+2*b^2)/a^5*ln(tan(d*x+c)))
```

3.65.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(138) = 276$.

Time = 0.29 (sec) , antiderivative size = 442, normalized size of antiderivative = 3.16

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{2\,(a^4+6\,a^2b^2)\cos{(dx+c)}^4+6\,a^2b^2-3\,(a^4+6\,a^2b^2)\cos{(dx+c)}^2+3\,\left((a^2b^2+2\,b^4)\cos{(dx+c)}^4+a^2b^2\right.}{}$$

input
```
integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(2*(a^4 + 6*a^2*b^2)*cos(d*x + c)^4 + 6*a^2*b^2 - 3*(a^4 + 6*a^2*b^2)*
cos(d*x + c)^2 + 3*((a^2*b^2 + 2*b^4)*cos(d*x + c)^4 + a^2*b^2 + 2*b^4 - 2
*(a^2*b^2 + 2*b^4)*cos(d*x + c)^2 - ((a^3*b + 2*a*b^3)*cos(d*x + c)^3 - (a
^3*b + 2*a*b^3)*cos(d*x + c))*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x
 + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 3*((a^2*b^2 + 2*b^4)*cos(d*x +
 c)^4 + a^2*b^2 + 2*b^4 - 2*(a^2*b^2 + 2*b^4)*cos(d*x + c)^2 - ((a^3*b + 2
*a*b^3)*cos(d*x + c)^3 - (a^3*b + 2*a*b^3)*cos(d*x + c))*sin(d*x + c))*log
(-1/4*cos(d*x + c)^2 + 1/4) - 2*(6*a*b^3*cos(d*x + c) - (a^3*b + 6*a*b^3)*
cos(d*x + c)^3)*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*b*d*cos(d*x
+ c)^2 + a^5*b*d - (a^6*d*cos(d*x + c)^3 - a^6*d*cos(d*x + c))*sin(d*x + c
))
```

3.65.6 Sympy [F]

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\csc^4{(c+dx)}}{(a+b\tan{(c+dx)})^2}\,dx$$

input
```
integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(csc(c + d*x)**4/(a + b*tan(c + d*x))**2, x)
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.03

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{2\,a^2b\tan(dx+c)-6\,(a^2b+2\,b^3)\tan(dx+c)^3-a^3-3\,(a^3+2\,ab^2)\tan(dx+c)^2}{a^4b\tan(dx+c)^4+a^5\tan(dx+c)^3}+\frac{6\,(a^2b+2\,b^3)\log(b\tan(dx+c)+a)}{a^5}-\frac{6\,(a^2b+2\,b^3)\log(\tan(dx+c))}{a^5}}{3\,d}$$

input
```
integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/3*((2*a^2*b*tan(d*x + c) - 6*(a^2*b + 2*b^3)*tan(d*x + c)^3 - a^3 - 3*(a
^3 + 2*a*b^2)*tan(d*x + c)^2)/(a^4*b*tan(d*x + c)^4 + a^5*tan(d*x + c)^3)
+ 6*(a^2*b + 2*b^3)*log(b*tan(d*x + c) + a)/a^5 - 6*(a^2*b + 2*b^3)*log(ta
n(d*x + c))/a^5)/d
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.45

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{6\,(a^2b+2\,b^3)\log(|\tan(dx+c)|)}{a^5}-\frac{6\,(a^2b^2+2\,b^4)\log(|b\tan(dx+c)+a|)}{a^5b}+\frac{3\,(2\,a^2b^2\tan(dx+c)+4\,b^4\tan(dx+c)+3\,a^3b+5\,ab^3)}{(b\tan(dx+c)+a)a^5}-\frac{11\,a^2b\tan(}{}}{3\,d}$$

input
```
integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/3*(6*(a^2*b + 2*b^3)*log(abs(tan(d*x + c)))/a^5 - 6*(a^2*b^2 + 2*b^4)*l
og(abs(b*tan(d*x + c) + a))/(a^5*b) + 3*(2*a^2*b^2*tan(d*x + c) + 4*b^4*ta
n(d*x + c) + 3*a^3*b + 5*a*b^3)/((b*tan(d*x + c) + a)*a^5) - (11*a^2*b*tan
(d*x + c)^3 + 22*b^3*tan(d*x + c)^3 - 3*a^3*tan(d*x + c)^2 - 9*a*b^2*tan(d
*x + c)^2 + 3*a^2*b*tan(d*x + c) - a^3)/(a^5*tan(d*x + c)^3))/d
```

3.65.9 Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{4\,b\,\mathrm{atanh}\left(\frac{2\,b\,(a^2+2\,b^2)\,(a+2\,b\tan(c+d\,x))}{a\,(2\,a^2\,b+4\,b^3)}\right)\,(a^2+2\,b^2)}{a^5\,d} - \frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,(a^2+2\,b^2)}{a^3}-\frac{2\,b\tan(c+d\,x)}{3\,a^2}+\frac{2\,b\tan(c+d\,x)^3\,(a^2+2\,b^2)}{a^4}}{d\,\left(b\tan\left(c+d\,x\right)^4+a\tan\left(c+d\,x\right)^3\right)}$$

input
```
int(1/(sin(c + d*x)^4*(a + b*tan(c + d*x))^2),x)
```

output
```
(4*b*atanh((2*b*(a^2 + 2*b^2)*(a + 2*b*tan(c + d*x)))/(a*(2*a^2*b + 4*b^3)
))*(a^2 + 2*b^2))/(a^5*d) - (1/(3*a) + (tan(c + d*x)^2*(a^2 + 2*b^2))/a^3
- (2*b*tan(c + d*x))/(3*a^2) + (2*b*tan(c + d*x)^3*(a^2 + 2*b^2))/a^4)/(d*
(a*tan(c + d*x)^3 + b*tan(c + d*x)^4))
```

3.66 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.66.1 Optimal result

Integrand size = 21, antiderivative size = 219

$$\begin{aligned}\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx =& -\frac{(a^2+b^2)(a^2+5b^2)\cot(c+dx)}{a^6d}+\frac{2b(a^2+b^2)\cot^2(c+dx)}{a^5d}\\&-\frac{(2a^2+3b^2)\cot^3(c+dx)}{3a^4d}+\frac{b\cot^4(c+dx)}{2a^3d}\\&-\frac{\cot^5(c+dx)}{5a^2d}-\frac{2b(a^2+b^2)(a^2+3b^2)\log(\tan(c+dx))}{a^7d}\\&+\frac{2b(a^2+b^2)(a^2+3b^2)\log(a+b\tan(c+dx))}{a^7d}\\&-\frac{b(a^2+b^2)^2}{a^6d(a+b\tan(c+dx))}\end{aligned}$$

output
```
-(a^2+b^2)*(a^2+5*b^2)*cot(d*x+c)/a^6/d+2*b*(a^2+b^2)*cot(d*x+c)^2/a^5/d-1
/3*(2*a^2+3*b^2)*cot(d*x+c)^3/a^4/d+1/2*b*cot(d*x+c)^4/a^3/d-1/5*cot(d*x+c
)^5/a^2/d-2*b*(a^2+b^2)*(a^2+3*b^2)*ln(tan(d*x+c))/a^7/d+2*b*(a^2+b^2)*(a^
2+3*b^2)*ln(a+b*tan(d*x+c))/a^7/d-b*(a^2+b^2)^2/a^6/d/(a+b*tan(d*x+c))
```

3.66.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 589 vs. $2(219) = 438$.

Time = 7.71 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.69

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\csc^5(c+dx)\sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{5a^2d(a+b\tan(c+dx))^2}$$
$$+\frac{(-8a^4\cos(c+dx)-75a^2b^2\cos(c+dx)-75b^4\cos(c+dx))\csc(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{15a^6d(a+b\tan(c+dx))^2}$$
$$+\frac{b(a^2+2b^2)\csc^2(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{a^5d(a+b\tan(c+dx))^2}$$
$$+\frac{(-4a^2\cos(c+dx)-15b^2\cos(c+dx))\csc^3(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{15a^4d(a+b\tan(c+dx))^2}$$
$$+\frac{b\csc^4(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{2a^3d(a+b\tan(c+dx))^2}$$
$$-\frac{2(a^4b+4a^2b^3+3b^5)\log(\sin(c+dx))\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{a^7d(a+b\tan(c+dx))^2}$$
$$+\frac{2(a^4b+4a^2b^3+3b^5)\log(a\cos(c+dx)+b\sin(c+dx))\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{a^7d(a+b\tan(c+dx))^2}$$
$$+\frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,(a^4b^2\sin(c+dx)+2a^2b^4\sin(c+dx)+b^6\sin(c+dx))}{a^7d(a+b\tan(c+dx))^2}$$

input
```
Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

output
```
-1/5*(Csc[c + d*x]^5*Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^
2*d*(a + b*Tan[c + d*x])^2) + ((-8*a^4*Cos[c + d*x] - 75*a^2*b^2*Cos[c + d
*x] - 75*b^4*Cos[c + d*x])*Csc[c + d*x]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b
*Sin[c + d*x])^2)/(15*a^6*d*(a + b*Tan[c + d*x])^2) + (b*(a^2 + 2*b^2)*Csc
[c + d*x]^2*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^5*d*(a
 + b*Tan[c + d*x])^2) + ((-4*a^2*Cos[c + d*x] - 15*b^2*Cos[c + d*x])*Csc[c
 + d*x]^3*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(15*a^4*d*(a
 + b*Tan[c + d*x])^2) + (b*Csc[c + d*x]^4*Sec[c + d*x]^2*(a*Cos[c + d*x] +
b*Sin[c + d*x])^2)/(2*a^3*d*(a + b*Tan[c + d*x])^2) - (2*(a^4*b + 4*a^2*b^
3 + 3*b^5)*Log[Sin[c + d*x]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*
x])^2)/(a^7*d*(a + b*Tan[c + d*x])^2) + (2*(a^4*b + 4*a^2*b^3 + 3*b^5)*Log
[a*Cos[c + d*x] + b*Sin[c + d*x]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^2)/(a^7*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c + d*
x] + b*Sin[c + d*x])*(a^4*b^2*Sin[c + d*x] + 2*a^2*b^4*Sin[c + d*x] + b^6*
Sin[c + d*x]))/(a^7*d*(a + b*Tan[c + d*x])^2)
```

3.66.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3999, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(c+dx)^6(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \ 3999$$

$$\frac{b\int \frac{\cot^6(c+dx)\left(\tan^2(c+dx)b^2+b^2\right)^2}{b^6(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \ 522$$

$$\frac{b\int \left(\frac{\cot^6(c+dx)}{a^2b^2}-\frac{2\cot^5(c+dx)}{a^3b}+\frac{\left(3b^4+2a^2b^2\right)\cot^4(c+dx)}{a^4b^4}-\frac{4\left(a^2+b^2\right)\cot^3(c+dx)}{a^5b}+\frac{\left(a^4+6b^2a^2+5b^4\right)\cot^2(c+dx)}{a^6b^2}-\frac{2(a^4+4b^2a^2+3b^4}{a^7b}\right.}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{b\left(\frac{\cot^4(c+dx)}{2a^3}-\frac{\cot^5(c+dx)}{5a^2b}-\frac{2\left(a^2+b^2\right)\left(a^2+3b^2\right)\log(b\tan(c+dx))}{a^7}+\frac{2\left(a^2+b^2\right)\left(a^2+3b^2\right)\log(a+b\tan(c+dx))}{a^7}-\frac{\left(a^2+b^2\right)^2}{a^6(a+b\tan(c+dx))}-\frac{(}{}\right.}{d}$$

input
```
Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

output
```
(b*(-(((a^2 + b^2)*(a^2 + 5*b^2)*Cot[c + d*x])/(a^6*b)) + (2*(a^2 + b^2)*C
ot[c + d*x]^2)/a^5 - ((2*a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*a^4*b) + Cot[c +
d*x]^4/(2*a^3) - Cot[c + d*x]^5/(5*a^2*b) - (2*(a^2 + b^2)*(a^2 + 3*b^2)*L
og[b*Tan[c + d*x]])/a^7 + (2*(a^2 + b^2)*(a^2 + 3*b^2)*Log[a + b*Tan[c + d
*x]])/a^7 - (a^2 + b^2)^2/(a^6*(a + b*Tan[c + d*x]))))/d
```

3.66.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.66.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{1}{5a^2\tan(dx+c)^5}-\frac{2a^2+3b^2}{3a^4\tan(dx+c)^3}-\frac{a^4+6a^2b^2+5b^4}{a^6\tan(dx+c)}+\frac{b}{2a^3\tan(dx+c)^4}+\frac{2b(a^2+b^2)}{a^5\tan(dx+c)^2}-\frac{2b(a^4+4a^2b^2+3b^4)\ln(\tan(dx+c))}{a^7}-\frac{(}{a^6}}{d}$
default	$\frac{-\frac{1}{5a^2\tan(dx+c)^5}-\frac{2a^2+3b^2}{3a^4\tan(dx+c)^3}-\frac{a^4+6a^2b^2+5b^4}{a^6\tan(dx+c)}+\frac{b}{2a^3\tan(dx+c)^4}+\frac{2b(a^2+b^2)}{a^5\tan(dx+c)^2}-\frac{2b(a^4+4a^2b^2+3b^4)\ln(\tan(dx+c))}{a^7}-\frac{(}{a^6}}{d}$
risch	$-\frac{4i(-45a^2b^3+4ia^5-45b^5-4a^4b+45iab^4e^{8i(dx+c)}-120ia^3b^2e^{6i(dx+c)}-180ia^3b^2e^{2i(dx+c)}-180iab^4e^{2i(dx+c)}-180ia}{}$

input
```
int(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/5/a^2/tan(d*x+c)^5-1/3*(2*a^2+3*b^2)/a^4/tan(d*x+c)^3-(a^4+6*a^2*b
^2+5*b^4)/a^6/tan(d*x+c)+1/2/a^3*b/tan(d*x+c)^4+2*b*(a^2+b^2)/a^5/tan(d*x+
c)^2-2*b*(a^4+4*a^2*b^2+3*b^4)/a^7*ln(tan(d*x+c))-(a^4+2*a^2*b^2+b^4)*b/a^
6/(a+b*tan(d*x+c))+2*b*(a^4+4*a^2*b^2+3*b^4)/a^7*ln(a+b*tan(d*x+c)))
```

3.66.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(213) = 426$.

Time = 0.32 (sec) , antiderivative size = 787, normalized size of antiderivative = 3.59

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{4\,(4\,a^6+45\,a^4b^2+45\,a^2b^4)\cos{(dx+c)}^6-75\,a^4b^2-90\,a^2b^4-10\,(4\,a^6+45\,a^4b^2+45\,a^2b^4)\cos{(dx+c)}^4}{}$$

input
```
integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/30*(4*(4*a^6 + 45*a^4*b^2 + 45*a^2*b^4)*cos(d*x + c)^6 - 75*a^4*b^2 - 90
*a^2*b^4 - 10*(4*a^6 + 45*a^4*b^2 + 45*a^2*b^4)*cos(d*x + c)^4 + 15*(2*a^6
 + 23*a^4*b^2 + 24*a^2*b^4)*cos(d*x + c)^2 + 30*((a^4*b^2 + 4*a^2*b^4 + 3*
b^6)*cos(d*x + c)^6 - a^4*b^2 - 4*a^2*b^4 - 3*b^6 - 3*(a^4*b^2 + 4*a^2*b^4
 + 3*b^6)*cos(d*x + c)^4 + 3*(a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^2
- ((a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^5 - 2*(a^5*b + 4*a^3*b^3 + 3
*a*b^5)*cos(d*x + c)^3 + (a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c))*sin(d
*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) - 30*((a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^6 - a^4*b^2 - 4*a^
2*b^4 - 3*b^6 - 3*(a^4*b^2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + 3*(a^4*b^
2 + 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 - ((a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos
(d*x + c)^5 - 2*(a^5*b + 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 + (a^5*b + 4*
a^3*b^3 + 3*a*b^5)*cos(d*x + c))*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1
/4) + (4*(4*a^5*b + 45*a^3*b^3 + 45*a*b^5)*cos(d*x + c)^5 - 10*(a^5*b + 33
*a^3*b^3 + 36*a*b^5)*cos(d*x + c)^3 - 15*(a^5*b - 10*a^3*b^3 - 12*a*b^5)*c
os(d*x + c))*sin(d*x + c))/(a^7*b*d*cos(d*x + c)^6 - 3*a^7*b*d*cos(d*x + c
)^4 + 3*a^7*b*d*cos(d*x + c)^2 - a^7*b*d - (a^8*d*cos(d*x + c)^5 - 2*a^8*d
*cos(d*x + c)^3 + a^8*d*cos(d*x + c))*sin(d*x + c))
```

3.66.6 Sympy [F]

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx=\int \frac{\csc^6{(c+dx)}}{(a+b\tan{(c+dx)})^2}\,dx$$

input
```
integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(csc(c + d*x)**6/(a + b*tan(c + d*x))**2, x)
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.03

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{9a^4b\tan(dx+c)-60(a^4b+4a^2b^3+3b^5)\tan(dx+c)^5-6a^5-30(a^5+4a^3b^2+3ab^4)\tan(dx+c)^4+10(4a^4b+3a^2b^3)\tan(dx+c)^3-5(4a^5+3a^3b^2)\,\mathrm{t}}{a^6b\tan(dx+c)^6+a^7\tan(dx+c)^5}}{30\,d}$$

input
```
integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/30*((9*a^4*b*tan(d*x + c) - 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*tan(d*x + c)^
5 - 6*a^5 - 30*(a^5 + 4*a^3*b^2 + 3*a*b^4)*tan(d*x + c)^4 + 10*(4*a^4*b +
3*a^2*b^3)*tan(d*x + c)^3 - 5*(4*a^5 + 3*a^3*b^2)*tan(d*x + c)^2)/(a^6*b*t
an(d*x + c)^6 + a^7*tan(d*x + c)^5) + 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*log(b
*tan(d*x + c) + a)/a^7 - 60*(a^4*b + 4*a^2*b^3 + 3*b^5)*log(tan(d*x + c))/
a^7)/d
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.52

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{60(a^4b+4a^2b^3+3b^5)\log(|\tan(dx+c)|)}{a^7}-\frac{60(a^4b^2+4a^2b^4+3b^6)\log(|b\tan(dx+c)+a|)}{a^7b}+\frac{30(2a^4b^2\tan(dx+c)+8a^2b^4\tan(dx+c)+6b^6\,\mathrm{ta}}{(b\tan(dx+c)+a)\mathrm{c}}}{}$$

input
```
integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/30*(60*(a^4*b + 4*a^2*b^3 + 3*b^5)*log(abs(tan(d*x + c)))/a^7 - 60*(a^4
*b^2 + 4*a^2*b^4 + 3*b^6)*log(abs(b*tan(d*x + c) + a))/(a^7*b) + 30*(2*a^4
*b^2*tan(d*x + c) + 8*a^2*b^4*tan(d*x + c) + 6*b^6*tan(d*x + c) + 3*a^5*b
+ 10*a^3*b^3 + 7*a*b^5)/((b*tan(d*x + c) + a)*a^7) - (137*a^4*b*tan(d*x +
c)^5 + 548*a^2*b^3*tan(d*x + c)^5 + 411*b^5*tan(d*x + c)^5 - 30*a^5*tan(d*
x + c)^4 - 180*a^3*b^2*tan(d*x + c)^4 - 150*a*b^4*tan(d*x + c)^4 + 60*a^4*
b*tan(d*x + c)^3 + 60*a^2*b^3*tan(d*x + c)^3 - 20*a^5*tan(d*x + c)^2 - 30*
a^3*b^2*tan(d*x + c)^2 + 15*a^4*b*tan(d*x + c) - 6*a^5)/(a^7*tan(d*x + c)^
5))/d
```

3.66.9 Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.08

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{4\,b\,\mathrm{atanh}\left(\frac{2\,b\,(a^2+3\,b^2)\,(a^2+b^2)\,(a+2\,b\tan(c+d\,x))}{a\,(2\,a^4\,b+8\,a^2\,b^3+6\,b^5)}\right)\,(a^2+3\,b^2)\,(a^2+b^2)}{a^7\,d} - \frac{\frac{1}{5\,a}+\frac{\tan(c+d\,x)^4\,(a^4+4\,a^2\,b^2+3\,b^4)}{a^5}+\frac{\tan(c+d\,x)^2\,(4\,a^2+3\,b^2)}{6\,a^3}-\frac{3\,b\tan(c+d\,x)}{10\,a^2}+\frac{2\,b\tan(c+d\,x)^5\,(a^4+4\,a^2\,b^2+3\,b^4)}{a^6}-\frac{b\tan(c+d\,x}{3}}{d\,\left(b\tan\left(c+d\,x\right)^6+a\tan\left(c+d\,x\right)^5\right)}$$

input
```
int(1/(sin(c + d*x)^6*(a + b*tan(c + d*x))^2),x)
```

output
```
(4*b*atanh((2*b*(a^2 + 3*b^2)*(a^2 + b^2)*(a + 2*b*tan(c + d*x)))/(a*(2*a^
4*b + 6*b^5 + 8*a^2*b^3)))*(a^2 + 3*b^2)*(a^2 + b^2))/(a^7*d) - (1/(5*a) +
 (tan(c + d*x)^4*(a^4 + 3*b^4 + 4*a^2*b^2))/a^5 + (tan(c + d*x)^2*(4*a^2 +
 3*b^2))/(6*a^3) - (3*b*tan(c + d*x))/(10*a^2) + (2*b*tan(c + d*x)^5*(a^4
+ 3*b^4 + 4*a^2*b^2))/a^6 - (b*tan(c + d*x)^3*(4*a^2 + 3*b^2))/(3*a^4))/(d
*(a*tan(c + d*x)^5 + b*tan(c + d*x)^6))
```

3.67 $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.67.1 Optimal result

Integrand size = 21, antiderivative size = 382

$$\begin{aligned}\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{a(5a^8-180a^6b^2+390a^4b^4-68a^2b^6-3b^8)\,x}{16\,(a^2+b^2)^6} \\ &\quad+\frac{a^4b(3a^4-22a^2b^2+15b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^6\,d} \\ &\quad-\frac{a^6b}{2\,(a^2+b^2)^4\,d(a+b\tan(c+dx))^2}-\frac{2a^5b(a^2-3b^2)}{(a^2+b^2)^5\,d(a+b\tan(c+dx))} \\ &\quad-\frac{\cos^6(c+dx)\,(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx))}{6\,(a^2+b^2)^3\,d} \\ &\quad+\frac{\cos^4(c+dx)\,(6b(9a^4-4a^2b^2-b^4)+a(13a^4-62a^2b^2-3b^4)\tan(c+dx))}{24\,(a^2+b^2)^4\,d} \\ &\quad-\frac{a\cos^2(c+dx)\,(24a^3b(3a^2-5b^2)+(11a^6-119a^4b^2+65a^2b^4+3b^6)\tan(c+dx))}{16\,(a^2+b^2)^5\,d}\end{aligned}$$

output
```
1/16*a*(5*a^8-180*a^6*b^2+390*a^4*b^4-68*a^2*b^6-3*b^8)*x/(a^2+b^2)^6+a^4*
b*(3*a^4-22*a^2*b^2+15*b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^6/d-1/
2*a^6*b/(a^2+b^2)^4/d/(a+b*tan(d*x+c))^2-2*a^5*b*(a^2-3*b^2)/(a^2+b^2)^5/d
/(a+b*tan(d*x+c))-1/6*cos(d*x+c)^6*(b*(3*a^2-b^2)+a*(a^2-3*b^2)*tan(d*x+c)
)/(a^2+b^2)^3/d+1/24*cos(d*x+c)^4*(6*b*(9*a^4-4*a^2*b^2-b^4)+a*(13*a^4-62*
a^2*b^2-3*b^4)*tan(d*x+c))/(a^2+b^2)^4/d-1/16*a*cos(d*x+c)^2*(24*a^3*b*(3*
a^2-5*b^2)+(11*a^6-119*a^4*b^2+65*a^2*b^4+3*b^6)*tan(d*x+c))/(a^2+b^2)^5/d
```

3.67.2 Mathematica [A] (verified)

Time = 6.72 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.95

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{b\left(-\frac{3a^5(a^2-7b^2)\arctan(\tan(c+dx))}{2b(a^2+b^2)^5} - \frac{5a(a^2-3b^2)\arctan(\tan(c+dx))}{16b(a^2+b^2)^3} + \frac{9a(a^4-4a^2b^2-b^4)\arctan(\tan(c+dx))}{8b(a^2+b^2)^4} - \frac{3a^4(3a^2-5b^2)\cos^2(c\cdots}{2(a^2+b^2)^5}\right.}{}$$

input
```
Integrate[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*((-3*a^5*(a^2 - 7*b^2)*ArcTan[Tan[c + d*x]])/(2*b*(a^2 + b^2)^5) - (5*a
*(a^2 - 3*b^2)*ArcTan[Tan[c + d*x]])/(16*b*(a^2 + b^2)^3) + (9*a*(a^4 - 4*
a^2*b^2 - b^4)*ArcTan[Tan[c + d*x]])/(8*b*(a^2 + b^2)^4) - (3*a^4*(3*a^2 -
 5*b^2)*Cos[c + d*x]^2)/(2*(a^2 + b^2)^5) + ((9*a^4 - 4*a^2*b^2 - b^4)*Cos
[c + d*x]^4)/(4*(a^2 + b^2)^4) - ((3*a^2 - b^2)*Cos[c + d*x]^6)/(6*(a^2 +
b^2)^3) - (a^4*(3*a^4 - 22*a^2*b^2 + 15*b^4 - (a^5 - 18*a^3*b^2 + 21*a*b^4
)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(2*(a^2 + b^2)^6) + (a^4*(
3*a^4 - 22*a^2*b^2 + 15*b^4)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^6 - (a^4
*(3*a^4 - 22*a^2*b^2 + 15*b^4 + (a^5 - 18*a^3*b^2 + 21*a*b^4)/Sqrt[-b^2])*
Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^6) - (3*a^5*(a^2 - 7*b^2)
*Cos[c + d*x]*Sin[c + d*x])/(2*b*(a^2 + b^2)^5) - (5*a*(a^2 - 3*b^2)*Cos[c
 + d*x]*Sin[c + d*x])/(16*b*(a^2 + b^2)^3) + (9*a*(a^4 - 4*a^2*b^2 - b^4)*
Cos[c + d*x]*Sin[c + d*x])/(8*b*(a^2 + b^2)^4) - (5*a*(a^2 - 3*b^2)*Cos[c
+ d*x]^3*Sin[c + d*x])/(24*b*(a^2 + b^2)^3) + (3*a*(a^4 - 4*a^2*b^2 - b^4)
*Cos[c + d*x]^3*Sin[c + d*x])/(4*b*(a^2 + b^2)^4) - (a*(a^2 - 3*b^2)*Cos[c
 + d*x]^5*Sin[c + d*x])/(6*b*(a^2 + b^2)^3) - a^6/(2*(a^2 + b^2)^4*(a + b*
Tan[c + d*x])^2) - (2*a^5*(a^2 - 3*b^2))/((a^2 + b^2)^5*(a + b*Tan[c + d*x
]))))/d
```

3.67.3 Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3999, 601, 25, 2178, 27, 2178, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \ \textcolor{blue}{3042}$$

$$\int \frac{\sin(c+dx)^6}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \ \textcolor{blue}{3999}$$

$$\frac{b\int \frac{b^6\tan^6(c+dx)}{(a+b\tan(c+dx))^3(\tan^2(c+dx)b^2+b^2)^4}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \ \textcolor{blue}{601}$$

$$\frac{b\left(-\frac{\int -\frac{-\frac{5a\left(a^2-3b^2\right)\tan^3(c+dx)b^9}{\left(a^2+b^2\right)^3}-\frac{3a^3\left(5a^2+b^2\right)\tan(c+dx)b^7}{\left(a^2+b^2\right)^3}+6\tan^4(c+dx)b^6-\frac{3a^2\left(2a^4+11b^2a^2-3b^4\right)\tan^2(c+dx)b^6}{\left(a^2+b^2\right)^3}+\frac{a^4\left(a^2-3b^2\right)b^6}{\left(a^2+b^2\right)^3}}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)^3}d(b\tan(c+dx)}{6b^2}\right.}{d}$$

$$\Big\downarrow \ \textcolor{blue}{25}$$

$$\frac{b\left(\frac{\int \frac{-\frac{5a\left(a^2-3b^2\right)\tan^3(c+dx)b^9}{\left(a^2+b^2\right)^3}-\frac{3a^3\left(5a^2+b^2\right)\tan(c+dx)b^7}{\left(a^2+b^2\right)^3}+6\tan^4(c+dx)b^6-\frac{3a^2\left(2a^4+11b^2a^2-3b^4\right)\tan^2(c+dx)b^6}{\left(a^2+b^2\right)^3}+\frac{a^4\left(a^2-3b^2\right)b^6}{\left(a^2+b^2\right)^3}}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)^3}d(b\tan(c+dx))}{6b^2}-\right.}{d}$$

$$\Big\downarrow \ \textcolor{blue}{2178}$$

$$\frac{b\left(\frac{\frac{b^4\left(ab\left(13a^4-62a^2b^2-3b^4\right)\tan(c+dx)+6b^2\left(9a^4-4a^2b^2-b^4\right)\right)}{4\left(a^2+b^2\right)^4\left(b^2\tan^2(c+dx)+b^2\right)^2}-\frac{3\int \frac{\left(-\frac{a\left(13a^4-62b^2a^2-3b^4\right)\tan^3(c+dx)b^9}{\left(a^2+b^2\right)^4}-\frac{3a^3\left(13a^4+2b^2a^2-3b^4\right)\tan(c+dx)b^7}{\left(a^2+b^2\right)^4}-\frac{a^2\left(8a\right.}{}\right.}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b\right.}}{4b^2}}{6b^2}\right.}{d}$$

$$\Big\downarrow \ \textcolor{blue}{27}$$

3.67. $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

$$\frac{b\left(\frac{\frac{b^4\left(ab\left(13a^4-62a^2b^2-3b^4\right)\tan(c+dx)+6b^2\left(9a^4-4a^2b^2-b^4\right)\right)}{4\left(a^2+b^2\right)^4\left(b^2\tan^2(c+dx)+b^2\right)^2}-\frac{3\int\frac{-\frac{a\left(13a^4-62b^2a^2-3b^4\right)\tan^3(c+dx)b^9}{\left(a^2+b^2\right)^4}-\frac{3a^3\left(13a^4+2b^2a^2-3b^4\right)\tan(c+dx)b^7}{\left(a^2+b^2\right)^4}-\frac{a^2\left(8a^6\cdots\right.}{\cdots}}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^{2}\cdots\right.}\cdots}{4b^2}\cdots}{6b^2}\cdots\right.}{d}$$

$\downarrow$ 2178

$$b\left(\frac{\frac{b^4\left(ab\left(13a^4-62a^2b^2-3b^4\right)\tan(c+dx)+6b^2\left(9a^4-4a^2b^2-b^4\right)\right)}{4\left(a^2+b^2\right)^4\left(b^2\tan^2(c+dx)+b^2\right)^2}-\frac{3\left(\frac{ab^4\left(24a^3b^2\left(3a^2-5b^2\right)+b\left(11a^6-119a^4b^2+65a^2b^4+3b^6\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^5\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{\int\frac{-\frac{a\left(11a^6-119\cdots\right.}{\cdots}}{\cdots}}{\cdots}\cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 2160

$$b\left(\frac{\frac{b^4\left(ab\left(13a^4-62a^2b^2-3b^4\right)\tan(c+dx)+6b^2\left(9a^4-4a^2b^2-b^4\right)\right)}{4\left(a^2+b^2\right)^4\left(b^2\tan^2(c+dx)+b^2\right)^2}-\frac{3\left(\frac{ab^4\left(24a^3b^2\left(3a^2-5b^2\right)+b\left(11a^6-119a^4b^2+65a^2b^4+3b^6\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^5\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{\int\left(\frac{16a^4\left(3a^4-22\cdots\right.}{\left(a^2+b^2\right)^6(a\cdots}\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 2009

3.67. $\int\frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

$$b\left(\frac{\frac{b^4\left(ab\left(13a^4-62a^2b^2-3b^4\right)\tan(c+dx)+6b^2\left(9a^4-4a^2b^2-b^4\right)\right)}{4\left(a^2+b^2\right)^4\left(b^2\tan^2(c+dx)+b^2\right)^2}-\frac{3\left(\frac{ab^4\left(24a^3b^2\left(3a^2-5b^2\right)+b\left(11a^6-119a^4b^2+65a^2b^4+3b^6\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^5\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{-\frac{8a^6}{\left(a^2+b^2\right)^4(a+b}}{}\right.}{}}{}\right.$$

input
```
Int[Sin[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*(-1/6*(b^4*(b^2*(3*a^2 - b^2) + a*b*(a^2 - 3*b^2)*Tan[c + d*x]))/((a^2
+ b^2)^3*(b^2 + b^2*Tan[c + d*x]^2)^3) + ((b^4*(6*b^2*(9*a^4 - 4*a^2*b^2 -
 b^4) + a*b*(13*a^4 - 62*a^2*b^2 - 3*b^4)*Tan[c + d*x]))/(4*(a^2 + b^2)^4*
(b^2 + b^2*Tan[c + d*x]^2)^2) - (3*((a*b^4*(24*a^3*b^2*(3*a^2 - 5*b^2) + b
*(11*a^6 - 119*a^4*b^2 + 65*a^2*b^4 + 3*b^6)*Tan[c + d*x]))/(2*(a^2 + b^2)
^5*(b^2 + b^2*Tan[c + d*x]^2)) - ((a*b^5*(5*a^8 - 180*a^6*b^2 + 390*a^4*b^
4 - 68*a^2*b^6 - 3*b^8)*ArcTan[Tan[c + d*x]])/(a^2 + b^2)^6 + (16*a^4*b^6*
(3*a^4 - 22*a^2*b^2 + 15*b^4)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^6 - (8*
a^4*b^6*(3*a^4 - 22*a^2*b^2 + 15*b^4)*Log[b^2 + b^2*Tan[c + d*x]^2])/(a^2
+ b^2)^6 - (8*a^6*b^6)/((a^2 + b^2)^4*(a + b*Tan[c + d*x])^2) - (32*a^5*b^
6*(a^2 - 3*b^2))/((a^2 + b^2)^5*(a + b*Tan[c + d*x])))/(2*b^2)))/(4*b^2))/
(6*b^2)))/d
```

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.67. $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.67.4 Maple [A] (verified)

Time = 70.94 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{-\frac{b\,a^{6}}{2\left(a^{2}+b^{2}\right)^{4}\left(a+b\tan\left(dx+c\right)\right)^{2}}+\frac{a^{4}b\left(3a^{4}-22a^{2}b^{2}+15b^{4}\right)\ln\left(a+b\tan\left(dx+c\right)\right)}{\left(a^{2}+b^{2}\right)^{6}}-\frac{2b\,a^{5}\left(a^{2}-3b^{2}\right)}{\left(a^{2}+b^{2}\right)^{5}\left(a+b\tan\left(dx+c\right)\right)}+\frac{\left(-\frac{11}{16}a^{9}+\frac{27}{4}b^{2}a^{7}+\frac{2}{8}\right.}{}}{}$
default	$\frac{-\frac{b\,a^{6}}{2\left(a^{2}+b^{2}\right)^{4}\left(a+b\tan\left(dx+c\right)\right)^{2}}+\frac{a^{4}b\left(3a^{4}-22a^{2}b^{2}+15b^{4}\right)\ln\left(a+b\tan\left(dx+c\right)\right)}{\left(a^{2}+b^{2}\right)^{6}}-\frac{2b\,a^{5}\left(a^{2}-3b^{2}\right)}{\left(a^{2}+b^{2}\right)^{5}\left(a+b\tan\left(dx+c\right)\right)}+\frac{\left(-\frac{11}{16}a^{9}+\frac{27}{4}b^{2}a^{7}+\frac{2}{8}\right.}{}}{}$
risch	Expression too large to display

input
```
int(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*b*a^6/(a^2+b^2)^4/(a+b*tan(d*x+c))^2+a^4*b*(3*a^4-22*a^2*b^2+15*
b^4)/(a^2+b^2)^6*ln(a+b*tan(d*x+c))-2*b*a^5*(a^2-3*b^2)/(a^2+b^2)^5/(a+b*t
an(d*x+c))+1/(a^2+b^2)^6*(((-11/16*a^9+27/4*b^2*a^7+27/8*b^4*a^5-17/4*b^6*
a^3-3/16*a*b^8)*tan(d*x+c)^5+(-9/2*a^8*b+3*a^6*b^3+15/2*a^4*b^5)*tan(d*x+c
)^4+(-5/6*a^9+12*b^2*a^7+2*b^4*a^5-34/3*b^6*a^3-1/2*a*b^8)*tan(d*x+c)^3+(-
27/4*a^8*b+19/2*a^6*b^3+15*a^4*b^5-3/2*a^2*b^7-1/4*b^9)*tan(d*x+c)^2+(-5/1
6*a^9+21/4*b^2*a^7-3/8*b^4*a^5-23/4*b^6*a^3+3/16*a*b^8)*tan(d*x+c)-11/4*a^
8*b+31/6*a^6*b^3+13/2*a^4*b^5-3/2*a^2*b^7-1/12*b^9)/(1+tan(d*x+c)^2)^3+1/1
6*a*(1/2*(-48*a^7*b+352*a^5*b^3-240*a^3*b^5)*ln(1+tan(d*x+c)^2)+(5*a^8-180
*a^6*b^2+390*a^4*b^4-68*a^2*b^6-3*b^8)*arctan(tan(d*x+c)))))
```

3.67.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. 2(372) = 744.

Time = 0.38 (sec) , antiderivative size = 932, normalized size of antiderivative = 2.44

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{195\,a^8b^3-427\,a^6b^5-165\,a^4b^7+27\,a^2b^9+2\,b^{11}-8\left(a^{10}b+5\,a^8b^3+10\,a^6b^5+10\,a^4b^7+5\,a^2b^9+b^{11}\right)\cos(}{}$$

input
```
integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/48*(195*a^8*b^3 - 427*a^6*b^5 - 165*a^4*b^7 + 27*a^2*b^9 + 2*b^11 - 8*(a
^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)*cos(d*x +
c)^8 + 20*(2*a^10*b + 9*a^8*b^3 + 16*a^6*b^5 + 14*a^4*b^7 + 6*a^2*b^9 + b^
11)*cos(d*x + c)^6 - 2*(49*a^10*b + 162*a^8*b^3 + 198*a^6*b^5 + 112*a^4*b^
7 + 33*a^2*b^9 + 6*b^11)*cos(d*x + c)^4 + 3*(5*a^9*b^2 - 180*a^7*b^4 + 390
*a^5*b^6 - 68*a^3*b^8 - 3*a*b^10)*d*x) + (9*a^10*b - 46*a^8*b^3 + 994*a^6*b
^5 + 144*a^4*b^7 - 43*a^2*b^9 - 2*b^11 + 3*(5*a^11 - 185*a^9*b^2 + 570*a^7
*b^4 - 458*a^5*b^6 + 65*a^3*b^8 + 3*a*b^10)*d*x)*cos(d*x + c)^2 + 24*(3*a^
8*b^3 - 22*a^6*b^5 + 15*a^4*b^7 + (3*a^10*b - 25*a^8*b^3 + 37*a^6*b^5 - 15
*a^4*b^7)*cos(d*x + c)^2 + 2*(3*a^9*b^2 - 22*a^7*b^4 + 15*a^5*b^6)*cos(d*x
 + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(
d*x + c)^2 + b^2) - (8*(a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*b^6 + 5*a^3
*b^8 + a*b^10)*cos(d*x + c)^7 - 2*(13*a^11 + 55*a^9*b^2 + 90*a^7*b^4 + 70*
a^5*b^6 + 25*a^3*b^8 + 3*a*b^10)*cos(d*x + c)^5 + (33*a^11 + 49*a^9*b^2 -
54*a^7*b^4 - 126*a^5*b^6 - 59*a^3*b^8 - 3*a*b^10)*cos(d*x + c)^3 - (261*a^
9*b^2 - 338*a^7*b^4 + 120*a^5*b^6 - 150*a^3*b^8 - 5*a*b^10 + 6*(5*a^10*b -
 180*a^8*b^3 + 390*a^6*b^5 - 68*a^4*b^7 - 3*a^2*b^9)*d*x)*cos(d*x + c))*si
n(d*x + c))/((a^14 + 5*a^12*b^2 + 9*a^10*b^4 + 5*a^8*b^6 - 5*a^6*b^8 - 9*a
^4*b^10 - 5*a^2*b^12 - b^14)*d*cos(d*x + c)^2 + 2*(a^13*b + 6*a^11*b^3 + 1
5*a^9*b^5 + 20*a^7*b^7 + 15*a^5*b^9 + 6*a^3*b^11 + a*b^13)*d*cos(d*x + ...
```

3.67.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(sin(d*x+c)**6/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.67.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. $2(372) = 744$.

Time = 0.34 (sec) , antiderivative size = 1088, normalized size of antiderivative = 2.85

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/48*(3*(5*a^9 - 180*a^7*b^2 + 390*a^5*b^4 - 68*a^3*b^6 - 3*a*b^8)*(d*x +
c)/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10
+ b^12) + 48*(3*a^8*b - 22*a^6*b^3 + 15*a^4*b^5)*log(b*tan(d*x + c) + a)/(
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^
12) - 24*(3*a^8*b - 22*a^6*b^3 + 15*a^4*b^5)*log(tan(d*x + c)^2 + 1)/(a^12
 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)
- (252*a^8*b - 644*a^6*b^3 + 68*a^4*b^5 + 4*a^2*b^7 + 3*(43*a^7*b^2 - 215*
a^5*b^4 + 65*a^3*b^6 + 3*a*b^8)*tan(d*x + c)^7 + 6*(31*a^8*b - 127*a^6*b^3
 + 5*a^4*b^5 + 3*a^2*b^7)*tan(d*x + c)^6 + (33*a^9 + 403*a^7*b^2 - 2005*a^
5*b^4 + 529*a^3*b^6 + 24*a*b^8)*tan(d*x + c)^5 + 4*(164*a^8*b - 515*a^6*b^
3 + 65*a^4*b^5 + 27*a^2*b^7 + 3*b^9)*tan(d*x + c)^4 + (40*a^9 + 335*a^7*b^
2 - 2171*a^5*b^4 + 429*a^3*b^6 + 15*a*b^8)*tan(d*x + c)^3 + 2*(357*a^8*b -
 987*a^6*b^3 + 125*a^4*b^5 + 31*a^2*b^7 + 2*b^9)*tan(d*x + c)^2 + (15*a^9
+ 93*a^7*b^2 - 763*a^5*b^4 + 127*a^3*b^6 + 8*a*b^8)*tan(d*x + c))/(a^12 +
5*a^10*b^2 + 10*a^8*b^4 + 10*a^6*b^6 + 5*a^4*b^8 + a^2*b^10 + (a^10*b^2 +
5*a^8*b^4 + 10*a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^10 + b^12)*tan(d*x + c)^8 +
2*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 + a*b^11)*tan(
d*x + c)^7 + (a^12 + 8*a^10*b^2 + 25*a^8*b^4 + 40*a^6*b^6 + 35*a^4*b^8 + 1
6*a^2*b^10 + 3*b^12)*tan(d*x + c)^6 + 6*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 +
 10*a^5*b^7 + 5*a^3*b^9 + a*b^11)*tan(d*x + c)^5 + 3*(a^12 + 6*a^10*b^2...
```

3.67.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(372) = 744$.

Time = 0.69 (sec) , antiderivative size = 923, normalized size of antiderivative = 2.42

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{3\,(5\,a^9-180\,a^7b^2+390\,a^5b^4-68\,a^3b^6-3\,ab^8)(dx+c)}{a^{12}+6\,a^{10}b^2+15\,a^8b^4+20\,a^6b^6+15\,a^4b^8+6\,a^2b^{10}+b^{12}} - \frac{24\,(3\,a^8b-22\,a^6b^3+15\,a^4b^5)\log\left(\tan(dx+c)^2+1\right)}{a^{12}+6\,a^{10}b^2+15\,a^8b^4+20\,a^6b^6+15\,a^4b^8+6\,a^2b^{10}+b^{12}} + \frac{48\,(3\,a^8b^2-22\,a^6b^4+15\,a^4 \cdots}{a^{12}b+6\,a^{10}b^3+15\,a^8b^5+20 \cdots}}{}$$

input
```
integrate(sin(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/48*(3*(5*a^9 - 180*a^7*b^2 + 390*a^5*b^4 - 68*a^3*b^6 - 3*a*b^8)*(d*x +
c)/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10
+ b^12) - 24*(3*a^8*b - 22*a^6*b^3 + 15*a^4*b^5)*log(tan(d*x + c)^2 + 1)/(
a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^
12) + 48*(3*a^8*b^2 - 22*a^6*b^4 + 15*a^4*b^6)*log(abs(b*tan(d*x + c) + a)
)/(a^12*b + 6*a^10*b^3 + 15*a^8*b^5 + 20*a^6*b^7 + 15*a^4*b^9 + 6*a^2*b^11
 + b^13) - 24*(9*a^8*b^3*tan(d*x + c)^2 - 66*a^6*b^5*tan(d*x + c)^2 + 45*a
^4*b^7*tan(d*x + c)^2 + 22*a^9*b^2*tan(d*x + c) - 140*a^7*b^4*tan(d*x + c)
 + 78*a^5*b^6*tan(d*x + c) + 14*a^10*b - 72*a^8*b^3 + 34*a^6*b^5)/((a^12 +
 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*(b
*tan(d*x + c) + a)^2) + (132*a^8*b*tan(d*x + c)^6 - 968*a^6*b^3*tan(d*x +
c)^6 + 660*a^4*b^5*tan(d*x + c)^6 - 33*a^9*tan(d*x + c)^5 + 324*a^7*b^2*ta
n(d*x + c)^5 + 162*a^5*b^4*tan(d*x + c)^5 - 204*a^3*b^6*tan(d*x + c)^5 - 9
*a*b^8*tan(d*x + c)^5 + 180*a^8*b*tan(d*x + c)^4 - 2760*a^6*b^3*tan(d*x +
c)^4 + 2340*a^4*b^5*tan(d*x + c)^4 - 40*a^9*tan(d*x + c)^3 + 576*a^7*b^2*t
an(d*x + c)^3 + 96*a^5*b^4*tan(d*x + c)^3 - 544*a^3*b^6*tan(d*x + c)^3 - 2
4*a*b^8*tan(d*x + c)^3 + 72*a^8*b*tan(d*x + c)^2 - 2448*a^6*b^3*tan(d*x +
c)^2 + 2700*a^4*b^5*tan(d*x + c)^2 - 72*a^2*b^7*tan(d*x + c)^2 - 12*b^9*ta
n(d*x + c)^2 - 15*a^9*tan(d*x + c) + 252*a^7*b^2*tan(d*x + c) - 18*a^5*b^4
*tan(d*x + c) - 276*a^3*b^6*tan(d*x + c) + 9*a*b^8*tan(d*x + c) - 720*a...
```

3.67.9 Mupad [B] (verification not implemented)

Time = 6.86 (sec) , antiderivative size = 1068, normalized size of antiderivative = 2.80

$$\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{3\,b}{{\left(a^2+b^2\right)}^2}-\frac{34\,b^3}{{\left(a^2+b^2\right)}^3}+\frac{99\,b^5}{{\left(a^2+b^2\right)}^4}-\frac{108\,b^7}{{\left(a^2+b^2\right)}^5}+\frac{40\,b^9}{{\left(a^2+b^2\right)}^6}\right)}{d}$$
$$-\frac{\frac{{\tan(c+d\,x)}^6\,\left(31\,a^8\,b-127\,a^6\,b^3+5\,a^4\,b^5+3\,a^2\,b^7\right)}{8\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10}\right)}+\frac{{\tan(c+d\,x)}^7\,\left(43\,a^7\,b^2-215\,a^5\,b^4+65\,a^3\,b^6+3\,a\,b^8\right)}{16\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+10\,a^4\,b^6+5\,a^2\,b^8+b^{10}\right)}+\frac{{\tan(c+d\,x)}^5\,\left(33\,a^9+403\,a^7\,b^2-20\ldots\right.}{48\,\left(a^{10}+5\,a^8\,b^2+10\,a^6\,b^4+\ldots\right.}}{d\,\left({\tan\left(c+d\,x\right)}^2\,\left(3\,a^2+b^2\right)\ldots\right.}$$
$$-\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(a^3\,5{}\mathrm{i}-18\,a^2\,b+a\,b^2\,3{}\mathrm{i}\right)}{32\,d\,\left(-a^6+a^5\,b\,6{}\mathrm{i}+15\,a^4\,b^2-a^3\,b^3\,20{}\mathrm{i}-15\,a^2\,b^4+a\,b^5\,6{}\mathrm{i}+b^6\right)}$$
$$-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(5\,a^3-a^2\,b\,18{}\mathrm{i}+3\,a\,b^2\right)}{32\,d\,\left(-a^6\,1{}\mathrm{i}+6\,a^5\,b+a^4\,b^2\,15{}\mathrm{i}-20\,a^3\,b^3-a^2\,b^4\,15{}\mathrm{i}+6\,a\,b^5+b^6\,1{}\mathrm{i}\right)}$$

input
```
int(sin(c + d*x)^6/(a + b*tan(c + d*x))^3,x)
```

output
```
(log(a + b*tan(c + d*x))*((3*b)/(a^2 + b^2)^2 - (34*b^3)/(a^2 + b^2)^3 + (
99*b^5)/(a^2 + b^2)^4 - (108*b^7)/(a^2 + b^2)^5 + (40*b^9)/(a^2 + b^2)^6))
/d - ((tan(c + d*x)^6*(31*a^8*b + 3*a^2*b^7 + 5*a^4*b^5 - 127*a^6*b^3))/(8
*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (tan(c
 + d*x)^7*(3*a*b^8 + 65*a^3*b^6 - 215*a^5*b^4 + 43*a^7*b^2))/(16*(a^10 + b
^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (tan(c + d*x)^5*
(24*a*b^8 + 33*a^9 + 529*a^3*b^6 - 2005*a^5*b^4 + 403*a^7*b^2))/(48*(a^10
+ b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (tan(c + d*x)
^4*(164*a^8*b + 3*b^9 + 27*a^2*b^7 + 65*a^4*b^5 - 515*a^6*b^3))/(12*(a^10
+ b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (a^2*(63*a^6*
b + b^7 + 17*a^2*b^5 - 161*a^4*b^3))/(12*(a^2 + b^2)*(a^8 + b^8 + 4*a^2*b^
6 + 6*a^4*b^4 + 4*a^6*b^2)) + (tan(c + d*x)^3*(15*a*b^8 + 40*a^9 + 429*a^3
*b^6 - 2171*a^5*b^4 + 335*a^7*b^2))/(48*(a^2 + b^2)*(a^8 + b^8 + 4*a^2*b^6
 + 6*a^4*b^4 + 4*a^6*b^2)) + (tan(c + d*x)^2*(357*a^8*b + 2*b^9 + 31*a^2*b
^7 + 125*a^4*b^5 - 987*a^6*b^3))/(24*(a^2 + b^2)*(a^8 + b^8 + 4*a^2*b^6 +
6*a^4*b^4 + 4*a^6*b^2)) + (a*tan(c + d*x)*(15*a^8 + 8*b^8 + 127*a^2*b^6 -
763*a^4*b^4 + 93*a^6*b^2))/(48*(a^2 + b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*
b^4 + 4*a^6*b^2)))/(d*(tan(c + d*x)^2*(3*a^2 + b^2) + tan(c + d*x)^6*(a^2
+ 3*b^2) + a^2 + tan(c + d*x)^4*(3*a^2 + 3*b^2) + b^2*tan(c + d*x)^8 + 2*a
*b*tan(c + d*x) + 6*a*b*tan(c + d*x)^3 + 6*a*b*tan(c + d*x)^5 + 2*a*b*t...
```

3.67. $\int \frac{\sin^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.68 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.68.1 Optimal result

Integrand size = 21, antiderivative size = 285

$$\begin{aligned}\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{3a(a^6-25a^4b^2+35a^2b^4-3b^6)\,x}{8\,(a^2+b^2)^5} \\ &+ \frac{3a^2b(a^4-5a^2b^2+2b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^5\,d} \\ &- \frac{a^4b}{2\,(a^2+b^2)^3\,d(a+b\tan(c+dx))^2} - \frac{2a^3b(a^2-2b^2)}{(a^2+b^2)^4\,d(a+b\tan(c+dx))} \\ &+ \frac{\cos^4(c+dx)\,(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx))}{4\,(a^2+b^2)^3\,d} \\ &- \frac{a\cos^2(c+dx)\,(24ab(a^2-b^2)+(5a^4-34a^2b^2+9b^4)\tan(c+dx))}{8\,(a^2+b^2)^4\,d}\end{aligned}$$

output
```
3/8*a*(a^6-25*a^4*b^2+35*a^2*b^4-3*b^6)*x/(a^2+b^2)^5+3*a^2*b*(a^4-5*a^2*b
^2+2*b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^5/d-1/2*a^4*b/(a^2+b^2)^
3/d/(a+b*tan(d*x+c))^2-2*a^3*b*(a^2-2*b^2)/(a^2+b^2)^4/d/(a+b*tan(d*x+c))+
1/4*cos(d*x+c)^4*(b*(3*a^2-b^2)+a*(a^2-3*b^2)*tan(d*x+c))/(a^2+b^2)^3/d-1/
8*a*cos(d*x+c)^2*(24*a*b*(a^2-b^2)+(5*a^4-34*a^2*b^2+9*b^4)*tan(d*x+c))/(a
^2+b^2)^4/d
```

3.68.2 Mathematica [A] (verified)

Time = 6.50 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.83

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{b\left(-\frac{a^3(a^2-5b^2)\arctan(\tan(c+dx))}{b(a^2+b^2)^4}+\frac{3a(a^2-3b^2)\arctan(\tan(c+dx))}{8b(a^2+b^2)^3}-\frac{3a^2(a-b)(a+b)\cos^2(c+dx)}{(a^2+b^2)^4}+\frac{(3a^2-b^2)\cos^4(c+dx)}{4(a^2+b^2)^3}-\frac{a^2\left(3a\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*(-((a^3*(a^2 - 5*b^2)*ArcTan[Tan[c + d*x]])/(b*(a^2 + b^2)^4)) + (3*a*(
a^2 - 3*b^2)*ArcTan[Tan[c + d*x]])/(8*b*(a^2 + b^2)^3) - (3*a^2*(a - b)*(a
 + b)*Cos[c + d*x]^2)/(a^2 + b^2)^4 + ((3*a^2 - b^2)*Cos[c + d*x]^4)/(4*(a
^2 + b^2)^3) - (a^2*(3*a^4 - 15*a^2*b^2 + 6*b^4 - (a^5 - 13*a^3*b^2 + 10*a
*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(2*(a^2 + b^2)^5) + (3
*a^2*(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^5 - (a
^2*(3*a^4 - 15*a^2*b^2 + 6*b^4 + (a^5 - 13*a^3*b^2 + 10*a*b^4)/Sqrt[-b^2])
*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^5) - (a^3*(a^2 - 5*b^2)*
Cos[c + d*x]*Sin[c + d*x])/(b*(a^2 + b^2)^4) + (3*a*(a^2 - 3*b^2)*Cos[c +
d*x]*Sin[c + d*x])/(8*b*(a^2 + b^2)^3) + (a*(a^2 - 3*b^2)*Cos[c + d*x]^3*S
in[c + d*x])/(4*b*(a^2 + b^2)^3) - a^4/(2*(a^2 + b^2)^3*(a + b*Tan[c + d*x
])^2) - (2*a^3*(a^2 - 2*b^2))/((a^2 + b^2)^4*(a + b*Tan[c + d*x]))))/d
```

3.68.3 Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3999, 601, 2178, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin(c+dx)^4}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \; 3999$$

$$\frac{b\int \frac{b^4\tan^4(c+dx)}{(a+b\tan(c+dx))^3(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 601$$

$$\frac{b\left(\frac{b^2(ab(a^2-3b^2)\tan(c+dx)+b^2(3a^2-b^2))}{4(a^2+b^2)^3(b^2\tan^2(c+dx)+b^2)^2}-\frac{\int \frac{-\frac{3a\left(a^2-3b^2\right)\tan^3(c+dx)b^7}{\left(a^2+b^2\right)^3}-\frac{a^3\left(9a^2+5b^2\right)\tan(c+dx)b^5}{\left(a^2+b^2\right)^3}-\frac{a^2\left(4a^4+21b^2a^2-3b^4\right)\tan^2(c+dx)b^4}{\left(a^2+b^2\right)^3}+\frac{a^4(\cdots}{(\cdots}}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)^2}}{4b^2}\cdots}{d}$$

$$\Big\downarrow \; 2178$$

$$\frac{b\left(\frac{b^2(ab(a^2-3b^2)\tan(c+dx)+b^2(3a^2-b^2))}{4(a^2+b^2)^3(b^2\tan^2(c+dx)+b^2)^2}-\frac{\frac{ab^2\left(24ab^2\left(a^2-b^2\right)+b\left(5a^4-34a^2b^2+9b^4\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^4\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{\int -\frac{a\left(5a^4-34b^2a^2+9b^4\right)\tan^3(c+dx)b^7}{\left(a^2+b^2\right)^4}-\frac{3a^2(5a\cdots}{\cdots}}{\cdots}}{\cdots}\cdots}{d}$$

$$\Big\downarrow \; 2160$$

$$\frac{b\left(\frac{b^2(ab(a^2-3b^2)\tan(c+dx)+b^2(3a^2-b^2))}{4(a^2+b^2)^3(b^2\tan^2(c+dx)+b^2)^2}-\frac{\frac{ab^2\left(24ab^2\left(a^2-b^2\right)+b\left(5a^4-34a^2b^2+9b^4\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^4\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{\int\left(\frac{24a^2\left(a^4-5b^2a^2+2b^4\right)b^4}{\left(a^2+b^2\right)^5(a+b\tan(c+dx))}+\frac{3a\left(a^6-25b^2a^4+\cdots\right.}{\cdots}\cdots\right)}{\cdots}}{\cdots}\cdots}{d}$$

$$\Big\downarrow \; 2009$$

$$b\left(\frac{b^2(ab(a^2-3b^2)\tan(c+dx)+b^2(3a^2-b^2))}{4(a^2+b^2)^3(b^2\tan^2(c+dx)+b^2)^2}-\frac{\frac{ab^2\left(24ab^2\left(a^2-b^2\right)+b\left(5a^4-34a^2b^2+9b^4\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^4\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{-\frac{4a^4b^4}{\left(a^2+b^2\right)^3(a+b\tan(c+dx))^2}-\frac{12a^2b^4\left(a^4-5a^2\cdots\right.}{\cdots}\cdots}{\cdots}}{\cdots}\cdots\right)$$

3.68. $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

input
```
Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*((b^2*(b^2*(3*a^2 - b^2) + a*b*(a^2 - 3*b^2)*Tan[c + d*x]))/(4*(a^2 + b
^2)^3*(b^2 + b^2*Tan[c + d*x]^2)^2) - ((a*b^2*(24*a*b^2*(a^2 - b^2) + b*(5
*a^4 - 34*a^2*b^2 + 9*b^4)*Tan[c + d*x]))/(2*(a^2 + b^2)^4*(b^2 + b^2*Tan[
c + d*x]^2)) - ((3*a*b^3*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*ArcTan[Ta
n[c + d*x]])/(a^2 + b^2)^5 + (24*a^2*b^4*(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a +
 b*Tan[c + d*x]])/(a^2 + b^2)^5 - (12*a^2*b^4*(a^4 - 5*a^2*b^2 + 2*b^4)*Lo
g[b^2 + b^2*Tan[c + d*x]^2])/(a^2 + b^2)^5 - (4*a^4*b^4)/((a^2 + b^2)^3*(a
 + b*Tan[c + d*x])^2) - (16*a^3*b^4*(a^2 - 2*b^2))/((a^2 + b^2)^4*(a + b*T
an[c + d*x])))/(2*b^2))/(4*b^2)))/d
```

3.68.3.1 Defintions of rubi rules used

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.68.4 Maple [A] (verified)

Time = 20.58 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\frac{\left(-\frac{5}{8}a^7+\frac{29}{8}a^5b^2+\frac{25}{8}a^3b^4-\frac{9}{8}ab^6\right)\left(\tan^3(dx+c)\right)+\left(-3a^6b+3b^5a^2\right)\left(\tan^2(dx+c)\right)+\left(-\frac{3}{8}a^7+\frac{27}{8}a^5b^2+\frac{15}{8}a^3b^4-\frac{15}{8}ab^6\right)\tan(dx+c)}{\left(1+\tan^2(dx+c)\right)^2}}{\left(a^2+\right.}$
default	$\frac{\frac{\left(-\frac{5}{8}a^7+\frac{29}{8}a^5b^2+\frac{25}{8}a^3b^4-\frac{9}{8}ab^6\right)\left(\tan^3(dx+c)\right)+\left(-3a^6b+3b^5a^2\right)\left(\tan^2(dx+c)\right)+\left(-\frac{3}{8}a^7+\frac{27}{8}a^5b^2+\frac{15}{8}a^3b^4-\frac{15}{8}ab^6\right)\tan(dx+c)}{\left(1+\tan^2(dx+c)\right)^2}}{\left(a^2+\right.}$
risch	$-\frac{9iaxb}{8(5ia^4b-10ia^2b^3+ib^5-a^5+10a^3b^2-5a\,b^4)}-\frac{3a^2x}{8(5ia^4b-10ia^2b^3+ib^5-a^5+10a^3b^2-5a\,b^4)}-\frac{12ia^2}{a^{10}+5a^8b^2+10a^6b^4+}$

input
```
int(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^5*(((-5/8*a^7+29/8*a^5*b^2+25/8*a^3*b^4-9/8*a*b^6)*tan(d*
x+c)^3+(-3*a^6*b+3*a^2*b^5)*tan(d*x+c)^2+(-3/8*a^7+27/8*a^5*b^2+15/8*a^3*b
^4-15/8*a*b^6)*tan(d*x+c)-9/4*a^6*b+5/4*a^4*b^3+13/4*b^5*a^2-1/4*b^7)/(1+t
an(d*x+c)^2)^2+3/8*a*(1/2*(-8*a^5*b+40*a^3*b^3-16*a*b^5)*ln(1+tan(d*x+c)^2
)+(a^6-25*a^4*b^2+35*a^2*b^4-3*b^6)*arctan(tan(d*x+c))))-1/2*b/(a^2+b^2)^3
*a^4/(a+b*tan(d*x+c))^2+3*a^2*b*(a^4-5*a^2*b^2+2*b^4)/(a^2+b^2)^5*ln(a+b*t
an(d*x+c))-2*a^3*b*(a^2-2*b^2)/(a^2+b^2)^4/(a+b*tan(d*x+c)))
```

3.68.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. 2(277) = 554.

Time = 0.34 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.47

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{119\,a^6b^3 - 159\,a^4b^5 - 51\,a^2b^7 + 3\,b^9 + 8\,(a^8b + 4\,a^6b^3 + 6\,a^4b^5 + 4\,a^2b^7 + b^9)\cos{(dx+c)}^6 - 8\,(5\,a^8b + 16}{}$$

input
```
integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/32*(119*a^6*b^3 - 159*a^4*b^5 - 51*a^2*b^7 + 3*b^9 + 8*(a^8*b + 4*a^6*b^
3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^6 - 8*(5*a^8*b + 16*a^6*b^3
+ 18*a^4*b^5 + 8*a^2*b^7 + b^9)*cos(d*x + c)^4 + 12*(a^7*b^2 - 25*a^5*b^4
+ 35*a^3*b^6 - 3*a*b^8)*d*x - (a^8*b + 110*a^6*b^3 - 420*a^4*b^5 - 78*a^2*
b^7 + 3*b^9 - 12*(a^9 - 26*a^7*b^2 + 60*a^5*b^4 - 38*a^3*b^6 + 3*a*b^8)*d*
x)*cos(d*x + c)^2 + 48*(a^6*b^3 - 5*a^4*b^5 + 2*a^2*b^7 + (a^8*b - 6*a^6*b
^3 + 7*a^4*b^5 - 2*a^2*b^7)*cos(d*x + c)^2 + 2*(a^7*b^2 - 5*a^5*b^4 + 2*a^
3*b^6)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a
^2 - b^2)*cos(d*x + c)^2 + b^2) + 2*(4*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^
3*b^6 + a*b^8)*cos(d*x + c)^5 - 2*(5*a^9 + 12*a^7*b^2 + 6*a^5*b^4 - 4*a^3*
b^6 - 3*a*b^8)*cos(d*x + c)^3 + (77*a^7*b^2 - 69*a^5*b^4 + 63*a^3*b^6 - 15
*a*b^8 + 12*(a^8*b - 25*a^6*b^3 + 35*a^4*b^5 - 3*a^2*b^7)*d*x)*cos(d*x + c
))*sin(d*x + c))/((a^12 + 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 - 4*a^2*b^10
- b^12)*d*cos(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5*b^7
 + 5*a^3*b^9 + a*b^11)*d*cos(d*x + c)*sin(d*x + c) + (a^10*b^2 + 5*a^8*b^4
 + 10*a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^10 + b^12)*d)
```

3.68.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(sin(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.68.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 744 vs. $2(277) = 554$.

Time = 0.55 (sec) , antiderivative size = 744, normalized size of antiderivative = 2.61

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{3\,(a^7-25\,a^5b^2+35\,a^3b^4-3\,ab^6)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} + \frac{24\,(a^6b-5\,a^4b^3+2\,a^2b^5)\log(b\tan(dx+c)+a)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - \frac{12\,(a^6b-5\,a^4b^3+2\,a^2b^5)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} -}{}$$

input
```
integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/8*(3*(a^7 - 25*a^5*b^2 + 35*a^3*b^4 - 3*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b
^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 24*(a^6*b - 5*a^4*b^3 +
 2*a^2*b^5)*log(b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^
4*b^6 + 5*a^2*b^8 + b^10) - 12*(a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*log(tan(d*x
 + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^1
0) - (38*a^6*b - 56*a^4*b^3 + 2*a^2*b^5 + 3*(7*a^5*b^2 - 22*a^3*b^4 + 3*a*
b^6)*tan(d*x + c)^5 + 6*(5*a^6*b - 12*a^4*b^3 - a^2*b^5)*tan(d*x + c)^4 +
(5*a^7 + 49*a^5*b^2 - 133*a^3*b^4 + 15*a*b^6)*tan(d*x + c)^3 + 2*(35*a^6*b
 - 61*a^4*b^3 + a^2*b^5 + b^7)*tan(d*x + c)^2 + (3*a^7 + 22*a^5*b^2 - 73*a
^3*b^4 + 4*a*b^6)*tan(d*x + c))/(a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6
+ a^2*b^8 + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*tan(d*x +
 c)^6 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)
^5 + (a^10 + 6*a^8*b^2 + 14*a^6*b^4 + 16*a^4*b^6 + 9*a^2*b^8 + 2*b^10)*tan
(d*x + c)^4 + 4*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*
x + c)^3 + (2*a^10 + 9*a^8*b^2 + 16*a^6*b^4 + 14*a^4*b^6 + 6*a^2*b^8 + b^1
0)*tan(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*
tan(d*x + c)))/d
```

3.68.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(277) = 554$.

Time = 0.68 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.06

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{3\,(a^7-25\,a^5b^2+35\,a^3b^4-3\,ab^6)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - \frac{12\,(a^6b-5\,a^4b^3+2\,a^2b^5)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} + \frac{24\,(a^6b^2-5\,a^4b^4+2\,a^2b^6)\log(|b\tan(dx+c)+a|)}{a^{10}b+5\,a^8b^3+10\,a^6b^5+10\,a^4b^7+5\,a^2b^9+b^{11}}}{}$$

input
```
integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(3*(a^7 - 25*a^5*b^2 + 35*a^3*b^4 - 3*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b
^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 12*(a^6*b - 5*a^4*b^3 +
 2*a^2*b^5)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^
4*b^6 + 5*a^2*b^8 + b^10) + 24*(a^6*b^2 - 5*a^4*b^4 + 2*a^2*b^6)*log(abs(b
*tan(d*x + c) + a))/(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*
b^9 + b^11) - (21*a^5*b^2*tan(d*x + c)^5 - 66*a^3*b^4*tan(d*x + c)^5 + 9*a
*b^6*tan(d*x + c)^5 + 30*a^6*b*tan(d*x + c)^4 - 72*a^4*b^3*tan(d*x + c)^4
- 6*a^2*b^5*tan(d*x + c)^4 + 5*a^7*tan(d*x + c)^3 + 49*a^5*b^2*tan(d*x + c
)^3 - 133*a^3*b^4*tan(d*x + c)^3 + 15*a*b^6*tan(d*x + c)^3 + 70*a^6*b*tan(
d*x + c)^2 - 122*a^4*b^3*tan(d*x + c)^2 + 2*a^2*b^5*tan(d*x + c)^2 + 2*b^7
*tan(d*x + c)^2 + 3*a^7*tan(d*x + c) + 22*a^5*b^2*tan(d*x + c) - 73*a^3*b^
4*tan(d*x + c) + 4*a*b^6*tan(d*x + c) + 38*a^6*b - 56*a^4*b^3 + 2*a^2*b^5)
/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c)^3 + a*ta
n(d*x + c)^2 + b*tan(d*x + c) + a)^2))/d
```

3.68.9 Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 717, normalized size of antiderivative = 2.52

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{3\,b}{{\left(a^2+b^2\right)}^2}-\frac{24\,b^3}{{\left(a^2+b^2\right)}^3}+\frac{45\,b^5}{{\left(a^2+b^2\right)}^4}-\frac{24\,b^7}{{\left(a^2+b^2\right)}^5}\right)}{d}$$
$$-\frac{\frac{19\,a^6\,b-28\,a^4\,b^3+a^2\,b^5}{4\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}+\frac{\tan(c+d\,x)^2\,\left(35\,a^6\,b-61\,a^4\,b^3+a^2\,b^5+b^7\right)}{4\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}-\frac{3\tan(c+d\,x)^4\,\left(-5\,a^6\,b+12\,a^4\,b^3+a^2\,b^5\right)}{4\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}+\frac{3\tan(c+d\,x}{8\,(a^8+4\,}}{d\,\left(\tan\left(c+d\,x\right)^2\,\left(2\,a^2+b^2\right)+\tan\left(c+d\,x\right)^4\,\left(a^2+2\,b^2\right)+a^2+b^2\tan\left(c+d\right.\right.}$$
$$-\frac{3\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a^2\,1\mathrm{i}+3\,b\,a\right)}{16\,d\,\left(a^5+a^4\,b\,5\mathrm{i}-10\,a^3\,b^2-a^2\,b^3\,10\mathrm{i}+5\,a\,b^4+b^5\,1\mathrm{i}\right)}$$
$$-\frac{3\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(3\,a\,b-a^2\,1\mathrm{i}\right)}{16\,d\,\left(a^5-a^4\,b\,5\mathrm{i}-10\,a^3\,b^2+a^2\,b^3\,10\mathrm{i}+5\,a\,b^4-b^5\,1\mathrm{i}\right)}$$

input
```
int(sin(c + d*x)^4/(a + b*tan(c + d*x))^3,x)
```

output
```
(log(a + b*tan(c + d*x))*((3*b)/(a^2 + b^2)^2 - (24*b^3)/(a^2 + b^2)^3 + (
45*b^5)/(a^2 + b^2)^4 - (24*b^7)/(a^2 + b^2)^5))/d - ((19*a^6*b + a^2*b^5
- 28*a^4*b^3)/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (tan(c
 + d*x)^2*(35*a^6*b + b^7 + a^2*b^5 - 61*a^4*b^3))/(4*(a^8 + b^8 + 4*a^2*b
^6 + 6*a^4*b^4 + 4*a^6*b^2)) - (3*tan(c + d*x)^4*(a^2*b^5 - 5*a^6*b + 12*a
^4*b^3))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (3*tan(c +
d*x)^5*(3*a*b^6 - 22*a^3*b^4 + 7*a^5*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a
^4*b^4 + 4*a^6*b^2)) + (tan(c + d*x)^3*(15*a*b^6 + 5*a^7 - 133*a^3*b^4 + 4
9*a^5*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (a*tan(c
 + d*x)*(3*a^6 + 4*b^6 - 73*a^2*b^4 + 22*a^4*b^2))/(8*(a^8 + b^8 + 4*a^2*b
^6 + 6*a^4*b^4 + 4*a^6*b^2)))/(d*(tan(c + d*x)^2*(2*a^2 + b^2) + tan(c + d
*x)^4*(a^2 + 2*b^2) + a^2 + b^2*tan(c + d*x)^6 + 2*a*b*tan(c + d*x) + 4*a*
b*tan(c + d*x)^3 + 2*a*b*tan(c + d*x)^5)) - (3*log(tan(c + d*x) - 1i)*(3*a
*b + a^2*1i))/(16*d*(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*
a^3*b^2)) - (3*log(tan(c + d*x) + 1i)*(3*a*b - a^2*1i))/(16*d*(5*a*b^4 - a
^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2))
```

3.68. $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.69 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.69.1 Optimal result

Integrand size = 21, antiderivative size = 206

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{a(a^4-14a^2b^2+9b^4)\,x}{2\,(a^2+b^2)^4} + \frac{b(3a^4-8a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d} - \frac{a^2b}{2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} - \frac{2ab(a^2-b^2)}{(a^2+b^2)^3\,d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\,(b(3a^2-b^2)+a(a^2-3b^2)\tan(c+dx))}{2\,(a^2+b^2)^3\,d}$$

output
```
1/2*a*(a^4-14*a^2*b^2+9*b^4)*x/(a^2+b^2)^4+b*(3*a^4-8*a^2*b^2+b^4)*ln(a*co
s(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^4/d-1/2*a^2*b/(a^2+b^2)^2/d/(a+b*tan(d*x+
c))^2-2*a*b*(a^2-b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))-1/2*cos(d*x+c)^2*(b*(
3*a^2-b^2)+a*(a^2-3*b^2)*tan(d*x+c))/(a^2+b^2)^3/d
```

3.69.2 Mathematica [A] (verified)

Time = 4.21 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.53

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$
$$-\frac{b\left(\frac{a(a^2-3b^2)(a^2+b^2)\arctan(\tan(c+dx))}{b}+(3a^2-b^2)(a^2+b^2)\cos^2(c+dx)+\left(3a^4-8a^2b^2+b^4-\frac{a^5-8a^3b^2+3ab^4}{\sqrt{-b^2}}\right.\right.}{}$$

input
```
Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

output
```
-1/2*(b*((a*(a^2 - 3*b^2)*(a^2 + b^2)*ArcTan[Tan[c + d*x]])/b + (3*a^2 - b
^2)*(a^2 + b^2)*Cos[c + d*x]^2 + (3*a^4 - 8*a^2*b^2 + b^4 - (a^5 - 8*a^3*b
^2 + 3*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*(3*a^4 - 8*
a^2*b^2 + b^4)*Log[a + b*Tan[c + d*x]] + (3*a^4 - 8*a^2*b^2 + b^4 + (a^5 -
 8*a^3*b^2 + 3*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (a*(a
^2 - 3*b^2)*(a^2 + b^2)*Sin[2*(c + d*x)])/(2*b) + (a^2*(a^2 + b^2)^2)/(a +
 b*Tan[c + d*x])^2 + (4*(a^5 - a*b^4))/(a + b*Tan[c + d*x])))/((a^2 + b^2)
^4*d)
```

3.69.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3999, 601, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin(c+dx)^2}{(a+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \; 3999$$
$$\frac{b\int \frac{b^2\tan^2(c+dx)}{(a+b\tan(c+dx))^3(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 601$$

$$\frac{b\left(-\frac{\int -\frac{-\frac{a\left(a^2-3b^2\right)\tan^3(c+dx)b^5}{\left(a^2+b^2\right)^3}-\frac{\left(3a^4-3b^2a^2-2b^4\right)\tan^2(c+dx)b^4}{\left(a^2+b^2\right)^3}-\frac{a^3\left(3a^2+7b^2\right)\tan(c+dx)b^3}{\left(a^2+b^2\right)^3}+\frac{a^4\left(a^2-3b^2\right)b^2}{\left(a^2+b^2\right)^3}}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2}-\frac{ab(a^2-3b^2)\tan}{2(a^2+b^2)^3(b}\right.}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{\int \frac{-\frac{a\left(a^2-3b^2\right)\tan^3(c+dx)b^5}{\left(a^2+b^2\right)^3}-\frac{\left(3a^4-3b^2a^2-2b^4\right)\tan^2(c+dx)b^4}{\left(a^2+b^2\right)^3}-\frac{a^3\left(3a^2+7b^2\right)\tan(c+dx)b^3}{\left(a^2+b^2\right)^3}+\frac{a^4\left(a^2-3b^2\right)b^2}{\left(a^2+b^2\right)^3}}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2}-\frac{ab(a^2-3b^2)\tan(c+}{2(a^2+b^2)^3(b^2\tan}\right.}{d}$$

$$\Big\downarrow \; 2160$$

$$\frac{b\left(\frac{\int\left(\frac{\left(a\left(a^4-14b^2a^2+9b^4\right)-2b\left(3a^4-8b^2a^2+b^4\right)\tan(c+dx)\right)b^2}{\left(a^2+b^2\right)^4\left(\tan^2(c+dx)b^2+b^2\right)}+\frac{4a\left(a^2-b^2\right)b^2}{\left(a^2+b^2\right)^3(a+b\tan(c+dx))^2}+\frac{2a^2b^2}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))^3}+\frac{2\left(b^6-8a^2b^4+3a^4b^2\right)}{\left(a^2+b^2\right)^4(a+b\tan(c+dx))}\right)d(}{2b^2}\right.}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\left(\frac{-\frac{4ab^2\left(a^2-b^2\right)}{\left(a^2+b^2\right)^3(a+b\tan(c+dx))}-\frac{a^2b^2}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}+\frac{ab\left(a^4-14a^2b^2+9b^4\right)\arctan(\tan(c+dx))}{\left(a^2+b^2\right)^4}-\frac{b^2\left(3a^4-8a^2b^2+b^4\right)\log\left(b^2\tan^2(c+dx)+b^2\right)}{\left(a^2+b^2\right)^4}+\frac{2b^2(}{}}{2b^2}\right.}{d}$$

input
```
Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*(-1/2*(b^2*(3*a^2 - b^2) + a*b*(a^2 - 3*b^2)*Tan[c + d*x])/((a^2 + b^2)
^3*(b^2 + b^2*Tan[c + d*x]^2)) + ((a*b*(a^4 - 14*a^2*b^2 + 9*b^4)*ArcTan[T
an[c + d*x]])/(a^2 + b^2)^4 + (2*b^2*(3*a^4 - 8*a^2*b^2 + b^4)*Log[a + b*T
an[c + d*x]])/(a^2 + b^2)^4 - (b^2*(3*a^4 - 8*a^2*b^2 + b^4)*Log[b^2 + b^2
*Tan[c + d*x]^2])/(a^2 + b^2)^4 - (a^2*b^2)/((a^2 + b^2)^2*(a + b*Tan[c +
d*x])^2) - (4*a*b^2*(a^2 - b^2))/((a^2 + b^2)^3*(a + b*Tan[c + d*x])))/(2*
b^2)))/d
```

3.69. $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.69.4 Maple [A] (verified)

Time = 5.73 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{-\frac{a^2 b}{2(a^2+b^2)^2(a+b\tan(dx+c))^2}+\frac{b(3a^4-8a^2b^2+b^4)\ln(a+b\tan(dx+c))}{(a^2+b^2)^4}-\frac{2ab(a^2-b^2)}{(a^2+b^2)^3(a+b\tan(dx+c))}+\frac{\frac{(-\frac{1}{2}a^5+a^3b^2+\frac{3}{2}ab^4)\tan(}{1+\tan^2(}}{}}{d}$
default	$\frac{-\frac{a^2 b}{2(a^2+b^2)^2(a+b\tan(dx+c))^2}+\frac{b(3a^4-8a^2b^2+b^4)\ln(a+b\tan(dx+c))}{(a^2+b^2)^4}-\frac{2ab(a^2-b^2)}{(a^2+b^2)^3(a+b\tan(dx+c))}+\frac{\frac{(-\frac{1}{2}a^5+a^3b^2+\frac{3}{2}ab^4)\tan(}{1+\tan^2(}}{}}{d}$
risch	$-\frac{ixb}{4ia^3b-4ia\,b^3-a^4+6a^2b^2-b^4}-\frac{xa}{2(4ia^3b-4ia\,b^3-a^4+6a^2b^2-b^4)}+\frac{ie^{2i(dx+c)}}{8(-3ib\,a^2+ib^3+a^3-3a\,b^2)d}-\frac{ie^{-2i(d}}{8(3ib\,a^2-ib^3+}$

input
```
int(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a^2*b/(a^2+b^2)^2/(a+b*tan(d*x+c))^2+b*(3*a^4-8*a^2*b^2+b^4)/(a^
2+b^2)^4*ln(a+b*tan(d*x+c))-2*a*b*(a^2-b^2)/(a^2+b^2)^3/(a+b*tan(d*x+c))+1
/(a^2+b^2)^4*(((-1/2*a^5+a^3*b^2+3/2*a*b^4)*tan(d*x+c)-3/2*a^4*b-a^2*b^3+1
/2*b^5)/(1+tan(d*x+c)^2)+1/4*(-6*a^4*b+16*a^2*b^3-2*b^5)*ln(1+tan(d*x+c)^2
)+1/2*(a^5-14*a^3*b^2+9*a*b^4)*arctan(tan(d*x+c))))
```

3.69.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(200) = 400$.

Time = 0.30 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.55

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{13\,a^4b^3-8\,a^2b^5-b^7-2\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\cos{(dx+c)}^4+2\,(a^5b^2-14\,a^3b^4+9\,ab^6)dx-(a^6b+}{}$$

input
```
integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(13*a^4*b^3 - 8*a^2*b^5 - b^7 - 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7
)*cos(d*x + c)^4 + 2*(a^5*b^2 - 14*a^3*b^4 + 9*a*b^6)*d*x - (a^6*b + 23*a^
4*b^3 - 21*a^2*b^5 - 3*b^7 - 2*(a^7 - 15*a^5*b^2 + 23*a^3*b^4 - 9*a*b^6)*d
*x)*cos(d*x + c)^2 + 2*(3*a^4*b^3 - 8*a^2*b^5 + b^7 + (3*a^6*b - 11*a^4*b^
3 + 9*a^2*b^5 - b^7)*cos(d*x + c)^2 + 2*(3*a^5*b^2 - 8*a^3*b^4 + a*b^6)*co
s(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)
*cos(d*x + c)^2 + b^2) - 2*((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x
+ c)^3 - 2*(4*a^5*b^2 - 3*a^3*b^4 + 3*a*b^6 + (a^6*b - 14*a^4*b^3 + 9*a^2*
b^5)*d*x)*cos(d*x + c))*sin(d*x + c))/((a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a
^4*b^6 - 3*a^2*b^8 - b^10)*d*cos(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5
*b^5 + 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) + (a^8*b^2 + 4*a^6*b
^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*d)
```

3.69.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.69.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(200) = 400.

Time = 0.45 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.25

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{(a^5-14\,a^3b^2+9\,ab^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{2\,(3\,a^4b-8\,a^2b^3+b^5)\log(b\tan(dx+c)+a)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{(3\,a^4b-8\,a^2b^3+b^5)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{}{a^8+3\,a^6b^2+3\,a^4b^4}}{}$$

input
```
integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*((a^5 - 14*a^3*b^2 + 9*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8) + 2*(3*a^4*b - 8*a^2*b^3 + b^5)*log(b*tan(d*x + c) + a)/
(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (3*a^4*b - 8*a^2*b^3 + b
^5)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
) - (8*a^4*b - 4*a^2*b^3 + (5*a^3*b^2 - 7*a*b^4)*tan(d*x + c)^3 + (7*a^4*b
 - 6*a^2*b^3 - b^5)*tan(d*x + c)^2 + (a^5 + 7*a^3*b^2 - 6*a*b^4)*tan(d*x +
 c))/(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2
*b^6 + b^8)*tan(d*x + c)^4 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan
(d*x + c)^3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*tan(d*x + c)
^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c)))/d
```

3.69.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(200) = 400$.

Time = 0.64 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.34

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{\frac{(a^5-14a^3b^2+9ab^4)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} - \frac{(3a^4b-8a^2b^3+b^5)\log\left(\tan(dx+c)^2+1\right)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} + \frac{2(3a^4b^2-8a^2b^4+b^6)\log(|b\tan(dx+c)+a|)}{a^8b+4a^6b^3+6a^4b^5+4a^2b^7+b^9} + \frac{3a^4b\tan(dx+c)^2}{}}{}$$

input
```
integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*((a^5 - 14*a^3*b^2 + 9*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8) - (3*a^4*b - 8*a^2*b^3 + b^5)*log(tan(d*x + c)^2 + 1)/(a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 2*(3*a^4*b^2 - 8*a^2*b^4 +
 b^6)*log(abs(b*tan(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*
b^7 + b^9) + (3*a^4*b*tan(d*x + c)^2 - 8*a^2*b^3*tan(d*x + c)^2 + b^5*tan(
d*x + c)^2 - a^5*tan(d*x + c) + 2*a^3*b^2*tan(d*x + c) + 3*a*b^4*tan(d*x +
 c) - 10*a^2*b^3 + 2*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)
*(tan(d*x + c)^2 + 1)) - (9*a^4*b^3*tan(d*x + c)^2 - 24*a^2*b^5*tan(d*x +
c)^2 + 3*b^7*tan(d*x + c)^2 + 22*a^5*b^2*tan(d*x + c) - 48*a^3*b^4*tan(d*x
 + c) + 2*a*b^6*tan(d*x + c) + 14*a^6*b - 22*a^4*b^3)/((a^8 + 4*a^6*b^2 +
6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c) + a)^2))/d
```

3.69.9 Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.10

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{\tan(c+d\,x)^2\,\left(-7\,a^4\,b+6\,a^2\,b^3+b^5\right)}{2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{\tan(c+d\,x)^3\,\left(7\,a\,b^4-5\,a^3\,b^2\right)}{2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{2\,a^2\,\left(2\,a^2\,b-b^3\right)}{\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{a\tan(c+d\,x)\,\left(a^4+7\,a^2\,b^2-6\,b^4\right)}{2\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(\tan\left(c+d\,x\right)^2\,\left(a^2+b^2\right)+a^2+b^2\tan\left(c+d\,x\right)^4+2\,a\,b\tan\left(c+d\,x\right)+2\,a\,b\tan\left(c+d\,x\right)^3\right)}$$
$$+\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{3\,b}{\left(a^2+b^2\right)^2}-\frac{14\,b^3}{\left(a^2+b^2\right)^3}+\frac{12\,b^5}{\left(a^2+b^2\right)^4}\right)}{d}$$
$$+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-2\,b+a\,1\mathrm{i}\right)}{4\,d\,\left(a^4-a^3\,b\,4\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4\mathrm{i}+b^4\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a-b\,2\mathrm{i}\right)}{4\,d\,\left(a^4\,1\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6\mathrm{i}+4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$

input
```
int(sin(c + d*x)^2/(a + b*tan(c + d*x))^3,x)
```

output
```
((tan(c + d*x)^2*(b^5 - 7*a^4*b + 6*a^2*b^3))/(2*(a^6 + b^6 + 3*a^2*b^4 +
3*a^4*b^2)) + (tan(c + d*x)^3*(7*a*b^4 - 5*a^3*b^2))/(2*(a^6 + b^6 + 3*a^2
*b^4 + 3*a^4*b^2)) - (2*a^2*(2*a^2*b - b^3))/((a^2 + b^2)*(a^4 + b^4 + 2*a
^2*b^2)) - (a*tan(c + d*x)*(a^4 - 6*b^4 + 7*a^2*b^2))/(2*(a^2 + b^2)*(a^4
+ b^4 + 2*a^2*b^2)))/(d*(tan(c + d*x)^2*(a^2 + b^2) + a^2 + b^2*tan(c + d*
x)^4 + 2*a*b*tan(c + d*x) + 2*a*b*tan(c + d*x)^3)) + (log(a + b*tan(c + d*
x))*((3*b)/(a^2 + b^2)^2 - (14*b^3)/(a^2 + b^2)^3 + (12*b^5)/(a^2 + b^2)^4
))/d + (log(tan(c + d*x) + 1i)*(a*1i - 2*b))/(4*d*(a*b^3*4i - a^3*b*4i + a
^4 + b^4 - 6*a^2*b^2)) + (log(tan(c + d*x) - 1i)*(a - b*2i))/(4*d*(4*a*b^3
 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i))
```

3.70 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.70.1 Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx=-\frac{\cot(c+dx)}{a^3d}-\frac{3b\log(\tan(c+dx))}{a^4d}+\frac{3b\log(a+b\tan(c+dx))}{a^4d}-\frac{b}{2a^2d(a+b\tan(c+dx))^2}-\frac{2b}{a^3d(a+b\tan(c+dx))}$$

output
```
-cot(d*x+c)/a^3/d-3*b*ln(tan(d*x+c))/a^4/d+3*b*ln(a+b*tan(d*x+c))/a^4/d-1/
2*b/a^2/d/(a+b*tan(d*x+c))^2-2*b/a^3/d/(a+b*tan(d*x+c))
```

3.70.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 241 vs. $2(95) = 190$.

Time = 4.19 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.54

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$=\frac{-2a^3(a^2+b^2)\cot(c+dx)+b(-2a^2(a^2+b^2)(2+3\log(\sin(c+dx))-3\log(a\cos(c+dx)+b\sin(c+dx)

output
```
(-2*a^3*(a^2 + b^2)*Cot[c + d*x] + b*(-2*a^2*(a^2 + b^2)*(2 + 3*Log[Sin[c
+ d*x]] - 3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]) - a^2*b^2*Sec[c + d*x]^2
 + 2*a*b*(2*a^2 + b^2 - 6*(a^2 + b^2)*Log[Sin[c + d*x]] + 6*(a^2 + b^2)*Lo
g[a*Cos[c + d*x] + b*Sin[c + d*x]])*Tan[c + d*x] - 2*b^2*(-3*a^2 - 2*b^2 +
 3*(a^2 + b^2)*Log[Sin[c + d*x]] - 3*(a^2 + b^2)*Log[a*Cos[c + d*x] + b*Si
n[c + d*x]])*Tan[c + d*x]^2))/(2*a^4*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
```

3.70.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3999, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^2(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3999}$$

$$\frac{b\int \frac{\cot^2(c+dx)}{b^2(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$\frac{b\int \left(\frac{\cot^2(c+dx)}{a^3b^2}-\frac{3\cot(c+dx)}{a^4b}+\frac{3}{a^4(a+b\tan(c+dx))}+\frac{2}{a^3(a+b\tan(c+dx))^2}+\frac{1}{a^2(a+b\tan(c+dx))^3}\right)d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{3\log(b\tan(c+dx))}{a^4}+\frac{3\log(a+b\tan(c+dx))}{a^4}-\frac{2}{a^3(a+b\tan(c+dx))}-\frac{\cot(c+dx)}{a^3b}-\frac{1}{2a^2(a+b\tan(c+dx))^2}\right)}{d}$$

input
```
Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

3.70. $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

output
```
(b*(-(Cot[c + d*x]/(a^3*b)) - (3*Log[b*Tan[c + d*x]])/a^4 + (3*Log[a + b*T
an[c + d*x]])/a^4 - 1/(2*a^2*(a + b*Tan[c + d*x])^2) - 2/(a^3*(a + b*Tan[c
 + d*x]))))/d
```

3.70.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.70.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{1}{a^{3}\tan(dx+c)}-\frac{3b\ln(\tan(dx+c))}{a^{4}}-\frac{b}{2a^{2}(a+b\tan(dx+c))^{2}}+\frac{3b\ln(a+b\tan(dx+c))}{a^{4}}-\frac{2b}{a^{3}(a+b\tan(dx+c))}}{d}$
default	$\frac{-\frac{1}{a^{3}\tan(dx+c)}-\frac{3b\ln(\tan(dx+c))}{a^{4}}-\frac{b}{2a^{2}(a+b\tan(dx+c))^{2}}+\frac{3b\ln(a+b\tan(dx+c))}{a^{4}}-\frac{2b}{a^{3}(a+b\tan(dx+c))}}{d}$
risch	$-\frac{2i\left(a^{4}e^{4i(dx+c)}-9a^{2}b^{2}e^{4i(dx+c)}+3b^{4}e^{4i(dx+c)}-4ia^{3}b\,e^{4i(dx+c)}+9ia\,b^{3}e^{4i(dx+c)}+2a^{4}e^{2i(dx+c)}-6b^{4}e^{2i(dx+c)}-4ia^{3}b\right.}{\left(e^{2i(dx+c)}-1\right)\left(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a\right)^{2}(-ib+a)^{2}a^{3}d}$

input
```
int(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^3/tan(d*x+c)-3/a^4*b*ln(tan(d*x+c))-1/2/a^2*b/(a+b*tan(d*x+c))^2
+3/a^4*b*ln(a+b*tan(d*x+c))-2/a^3*b/(a+b*tan(d*x+c)))
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(93) = 186$.

Time = 0.28 (sec) , antiderivative size = 565, normalized size of antiderivative = 5.95

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{2\left(a^7+4a^5b^2-2a^3b^4-3ab^6\right)\cos\left(dx+c\right)^3-2\left(2a^5b^2-3a^3b^4-3ab^6\right)\cos\left(dx+c\right)+3\left(2\left(a^5b^2+2a^3b^4\right.\right.}{}$$

input
```
integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(2*(a^7 + 4*a^5*b^2 - 2*a^3*b^4 - 3*a*b^6)*cos(d*x + c)^3 - 2*(2*a^5*b
^2 - 3*a^3*b^4 - 3*a*b^6)*cos(d*x + c) + 3*(2*(a^5*b^2 + 2*a^3*b^4 + a*b^6
)*cos(d*x + c)^3 - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c) - (a^4*b^3
 + 2*a^2*b^5 + b^7 + (a^6*b + a^4*b^3 - a^2*b^5 - b^7)*cos(d*x + c)^2)*sin
(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^
2 + b^2) - 3*(2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 2*(a^5*b^2
+ 2*a^3*b^4 + a*b^6)*cos(d*x + c) - (a^4*b^3 + 2*a^2*b^5 + b^7 + (a^6*b +
a^4*b^3 - a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/4*cos(d*x +
c)^2 + 1/4) - (5*a^4*b^3 + 3*a^2*b^5 - 4*(a^6*b + 5*a^4*b^3 + 3*a^2*b^5)*c
os(d*x + c)^2)*sin(d*x + c))/(2*(a^9*b + 2*a^7*b^3 + a^5*b^5)*d*cos(d*x +
c)^3 - 2*(a^9*b + 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c) - ((a^10 + a^8*b^2 -
 a^6*b^4 - a^4*b^6)*d*cos(d*x + c)^2 + (a^8*b^2 + 2*a^6*b^4 + a^4*b^6)*d)*
sin(d*x + c))
```

3.70.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\csc^2\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(csc(c + d*x)**2/(a + b*tan(c + d*x))**3, x)
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{6\,b^2\tan(dx+c)^2+9\,ab\tan(dx+c)+2\,a^2}{a^3b^2\tan(dx+c)^3+2\,a^4b\tan(dx+c)^2+a^5\tan(dx+c)} - \frac{6\,b\log(b\tan(dx+c)+a)}{a^4} + \frac{6\,b\log(\tan(dx+c))}{a^4}}{2\,d}$$

input
```
integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*((6*b^2*tan(d*x + c)^2 + 9*a*b*tan(d*x + c) + 2*a^2)/(a^3*b^2*tan(d*x
 + c)^3 + 2*a^4*b*tan(d*x + c)^2 + a^5*tan(d*x + c)) - 6*b*log(b*tan(d*x +
 c) + a)/a^4 + 6*b*log(tan(d*x + c))/a^4)/d
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{6\,b\log(|b\tan(dx+c)+a|)}{a^4} - \frac{6\,b\log(|\tan(dx+c)|)}{a^4} + \frac{2\,(3\,b\tan(dx+c)-a)}{a^4\tan(dx+c)} - \frac{9\,b^3\tan(dx+c)^2+22\,ab^2\tan(dx+c)+14\,a^2b}{(b\tan(dx+c)+a)^2a^4}}{2\,d}$$

input
```
integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(6*b*log(abs(b*tan(d*x + c) + a))/a^4 - 6*b*log(abs(tan(d*x + c)))/a^4
 + 2*(3*b*tan(d*x + c) - a)/(a^4*tan(d*x + c)) - (9*b^3*tan(d*x + c)^2 + 2
2*a*b^2*tan(d*x + c) + 14*a^2*b)/((b*tan(d*x + c) + a)^2*a^4))/d
```

3.70.9 Mupad [B] (verification not implemented)

Time = 4.87 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{6\,b\,\mathrm{atanh}\left(\frac{2\,b\tan(c+d\,x)}{a}+1\right)}{a^4\,d} - \frac{\frac{1}{a}+\frac{3\,b^2\tan(c+d\,x)^2}{a^3}+\frac{9\,b\tan(c+d\,x)}{2\,a^2}}{d\,\left(a^2\tan\left(c+d\,x\right)+2\,a\,b\tan\left(c+d\,x\right)^2+b^2\tan\left(c+d\,x\right)^3\right)}$$

input
```
int(1/(sin(c + d*x)^2*(a + b*tan(c + d*x))^3),x)
```

output
```
(6*b*atanh((2*b*tan(c + d*x))/a + 1))/(a^4*d) - (1/a + (3*b^2*tan(c + d*x)
^2)/a^3 + (9*b*tan(c + d*x))/(2*a^2))/(d*(a^2*tan(c + d*x) + b^2*tan(c + d
*x)^3 + 2*a*b*tan(c + d*x)^2))
```

3.71 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.71.1 Optimal result

Integrand size = 21, antiderivative size = 178

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{(a^2+6b^2)\cot(c+dx)}{a^5d}+\frac{3b\cot^2(c+dx)}{2a^4d} - \frac{\cot^3(c+dx)}{3a^3d}-\frac{b(3a^2+10b^2)\log(\tan(c+dx))}{a^6d} + \frac{b(3a^2+10b^2)\log(a+b\tan(c+dx))}{a^6d} - \frac{b(a^2+b^2)}{2a^4d(a+b\tan(c+dx))^2}-\frac{2b(a^2+2b^2)}{a^5d(a+b\tan(c+dx))}$$

output
```
-(a^2+6*b^2)*cot(d*x+c)/a^5/d+3/2*b*cot(d*x+c)^2/a^4/d-1/3*cot(d*x+c)^3/a^
3/d-b*(3*a^2+10*b^2)*ln(tan(d*x+c))/a^6/d+b*(3*a^2+10*b^2)*ln(a+b*tan(d*x+
c))/a^6/d-1/2*b*(a^2+b^2)/a^4/d/(a+b*tan(d*x+c))^2-2*b*(a^2+2*b^2)/a^5/d/(
a+b*tan(d*x+c))
```

3.71.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 456 vs. $2(178) = 356$.

Time = 6.67 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.56

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{b^3\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))}{2a^4d(a+b\tan(c+dx))^3}$$
$$-\frac{\csc^3(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3}{3a^3d(a+b\tan(c+dx))^3}$$
$$-\frac{2(a^2\cos(c+dx)+9b^2\cos(c+dx))\csc(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3}{3a^5d(a+b\tan(c+dx))^3}$$
$$+\frac{3b\csc^2(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3}{2a^4d(a+b\tan(c+dx))^3}$$
$$+\frac{(-3a^2b-10b^3)\log(\sin(c+dx))\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3}{a^6d(a+b\tan(c+dx))^3}$$
$$+\frac{(3a^2b+10b^3)\log(a\cos(c+dx)+b\sin(c+dx))\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3}{a^6d(a+b\tan(c+dx))^3}$$
$$+\frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2(3a^2b^2\sin(c+dx)+4b^4\sin(c+dx))}{a^6d(a+b\tan(c+dx))^3}$$

input
```
Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

output
```
-1/2*(b^3*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(a^4*d*(a + b*
Tan[c + d*x])^3) - (Csc[c + d*x]^3*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3)/(3*a^3*d*(a + b*Tan[c + d*x])^3) - (2*(a^2*Cos[c + d*x] + 9*b
^2*Cos[c + d*x])*Csc[c + d*x]*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d
*x])^3)/(3*a^5*d*(a + b*Tan[c + d*x])^3) + (3*b*Csc[c + d*x]^2*Sec[c + d*x
]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(2*a^4*d*(a + b*Tan[c + d*x])^3)
+ ((-3*a^2*b - 10*b^3)*Log[Sin[c + d*x]]*Sec[c + d*x]^3*(a*Cos[c + d*x] +
b*Sin[c + d*x])^3)/(a^6*d*(a + b*Tan[c + d*x])^3) + ((3*a^2*b + 10*b^3)*Lo
g[a*Cos[c + d*x] + b*Sin[c + d*x]]*Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3)/(a^6*d*(a + b*Tan[c + d*x])^3) + (Sec[c + d*x]^3*(a*Cos[c + d
*x] + b*Sin[c + d*x])^2*(3*a^2*b^2*Sin[c + d*x] + 4*b^4*Sin[c + d*x]))/(a^
6*d*(a + b*Tan[c + d*x])^3)
```

3.71.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3999, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin(c+dx)^4(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \; 3999$$

$$\frac{b\int \frac{\cot^4(c+dx)\left(\tan^2(c+dx)b^2+b^2\right)}{b^4(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 522$$

$$\frac{b\int \left(\frac{\cot^4(c+dx)}{a^3b^2}-\frac{3\cot^3(c+dx)}{a^4b}+\frac{(a^2+6b^2)\cot^2(c+dx)}{a^5b^2}+\frac{(-3a^2-10b^2)\cot(c+dx)}{a^6b}+\frac{3a^2+10b^2}{a^6(a+b\tan(c+dx))}+\frac{2(a^2+2b^2)}{a^5(a+b\tan(c+dx))^2}+\frac{\ldots}{a^4(\ldots}\right.}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\left(\frac{3\cot^2(c+dx)}{2a^4}-\frac{\cot^3(c+dx)}{3a^3b}-\frac{(3a^2+10b^2)\log(b\tan(c+dx))}{a^6}+\frac{(3a^2+10b^2)\log(a+b\tan(c+dx))}{a^6}-\frac{2(a^2+2b^2)}{a^5(a+b\tan(c+dx))}-\frac{(a^2+6b^2)\cot(c\ldots}{a^5b}\right.}{d}$$

input
```
Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*(-(((a^2 + 6*b^2)*Cot[c + d*x])/(a^5*b)) + (3*Cot[c + d*x]^2)/(2*a^4) -
 Cot[c + d*x]^3/(3*a^3*b) - ((3*a^2 + 10*b^2)*Log[b*Tan[c + d*x]])/a^6 + (
(3*a^2 + 10*b^2)*Log[a + b*Tan[c + d*x]])/a^6 - (a^2 + b^2)/(2*a^4*(a + b*
Tan[c + d*x])^2) - (2*(a^2 + 2*b^2))/(a^5*(a + b*Tan[c + d*x]))))/d
```

3.71.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.71.4 Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{1}{3a^3\tan(dx+c)^3}-\frac{a^2+6b^2}{a^5\tan(dx+c)}+\frac{3b}{2a^4\tan(dx+c)^2}-\frac{b\left(3a^2+10b^2\right)\ln(\tan(dx+c))}{a^6}+\frac{b\left(3a^2+10b^2\right)\ln(a+b\tan(dx+c))}{a^6}-\frac{\left(a^2+\right.}{2a^4(a+b\tan}}{d}$
default	$\frac{-\frac{1}{3a^3\tan(dx+c)^3}-\frac{a^2+6b^2}{a^5\tan(dx+c)}+\frac{3b}{2a^4\tan(dx+c)^2}-\frac{b\left(3a^2+10b^2\right)\ln(\tan(dx+c))}{a^6}+\frac{b\left(3a^2+10b^2\right)\ln(a+b\tan(dx+c))}{a^6}-\frac{\left(a^2+\right.}{2a^4(a+b\tan}}{d}$
risch	$-\frac{2i\left(29a^2b^3-2ia^5+30b^5+90ia\,b^4e^{4i(dx+c)}+18ia^3b^2e^{8i(dx+c)}-150ia\,b^4e^{6i(dx+c)}-9a^4b\,e^{8i(dx+c)}-a^4b\,e^{2i(dx+c)}+15a^4\right.}{}$

input
```
int(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3/a^3/tan(d*x+c)^3-(a^2+6*b^2)/a^5/tan(d*x+c)+3/2/a^4*b/tan(d*x+c)
^2-b*(3*a^2+10*b^2)/a^6*ln(tan(d*x+c))+b*(3*a^2+10*b^2)/a^6*ln(a+b*tan(d*x
+c))-1/2*(a^2+b^2)*b/a^4/(a+b*tan(d*x+c))^2-2*b*(a^2+2*b^2)/a^5/(a+b*tan(d
*x+c)))
```

3.71.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(172) = 344$.

Time = 0.30 (sec) , antiderivative size = 811, normalized size of antiderivative = 4.56

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{2\,(2\,a^7 + 27\,a^5b^2 + a^3b^4 - 30\,ab^6)\cos{(dx+c)}^5 - 2\,(3\,a^7 + 43\,a^5b^2 - 8\,a^3b^4 - 60\,ab^6)\cos{(dx+c)}^3 + 6\,(5\,a$$

input
```
integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/6*(2*(2*a^7 + 27*a^5*b^2 + a^3*b^4 - 30*a*b^6)*cos(d*x + c)^5 - 2*(3*a^7
 + 43*a^5*b^2 - 8*a^3*b^4 - 60*a*b^6)*cos(d*x + c)^3 + 6*(5*a^5*b^2 - 3*a^
3*b^4 - 10*a*b^6)*cos(d*x + c) + 3*(2*(3*a^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*
cos(d*x + c)^5 - 4*(3*a^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^3 + 2*
(3*a^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x + c) + (3*a^4*b^3 + 13*a^2*b^5
 + 10*b^7 - (3*a^6*b + 10*a^4*b^3 - 3*a^2*b^5 - 10*b^7)*cos(d*x + c)^4 + (
3*a^6*b + 7*a^4*b^3 - 16*a^2*b^5 - 20*b^7)*cos(d*x + c)^2)*sin(d*x + c))*l
og(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 3
*(2*(3*a^5*b^2 + 13*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^5 - 4*(3*a^5*b^2 + 13
*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^3 + 2*(3*a^5*b^2 + 13*a^3*b^4 + 10*a*b^6
)*cos(d*x + c) + (3*a^4*b^3 + 13*a^2*b^5 + 10*b^7 - (3*a^6*b + 10*a^4*b^3
- 3*a^2*b^5 - 10*b^7)*cos(d*x + c)^4 + (3*a^6*b + 7*a^4*b^3 - 16*a^2*b^5 -
 20*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/4*cos(d*x + c)^2 + 1/4) + (2
4*a^4*b^3 + 30*a^2*b^5 + 4*(2*a^6*b + 29*a^4*b^3 + 30*a^2*b^5)*cos(d*x + c
)^4 - 3*(a^6*b + 45*a^4*b^3 + 50*a^2*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(2
*(a^9*b + a^7*b^3)*d*cos(d*x + c)^5 - 4*(a^9*b + a^7*b^3)*d*cos(d*x + c)^3
 + 2*(a^9*b + a^7*b^3)*d*cos(d*x + c) - ((a^10 - a^6*b^4)*d*cos(d*x + c)^4
 - (a^10 - a^8*b^2 - 2*a^6*b^4)*d*cos(d*x + c)^2 - (a^8*b^2 + a^6*b^4)*d)*
sin(d*x + c))
```

3.71.6 Sympy [F]

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

input
```
integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(csc(c + d*x)**4/(a + b*tan(c + d*x))**3, x)
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.08

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{5a^3b\tan(dx+c)-6(3a^2b^2+10b^4)\tan(dx+c)^4-2a^4-9(3a^3b+10ab^3)\tan(dx+c)^3-2(3a^4+10a^2b^2)\tan(dx+c)^2}{a^5b^2\tan(dx+c)^5+2a^6b\tan(dx+c)^4+a^7\tan(dx+c)^3} + \frac{6(3a^2b+10b^3)\log(b\tan(}{a^6}}{6d}$$

input
```
integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/6*((5*a^3*b*tan(d*x + c) - 6*(3*a^2*b^2 + 10*b^4)*tan(d*x + c)^4 - 2*a^4
 - 9*(3*a^3*b + 10*a*b^3)*tan(d*x + c)^3 - 2*(3*a^4 + 10*a^2*b^2)*tan(d*x
+ c)^2)/(a^5*b^2*tan(d*x + c)^5 + 2*a^6*b*tan(d*x + c)^4 + a^7*tan(d*x + c
)^3) + 6*(3*a^2*b + 10*b^3)*log(b*tan(d*x + c) + a)/a^6 - 6*(3*a^2*b + 10*
b^3)*log(tan(d*x + c))/a^6)/d
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.33

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{6(3a^2b+10b^3)\log(|\tan(dx+c)|)}{a^6} - \frac{6(3a^2b^2+10b^4)\log(|b\tan(dx+c)+a|)}{a^6b} + \frac{3\left(9a^2b^3\tan(dx+c)^2+30b^5\tan(dx+c)^2+22a^3b^2\tan(dx+c)\right.}{(b\tan(dx+c)+a)^2a^6}}{6}$$

input
```
integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/6*(6*(3*a^2*b + 10*b^3)*log(abs(tan(d*x + c)))/a^6 - 6*(3*a^2*b^2 + 10*
b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b) + 3*(9*a^2*b^3*tan(d*x + c)^2 +
30*b^5*tan(d*x + c)^2 + 22*a^3*b^2*tan(d*x + c) + 68*a*b^4*tan(d*x + c) +
14*a^4*b + 39*a^2*b^3)/((b*tan(d*x + c) + a)^2*a^6) - (33*a^2*b*tan(d*x +
c)^3 + 110*b^3*tan(d*x + c)^3 - 6*a^3*tan(d*x + c)^2 - 36*a*b^2*tan(d*x +
c)^2 + 9*a^2*b*tan(d*x + c) - 2*a^3)/(a^6*tan(d*x + c)^3))/d
```

3.71.9 Mupad [B] (verification not implemented)

Time = 5.05 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.12

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{2\,b\,\mathrm{atanh}\left(\frac{b\,(3\,a^2+10\,b^2)\,(a+2\,b\tan(c+d\,x))}{a\,(3\,a^2\,b+10\,b^3)}\right)\,(3\,a^2+10\,b^2)}{a^6\,d} - \frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,(3\,a^2+10\,b^2)}{3\,a^3}-\frac{5\,b\tan(c+d\,x)}{6\,a^2}+\frac{b^2\tan(c+d\,x)^4\,(3\,a^2+10\,b^2)}{a^5}+\frac{3\,b\tan(c+d\,x)^3\,(3\,a^2+10\,b^2)}{2\,a^4}}{d\,\left(a^2\tan\left(c+d\,x\right)^3+2\,a\,b\tan\left(c+d\,x\right)^4+b^2\tan\left(c+d\,x\right)^5\right)}$$

input
```
int(1/(sin(c + d*x)^4*(a + b*tan(c + d*x))^3),x)
```

output
```
(2*b*atanh((b*(3*a^2 + 10*b^2)*(a + 2*b*tan(c + d*x)))/(a*(3*a^2*b + 10*b^
3)))*(3*a^2 + 10*b^2))/(a^6*d) - (1/(3*a) + (tan(c + d*x)^2*(3*a^2 + 10*b^
2))/(3*a^3) - (5*b*tan(c + d*x))/(6*a^2) + (b^2*tan(c + d*x)^4*(3*a^2 + 10
*b^2))/a^5 + (3*b*tan(c + d*x)^3*(3*a^2 + 10*b^2))/(2*a^4))/(d*(a^2*tan(c
+ d*x)^3 + b^2*tan(c + d*x)^5 + 2*a*b*tan(c + d*x)^4))
```

3.72 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.72.1 Optimal result

Integrand size = 21, antiderivative size = 265

$$\begin{aligned}\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx =& -\frac{(a^4+12a^2b^2+15b^4)\cot(c+dx)}{a^7d}+\frac{b(3a^2+5b^2)\cot^2(c+dx)}{a^6d}\\&-\frac{2(a^2+3b^2)\cot^3(c+dx)}{3a^5d}+\frac{3b\cot^4(c+dx)}{4a^4d}\\&-\frac{\cot^5(c+dx)}{5a^3d}-\frac{b(3a^4+20a^2b^2+21b^4)\log(\tan(c+dx))}{a^8d}\\&+\frac{b(3a^4+20a^2b^2+21b^4)\log(a+b\tan(c+dx))}{a^8d}\\&-\frac{b(a^2+b^2)^2}{2a^6d(a+b\tan(c+dx))^2}-\frac{2b(a^2+b^2)(a^2+3b^2)}{a^7d(a+b\tan(c+dx))}\end{aligned}$$

output
```
-(a^4+12*a^2*b^2+15*b^4)*cot(d*x+c)/a^7/d+b*(3*a^2+5*b^2)*cot(d*x+c)^2/a^6
/d-2/3*(a^2+3*b^2)*cot(d*x+c)^3/a^5/d+3/4*b*cot(d*x+c)^4/a^4/d-1/5*cot(d*x
+c)^5/a^3/d-b*(3*a^4+20*a^2*b^2+21*b^4)*ln(tan(d*x+c))/a^8/d+b*(3*a^4+20*a
^2*b^2+21*b^4)*ln(a+b*tan(d*x+c))/a^8/d-1/2*b*(a^2+b^2)^2/a^6/d/(a+b*tan(d
*x+c))^2-2*b*(a^2+b^2)*(a^2+3*b^2)/a^7/d/(a+b*tan(d*x+c))
```

3.72.2 Mathematica [A] (verified)

Time = 6.95 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.86

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$
$$-\frac{\csc^5(c+dx)\left(\sec^2(c+dx)\left((8a^7+567a^5b^2+630a^3b^4-1215ab^6)\cos(3(c+dx))-(24a^7+619a^5b^2+6\right.\right.}{}$$

input
```
Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

output
```
-1/960*(Csc[c + d*x]^5*(Sec[c + d*x]^2*((8*a^7 + 567*a^5*b^2 + 630*a^3*b^4
 - 1215*a*b^6)*Cos[3*(c + d*x)] - (24*a^7 + 619*a^5*b^2 + 630*a^3*b^4 - 67
5*a*b^6)*Cos[5*(c + d*x)] + 8*a^7*Cos[7*(c + d*x)] + 187*a^5*b^2*Cos[7*(c
+ d*x)] + 210*a^3*b^4*Cos[7*(c + d*x)] - 135*a*b^6*Cos[7*(c + d*x)] - 126*
a^6*b*Sin[3*(c + d*x)] + 1665*a^4*b^3*Sin[3*(c + d*x)] + 4635*a^2*b^5*Sin[
3*(c + d*x)] + 1890*b^7*Sin[3*(c + d*x)] + 10*a^6*b*Sin[5*(c + d*x)] - 121
5*a^4*b^3*Sin[5*(c + d*x)] - 2565*a^2*b^5*Sin[5*(c + d*x)] - 630*b^7*Sin[5
*(c + d*x)] + 16*a^6*b*Sin[7*(c + d*x)] + 345*a^4*b^3*Sin[7*(c + d*x)] + 5
85*a^2*b^5*Sin[7*(c + d*x)] + 90*b^7*Sin[7*(c + d*x)]) + 960*b*(3*a^4 + 20
*a^2*b^2 + 21*b^4)*(Log[Sin[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c + d*x
]])*Sin[c + d*x]^5*(a + b*Tan[c + d*x])^2 + 5*Sec[c + d*x]*(40*a^7 - 27*a^
5*b^2 - 42*a^3*b^4 + 135*a*b^6 - 3*b*(8*a^6 + 89*a^4*b^2 + 345*a^2*b^4 + 2
10*b^6)*Tan[c + d*x])))/(a^8*d*(a + b*Tan[c + d*x])^2)
```

3.72.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3999, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sin(c+dx)^6(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \; 3999$$

$$\frac{b\int \frac{\cot^6(c+dx)\left(\tan^2(c+dx)b^2+b^2\right)^2}{b^6(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 522$$

$$\frac{b\int \left(\frac{\cot^6(c+dx)}{a^3b^2}-\frac{3\cot^5(c+dx)}{a^4b}+\frac{2(a^2+3b^2)\cot^4(c+dx)}{a^5b^2}-\frac{2(5b^4+3a^2b^2)\cot^3(c+dx)}{a^6b^3}+\frac{(a^4+12b^2a^2+15b^4)\cot^2(c+dx)}{a^7b^2}+\frac{(-3a^4-20b^2}{}\right.}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\left(\frac{3\cot^4(c+dx)}{4a^4}-\frac{\cot^5(c+dx)}{5a^3b}-\frac{2(a^2+b^2)(a^2+3b^2)}{a^7(a+b\tan(c+dx))}-\frac{(a^2+b^2)^2}{2a^6(a+b\tan(c+dx))^2}+\frac{(3a^2+5b^2)\cot^2(c+dx)}{a^6}-\frac{2(a^2+3b^2)\cot^3(c+dx)}{3a^5b}-\frac{(3a^4}{}\right.}{d}$$

input
```
Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*(-(((a^4 + 12*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(a^7*b)) + ((3*a^2 + 5*b^
2)*Cot[c + d*x]^2)/a^6 - (2*(a^2 + 3*b^2)*Cot[c + d*x]^3)/(3*a^5*b) + (3*C
ot[c + d*x]^4)/(4*a^4) - Cot[c + d*x]^5/(5*a^3*b) - ((3*a^4 + 20*a^2*b^2 +
 21*b^4)*Log[b*Tan[c + d*x]])/a^8 + ((3*a^4 + 20*a^2*b^2 + 21*b^4)*Log[a +
 b*Tan[c + d*x]])/a^8 - (a^2 + b^2)^2/(2*a^6*(a + b*Tan[c + d*x])^2) - (2*
(a^2 + b^2)*(a^2 + 3*b^2))/(a^7*(a + b*Tan[c + d*x]))))/d
```

3.72.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.72.4 Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\frac{b\left(3a^4+20a^2b^2+21b^4\right)\ln(a+b\tan(dx+c))}{a^8}-\frac{\left(a^4+2a^2b^2+b^4\right)b}{2a^6(a+b\tan(dx+c))^2}-\frac{2b\left(a^4+4a^2b^2+3b^4\right)}{a^7(a+b\tan(dx+c))}-\frac{1}{5a^3\tan(dx+c)^5}-\frac{2a^2+6b^2}{3a^5\tan(dx+c)^3}-\frac{a^4+}{a^7}}{d}$
default	$\frac{\frac{b\left(3a^4+20a^2b^2+21b^4\right)\ln(a+b\tan(dx+c))}{a^8}-\frac{\left(a^4+2a^2b^2+b^4\right)b}{2a^6(a+b\tan(dx+c))^2}-\frac{2b\left(a^4+4a^2b^2+3b^4\right)}{a^7(a+b\tan(dx+c))}-\frac{1}{5a^3\tan(dx+c)^5}-\frac{2a^2+6b^2}{3a^5\tan(dx+c)^3}-\frac{a^4+}{a^7}}{d}$
risch	$-\frac{2i(-315b^6+187a^4b^2+120a^2b^4+8a^6+120a^6e^{6i(dx+c)}+1890b^6e^{2i(dx+c)}-4725b^6e^{8i(dx+c)}-24a^6e^{2i(dx+c)}-315b^6e^{12i}}{}$

input
```
int(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*(3*a^4+20*a^2*b^2+21*b^4)/a^8*ln(a+b*tan(d*x+c))-1/2*(a^4+2*a^2*b^2
+b^4)*b/a^6/(a+b*tan(d*x+c))^2-2*b*(a^4+4*a^2*b^2+3*b^4)/a^7/(a+b*tan(d*x+
c))-1/5/a^3/tan(d*x+c)^5-1/3*(2*a^2+6*b^2)/a^5/tan(d*x+c)^3-(a^4+12*a^2*b^
2+15*b^4)/a^7/tan(d*x+c)+3/4/a^4*b/tan(d*x+c)^4+b*(3*a^2+5*b^2)/a^6/tan(d*
x+c)^2-b*(3*a^4+20*a^2*b^2+21*b^4)/a^8*ln(tan(d*x+c)))
```

3.72.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. 2(257) = 514.

Time = 0.31 (sec) , antiderivative size = 1018, normalized size of antiderivative = 3.84

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(4*(8*a^7 + 187*a^5*b^2 + 120*a^3*b^4 - 315*a*b^6)*cos(d*x + c)^7 - 4
*(20*a^7 + 482*a^5*b^2 + 255*a^3*b^4 - 945*a*b^6)*cos(d*x + c)^5 + 10*(6*a
^7 + 157*a^5*b^2 + 60*a^3*b^4 - 378*a*b^6)*cos(d*x + c)^3 - 30*(13*a^5*b^2
 + 2*a^3*b^4 - 42*a*b^6)*cos(d*x + c) + 30*(2*(3*a^5*b^2 + 20*a^3*b^4 + 21
*a*b^6)*cos(d*x + c)^7 - 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c
)^5 + 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^3 - 2*(3*a^5*b^2
+ 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c) - (3*a^4*b^3 + 20*a^2*b^5 + 21*b^7 +
 (3*a^6*b + 17*a^4*b^3 + a^2*b^5 - 21*b^7)*cos(d*x + c)^6 - (6*a^6*b + 31*
a^4*b^3 - 18*a^2*b^5 - 63*b^7)*cos(d*x + c)^4 + (3*a^6*b + 11*a^4*b^3 - 39
*a^2*b^5 - 63*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*si
n(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 30*(2*(3*a^5*b^2 + 20*a^3
*b^4 + 21*a*b^6)*cos(d*x + c)^7 - 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*co
s(d*x + c)^5 + 6*(3*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c)^3 - 2*(3
*a^5*b^2 + 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c) - (3*a^4*b^3 + 20*a^2*b^5 +
 21*b^7 + (3*a^6*b + 17*a^4*b^3 + a^2*b^5 - 21*b^7)*cos(d*x + c)^6 - (6*a^
6*b + 31*a^4*b^3 - 18*a^2*b^5 - 63*b^7)*cos(d*x + c)^4 + (3*a^6*b + 11*a^4
*b^3 - 39*a^2*b^5 - 63*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/4*cos(d*x
 + c)^2 + 1/4) - (285*a^4*b^3 + 630*a^2*b^5 - 8*(8*a^6*b + 195*a^4*b^3 + 3
15*a^2*b^5)*cos(d*x + c)^6 + 10*(7*a^6*b + 330*a^4*b^3 + 567*a^2*b^5)*cos(
d*x + c)^4 + 15*(a^6*b - 135*a^4*b^3 - 252*a^2*b^5)*cos(d*x + c)^2)*sin...
```

3.72.6 Sympy [F]

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\csc^6\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(csc(c + d*x)**6/(a + b*tan(c + d*x))**3, x)
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.06

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{21\,a^5b\tan(dx+c)-60\,(3\,a^4b^2+20\,a^2b^4+21\,b^6)\tan(dx+c)^6-12\,a^6-90\,(3\,a^5b+20\,a^3b^3+21\,ab^5)\tan(dx+c)^5-20\,(3\,a^6+20\,a^4b^2+21\,a^2b^4)\tan(dx+c)}{a^7b^2\tan(dx+c)^7+2\,a^8b\tan(dx+c)^6+a^9\tan(dx+c)^5}}{}$$

input
```
integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/60*((21*a^5*b*tan(d*x + c) - 60*(3*a^4*b^2 + 20*a^2*b^4 + 21*b^6)*tan(d*
x + c)^6 - 12*a^6 - 90*(3*a^5*b + 20*a^3*b^3 + 21*a*b^5)*tan(d*x + c)^5 -
20*(3*a^6 + 20*a^4*b^2 + 21*a^2*b^4)*tan(d*x + c)^4 + 5*(20*a^5*b + 21*a^3
*b^3)*tan(d*x + c)^3 - 2*(20*a^6 + 21*a^4*b^2)*tan(d*x + c)^2)/(a^7*b^2*ta
n(d*x + c)^7 + 2*a^8*b*tan(d*x + c)^6 + a^9*tan(d*x + c)^5) + 60*(3*a^4*b
+ 20*a^2*b^3 + 21*b^5)*log(b*tan(d*x + c) + a)/a^8 - 60*(3*a^4*b + 20*a^2*
b^3 + 21*b^5)*log(tan(d*x + c))/a^8)/d
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.44

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{60\,(3\,a^4b+20\,a^2b^3+21\,b^5)\log(|\tan(dx+c)|)}{a^8}-\frac{60\,(3\,a^4b^2+20\,a^2b^4+21\,b^6)\log(|b\tan(dx+c)+a|)}{a^8b}+\frac{30\left(9\,a^4b^3\tan(dx+c)^2+60\,a^2b^5\tan(dx+\right.}{}}{}$$

input
```
integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/60*(60*(3*a^4*b + 20*a^2*b^3 + 21*b^5)*log(abs(tan(d*x + c)))/a^8 - 60*
(3*a^4*b^2 + 20*a^2*b^4 + 21*b^6)*log(abs(b*tan(d*x + c) + a))/(a^8*b) + 3
0*(9*a^4*b^3*tan(d*x + c)^2 + 60*a^2*b^5*tan(d*x + c)^2 + 63*b^7*tan(d*x +
 c)^2 + 22*a^5*b^2*tan(d*x + c) + 136*a^3*b^4*tan(d*x + c) + 138*a*b^6*tan
(d*x + c) + 14*a^6*b + 78*a^4*b^3 + 76*a^2*b^5)/((b*tan(d*x + c) + a)^2*a^
8) - (411*a^4*b*tan(d*x + c)^5 + 2740*a^2*b^3*tan(d*x + c)^5 + 2877*b^5*ta
n(d*x + c)^5 - 60*a^5*tan(d*x + c)^4 - 720*a^3*b^2*tan(d*x + c)^4 - 900*a*
b^4*tan(d*x + c)^4 + 180*a^4*b*tan(d*x + c)^3 + 300*a^2*b^3*tan(d*x + c)^3
 - 40*a^5*tan(d*x + c)^2 - 120*a^3*b^2*tan(d*x + c)^2 + 45*a^4*b*tan(d*x +
 c) - 12*a^5)/(a^8*tan(d*x + c)^5))/d
```

3.72.9 Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.12

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{2\,b\,\mathrm{atanh}\left(\frac{b\,(a+2\,b\tan(c+d\,x))\,(3\,a^4+20\,a^2\,b^2+21\,b^4)}{a\,(3\,a^4\,b+20\,a^2\,b^3+21\,b^5)}\right)\,(3\,a^4+20\,a^2\,b^2+21\,b^4)}{a^8\,d}$$
$$-\frac{\frac{1}{5\,a}+\frac{\tan(c+d\,x)^4\,(3\,a^4+20\,a^2\,b^2+21\,b^4)}{3\,a^5}+\frac{\tan(c+d\,x)^2\,(20\,a^2+21\,b^2)}{30\,a^3}-\frac{7\,b\tan(c+d\,x)}{20\,a^2}+\frac{b^2\tan(c+d\,x)^6\,(3\,a^4+20\,a^2\,b^2+21\,b^4)}{a^7}+\frac{3\,b}{}}{d\,\left(a^2\tan\left(c+d\,x\right)^5+2\,a\,b\tan\left(c+d\,x\right)^6+b^2\tan\left(c+d\,x\right)\right.}$$

input
```
int(1/(sin(c + d*x)^6*(a + b*tan(c + d*x))^3),x)
```

output
```
(2*b*atanh((b*(a + 2*b*tan(c + d*x))*(3*a^4 + 21*b^4 + 20*a^2*b^2))/(a*(3*
a^4*b + 21*b^5 + 20*a^2*b^3)))*(3*a^4 + 21*b^4 + 20*a^2*b^2))/(a^8*d) - (1
/(5*a) + (tan(c + d*x)^4*(3*a^4 + 21*b^4 + 20*a^2*b^2))/(3*a^5) + (tan(c +
 d*x)^2*(20*a^2 + 21*b^2))/(30*a^3) - (7*b*tan(c + d*x))/(20*a^2) + (b^2*t
an(c + d*x)^6*(3*a^4 + 21*b^4 + 20*a^2*b^2))/a^7 + (3*b*tan(c + d*x)^5*(3*
a^4 + 21*b^4 + 20*a^2*b^2))/(2*a^6) - (b*tan(c + d*x)^3*(20*a^2 + 21*b^2))
/(12*a^4))/(d*(a^2*tan(c + d*x)^5 + b^2*tan(c + d*x)^7 + 2*a*b*tan(c + d*x
)^6))
```

3.73 $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

3.73.1 Optimal result

Integrand size = 21, antiderivative size = 366

$$\begin{aligned}\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx &= \frac{(3a^8-132a^6b^2+370a^4b^4-132a^2b^6+3b^8)\,x}{8\,(a^2+b^2)^6} \\ &+ \frac{4ab(a^2-b^2)\,(a^4-8a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^6\,d} \\ &- \frac{a^4b}{3\,(a^2+b^2)^3\,d(a+b\tan(c+dx))^3} \\ &- \frac{a^3b(a^2-2b^2)}{(a^2+b^2)^4\,d(a+b\tan(c+dx))^2} - \frac{3a^2b(a^4-5a^2b^2+2b^4)}{(a^2+b^2)^5\,d(a+b\tan(c+dx))} \\ &+ \frac{\cos^4(c+dx)\,(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx))}{4\,(a^2+b^2)^4\,d} \\ &- \frac{\cos^2(c+dx)\,(16ab(2a^4-5a^2b^2+b^4)+(5a^6-65a^4b^2+55a^2b^4-3b^6)\tan(c+dx))}{8\,(a^2+b^2)^5\,d}\end{aligned}$$

output
```
1/8*(3*a^8-132*a^6*b^2+370*a^4*b^4-132*a^2*b^6+3*b^8)*x/(a^2+b^2)^6+4*a*b*
(a^2-b^2)*(a^4-8*a^2*b^2+b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^6/d-
1/3*a^4*b/(a^2+b^2)^3/d/(a+b*tan(d*x+c))^3-a^3*b*(a^2-2*b^2)/(a^2+b^2)^4/d
/(a+b*tan(d*x+c))^2-3*a^2*b*(a^4-5*a^2*b^2+2*b^4)/(a^2+b^2)^5/d/(a+b*tan(d
*x+c))+1/4*cos(d*x+c)^4*(4*a*b*(a^2-b^2)+(a^4-6*a^2*b^2+b^4)*tan(d*x+c))/(
a^2+b^2)^4/d-1/8*cos(d*x+c)^2*(16*a*b*(2*a^4-5*a^2*b^2+b^4)+(5*a^6-65*a^4*
b^2+55*a^2*b^4-3*b^6)*tan(d*x+c))/(a^2+b^2)^5/d
```

3.73.2 Mathematica [A] (verified)

Time = 5.76 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.61

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{b\left(-\frac{9(a^2+b^2)^2(a^4-6a^2b^2+b^4)\arctan(\tan(c+dx))}{b}+\frac{24a^2(a^2+b^2)(a^4-10a^2b^2+5b^4)\arctan(\tan(c+dx))}{b}+48a(a^2+b^2)(2a^4-5\cdots\right.}{\cdots}$$

input
```
Integrate[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

output
```
-1/24*(b*((-9*(a^2 + b^2)^2*(a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/
b + (24*a^2*(a^2 + b^2)*(a^4 - 10*a^2*b^2 + 5*b^4)*ArcTan[Tan[c + d*x]])/b
 + 48*a*(a^2 + b^2)*(2*a^4 - 5*a^2*b^2 + b^4)*Cos[c + d*x]^2 - 24*a*(a - b
)*(a + b)*(a^2 + b^2)^2*Cos[c + d*x]^4 + 12*a*(4*a^6 - 36*a^4*b^2 + 36*a^2
*b^4 - 4*b^6 + (-a^7 + 24*a^5*b^2 - 45*a^3*b^4 + 10*a*b^6)/Sqrt[-b^2])*Log
[Sqrt[-b^2] - b*Tan[c + d*x]] - 96*a*(a - b)*(a + b)*(a^4 - 8*a^2*b^2 + b^
4)*Log[a + b*Tan[c + d*x]] + 12*a*(4*a^6 - 36*a^4*b^2 + 36*a^2*b^4 - 4*b^6
 + (a^7 - 24*a^5*b^2 + 45*a^3*b^4 - 10*a*b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] +
 b*Tan[c + d*x]] - (6*(a^2 + b^2)^2*(a^4 - 6*a^2*b^2 + b^4)*Cos[c + d*x]^3
*Sin[c + d*x])/b - (9*(a^2 + b^2)^2*(a^4 - 6*a^2*b^2 + b^4)*Sin[2*(c + d*x
)])/(2*b) + (12*a^2*(a^2 + b^2)*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[2*(c + d*x)
])/b + (8*a^4*(a^2 + b^2)^3)/(a + b*Tan[c + d*x])^3 + (24*a^3*(a^2 - 2*b^2
)*(a^2 + b^2)^2)/(a + b*Tan[c + d*x])^2 + (72*a^2*(a^2 + b^2)*(a^4 - 5*a^2
*b^2 + 2*b^4))/(a + b*Tan[c + d*x])))/((a^2 + b^2)^6*d)
```

3.73.3 Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3999, 601, 2178, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^4}{(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \; 3999$$

$$\frac{b\int \frac{b^4\tan^4(c+dx)}{(a+b\tan(c+dx))^4(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 601$$

$$\frac{b\left(\frac{b^2\left(4ab^2\left(a^2-b^2\right)+b\left(a^4-6a^2b^2+b^4\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^4\left(b^2\tan^2(c+dx)+b^2\right)^2}-\frac{\int \frac{-\frac{3\left(a^4-6b^2a^2+b^4\right)\tan^4(c+dx)b^8}{\left(a^2+b^2\right)^4}-\frac{4a\left(3a^4-14b^2a^2-b^4\right)\tan^3(c+dx)b^7}{\left(a^2+b^2\right)^4}-\frac{4a^3\left(3a^2-b^2\right)\tan(c+d}{\left(a^2+b^2\right)^3}}{(a+b\tan(c+dx))^4\left(\tan^2(c+da\right.}}{4b^2}\right.}{d}$$

$$\Big\downarrow \; 2178$$

$$b\left(\frac{b^2\left(4ab^2\left(a^2-b^2\right)+b\left(a^4-6a^2b^2+b^4\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^4\left(b^2\tan^2(c+dx)+b^2\right)^2}-\frac{\frac{b^2\left(16ab^2\left(2a^4-5a^2b^2+b^4\right)+b\left(5a^6-65a^4b^2+55a^2b^4-3b^6\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^5\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{\int \frac{-\frac{\left(5a^6-65b^2a^4+55b^4a\right.}{\left(a^2+\right.}}{}}{}}{}\right.$$

$$\Big\downarrow \; 2160$$

$$b\left(\frac{b^2\left(4ab^2\left(a^2-b^2\right)+b\left(a^4-6a^2b^2+b^4\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^4\left(b^2\tan^2(c+dx)+b^2\right)^2}-\frac{\frac{b^2\left(16ab^2\left(2a^4-5a^2b^2+b^4\right)+b\left(5a^6-65a^4b^2+55a^2b^4-3b^6\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^5\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{\int\left(\frac{32a\left(a^2-b^2\right)\left(a^4-8b^2\right.}{\left(a^2+b^2\right)^6(a+b\tan(}\right.}{}}{}\right.$$

$$\Big\downarrow \; 2009$$

$$b\left(\frac{b^2\left(4ab^2\left(a^2-b^2\right)+b\left(a^4-6a^2b^2+b^4\right)\tan(c+dx)\right)}{4\left(a^2+b^2\right)^4\left(b^2\tan^2(c+dx)+b^2\right)^2}-\frac{\frac{b^2\left(16ab^2\left(2a^4-5a^2b^2+b^4\right)+b\left(5a^6-65a^4b^2+55a^2b^4-3b^6\right)\tan(c+dx)\right)}{2\left(a^2+b^2\right)^5\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{-\frac{24a^2b^4\left(a^4-5a^2b^2+2b\right.}{\left(a^2+b^2\right)^5(a+b\tan(c+d}}{}}{}\right.$$

3.73. $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

input
```
Int[Sin[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

output
```
(b*((b^2*(4*a*b^2*(a^2 - b^2) + b*(a^4 - 6*a^2*b^2 + b^4)*Tan[c + d*x]))/(
4*(a^2 + b^2)^4*(b^2 + b^2*Tan[c + d*x]^2)^2) - ((b^2*(16*a*b^2*(2*a^4 - 5
*a^2*b^2 + b^4) + b*(5*a^6 - 65*a^4*b^2 + 55*a^2*b^4 - 3*b^6)*Tan[c + d*x]
))/(2*(a^2 + b^2)^5*(b^2 + b^2*Tan[c + d*x]^2)) - ((b^3*(3*a^8 - 132*a^6*b
^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*ArcTan[Tan[c + d*x]])/(a^2 + b^2)^
6 + (32*a*b^4*(a^2 - b^2)*(a^4 - 8*a^2*b^2 + b^4)*Log[a + b*Tan[c + d*x]])
/(a^2 + b^2)^6 - (16*a*b^4*(a^2 - b^2)*(a^4 - 8*a^2*b^2 + b^4)*Log[b^2 + b
^2*Tan[c + d*x]^2])/(a^2 + b^2)^6 - (8*a^4*b^4)/(3*(a^2 + b^2)^3*(a + b*Ta
n[c + d*x])^3) - (8*a^3*b^4*(a^2 - 2*b^2))/((a^2 + b^2)^4*(a + b*Tan[c + d
*x])^2) - (24*a^2*b^4*(a^4 - 5*a^2*b^2 + 2*b^4))/((a^2 + b^2)^5*(a + b*Tan
[c + d*x]))/(2*b^2))/(4*b^2)))/d
```

3.73.3.1 Defintions of rubi rules used

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.73.4 Maple [A] (verified)

Time = 55.09 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\frac{\left(-\frac{5}{8}a^8+\frac{15}{2}a^6b^2+\frac{5}{4}a^4b^4-\frac{13}{2}b^6a^2+\frac{3}{8}b^8\right)\left(\tan^3(dx+c)\right)+\left(-4b\,a^7+6b^3a^5+8a^3b^5-2a\,b^7\right)\left(\tan^2(dx+c)\right)+\left(-\frac{3}{8}a^8+\frac{13}{2}a^6b^2-\frac{15}{2}b^6a^2\right.}{\left(1+\tan^2(dx+c)\right)^2}}{}$
default	$\frac{\frac{\left(-\frac{5}{8}a^8+\frac{15}{2}a^6b^2+\frac{5}{4}a^4b^4-\frac{13}{2}b^6a^2+\frac{3}{8}b^8\right)\left(\tan^3(dx+c)\right)+\left(-4b\,a^7+6b^3a^5+8a^3b^5-2a\,b^7\right)\left(\tan^2(dx+c)\right)+\left(-\frac{3}{8}a^8+\frac{13}{2}a^6b^2-\frac{15}{2}b^6a^2\right.}{\left(1+\tan^2(dx+c)\right)^2}}{}$
risch	Expression too large to display

input
```
int(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^6*(((-5/8*a^8+15/2*a^6*b^2+5/4*a^4*b^4-13/2*b^6*a^2+3/8*b
^8)*tan(d*x+c)^3+(-4*a^7*b+6*a^5*b^3+8*a^3*b^5-2*a*b^7)*tan(d*x+c)^2+(-3/8
*a^8+13/2*a^6*b^2-15/2*b^6*a^2+5/8*b^8-5/4*a^4*b^4)*tan(d*x+c)-3*b*a^7+7*b
^3*a^5+7*a^3*b^5-3*a*b^7)/(1+tan(d*x+c)^2)^2+1/16*(-32*a^7*b+288*a^5*b^3-2
88*a^3*b^5+32*a*b^7)*ln(1+tan(d*x+c)^2)+1/8*(3*a^8-132*a^6*b^2+370*a^4*b^4
-132*a^2*b^6+3*b^8)*arctan(tan(d*x+c)))-1/3*a^4*b/(a^2+b^2)^3/(a+b*tan(d*x
+c))^3-3*a^2*b*(a^4-5*a^2*b^2+2*b^4)/(a^2+b^2)^5/(a+b*tan(d*x+c))-a^3*b*(a
^2-2*b^2)/(a^2+b^2)^4/(a+b*tan(d*x+c))^2+4*b*a*(a^6-9*a^4*b^2+9*a^2*b^4-b^
6)/(a^2+b^2)^6*ln(a+b*tan(d*x+c)))
```

3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1053 vs. $2(358) = 716$.

Time = 0.37 (sec) , antiderivative size = 1053, normalized size of antiderivative = 2.88

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*(6*(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11)*
cos(d*x + c)^7 - 3*(11*a^10*b + 45*a^8*b^3 + 70*a^6*b^5 + 50*a^4*b^7 + 15*
a^2*b^9 + b^11)*cos(d*x + c)^5 - (6*a^10*b + 342*a^8*b^3 - 1830*a^6*b^5 +
614*a^4*b^7 - 216*a^2*b^9 + 12*b^11 - 3*(3*a^11 - 141*a^9*b^2 + 766*a^7*b^
4 - 1242*a^5*b^6 + 399*a^3*b^8 - 9*a*b^10)*d*x)*cos(d*x + c)^3 + 3*(114*a^
8*b^3 - 381*a^6*b^5 + 187*a^4*b^7 - 67*a^2*b^9 + 3*b^11 + 3*(3*a^9*b^2 - 1
32*a^7*b^4 + 370*a^5*b^6 - 132*a^3*b^8 + 3*a*b^10)*d*x)*cos(d*x + c) + 48*
((a^10*b - 12*a^8*b^3 + 36*a^6*b^5 - 28*a^4*b^7 + 3*a^2*b^9)*cos(d*x + c)^
3 + 3*(a^8*b^3 - 9*a^6*b^5 + 9*a^4*b^7 - a^2*b^9)*cos(d*x + c) + (a^7*b^4
- 9*a^5*b^6 + 9*a^3*b^8 - a*b^10 + (3*a^9*b^2 - 28*a^7*b^4 + 36*a^5*b^6 -
12*a^3*b^8 + a*b^10)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*
sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) + (143*a^7*b^4 - 537*a^5*
b^6 + 105*a^3*b^8 + 33*a*b^10 + 6*(a^11 + 5*a^9*b^2 + 10*a^7*b^4 + 10*a^5*
b^6 + 5*a^3*b^8 + a*b^10)*cos(d*x + c)^6 - 15*(a^11 + 3*a^9*b^2 + 2*a^7*b^
4 - 2*a^5*b^6 - 3*a^3*b^8 - a*b^10)*cos(d*x + c)^4 + 3*(3*a^8*b^3 - 132*a^
6*b^5 + 370*a^4*b^7 - 132*a^2*b^9 + 3*b^11)*d*x + (216*a^9*b^2 - 734*a^7*b
^4 + 1590*a^5*b^6 - 522*a^3*b^8 - 54*a*b^10 + 3*(9*a^10*b - 399*a^8*b^3 +
1242*a^6*b^5 - 766*a^4*b^7 + 141*a^2*b^9 - 3*b^11)*d*x)*cos(d*x + c)^2)*si
n(d*x + c))/((a^15 + 3*a^13*b^2 - 3*a^11*b^4 - 25*a^9*b^6 - 45*a^7*b^8 - 3
9*a^5*b^10 - 17*a^3*b^12 - 3*a*b^14)*d*cos(d*x + c)^3 + 3*(a^13*b^2 + 6...
```

3.73.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(sin(d*x+c)**4/(a+b*tan(d*x+c))**4,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.73.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. 2(358) = 716.

Time = 0.42 (sec) , antiderivative size = 997, normalized size of antiderivative = 2.72

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{\frac{3\,(3\,a^8-132\,a^6b^2+370\,a^4b^4-132\,a^2b^6+3\,b^8)(dx+c)}{a^{12}+6\,a^{10}b^2+15\,a^8b^4+20\,a^6b^6+15\,a^4b^8+6\,a^2b^{10}+b^{12}} + \frac{96\,(a^7b-9\,a^5b^3+9\,a^3b^5-ab^7)\log(b\tan(dx+c)+a)}{a^{12}+6\,a^{10}b^2+15\,a^8b^4+20\,a^6b^6+15\,a^4b^8+6\,a^2b^{10}+b^{12}} - \frac{48\,(a^7b-9\,a^5b^3+9\,a^3b^5-c}{a^{12}+6\,a^{10}b^2+15\,a^8b^4+20\,a}}{}$$

input
```
integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/24*(3*(3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*(d*x + c
)/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
 b^12) + 96*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*log(b*tan(d*x + c) + a
)/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
 b^12) - 48*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*log(tan(d*x + c)^2 + 1
)/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
 b^12) - (176*a^8*b - 608*a^6*b^3 + 176*a^4*b^5 + 3*(29*a^6*b^3 - 185*a^4*
b^5 + 103*a^2*b^7 - 3*b^9)*tan(d*x + c)^6 + 3*(71*a^7*b^2 - 411*a^5*b^4 +
165*a^3*b^6 + 7*a*b^8)*tan(d*x + c)^5 + (149*a^8*b - 512*a^6*b^3 - 1006*a^
4*b^5 + 600*a^2*b^7 - 15*b^9)*tan(d*x + c)^4 + 3*(5*a^9 + 152*a^7*b^2 - 82
2*a^5*b^4 + 320*a^3*b^6 + 9*a*b^8)*tan(d*x + c)^3 + (331*a^8*b - 1183*a^6*
b^3 - 239*a^4*b^5 + 315*a^2*b^7)*tan(d*x + c)^2 + 3*(3*a^9 + 73*a^7*b^2 -
423*a^5*b^4 + 147*a^3*b^6)*tan(d*x + c))/(a^13 + 5*a^11*b^2 + 10*a^9*b^4 +
 10*a^7*b^6 + 5*a^5*b^8 + a^3*b^10 + (a^10*b^3 + 5*a^8*b^5 + 10*a^6*b^7 +
10*a^4*b^9 + 5*a^2*b^11 + b^13)*tan(d*x + c)^7 + 3*(a^11*b^2 + 5*a^9*b^4 +
 10*a^7*b^6 + 10*a^5*b^8 + 5*a^3*b^10 + a*b^12)*tan(d*x + c)^6 + (3*a^12*b
 + 17*a^10*b^3 + 40*a^8*b^5 + 50*a^6*b^7 + 35*a^4*b^9 + 13*a^2*b^11 + 2*b^
13)*tan(d*x + c)^5 + (a^13 + 11*a^11*b^2 + 40*a^9*b^4 + 70*a^7*b^6 + 65*a^
5*b^8 + 31*a^3*b^10 + 6*a*b^12)*tan(d*x + c)^4 + (6*a^12*b + 31*a^10*b^3 +
 65*a^8*b^5 + 70*a^6*b^7 + 40*a^4*b^9 + 11*a^2*b^11 + b^13)*tan(d*x + c...
```

3.73.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(358) = 716$.

Time = 0.91 (sec) , antiderivative size = 902, normalized size of antiderivative = 2.46

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

$$= \frac{\frac{3\left(3a^8-132a^6b^2+370a^4b^4-132a^2b^6+3b^8\right)(dx+c)}{a^{12}+6a^{10}b^2+15a^8b^4+20a^6b^6+15a^4b^8+6a^2b^{10}+b^{12}} - \frac{48\left(a^7b-9a^5b^3+9a^3b^5-ab^7\right)\log\left(\tan(dx+c)^2+1\right)}{a^{12}+6a^{10}b^2+15a^8b^4+20a^6b^6+15a^4b^8+6a^2b^{10}+b^{12}} + \frac{96\left(a^7b^2-9a^5b^4+9a^3b^6-\cdots\right.}{a^{12}b+6a^{10}b^3+15a^8b^5+20\cdots}}{}$$

input
```
integrate(sin(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/24*(3*(3*a^8 - 132*a^6*b^2 + 370*a^4*b^4 - 132*a^2*b^6 + 3*b^8)*(d*x + c
)/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
 b^12) - 48*(a^7*b - 9*a^5*b^3 + 9*a^3*b^5 - a*b^7)*log(tan(d*x + c)^2 + 1
)/(a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 +
 b^12) + 96*(a^7*b^2 - 9*a^5*b^4 + 9*a^3*b^6 - a*b^8)*log(abs(b*tan(d*x +
c) + a))/(a^12*b + 6*a^10*b^3 + 15*a^8*b^5 + 20*a^6*b^7 + 15*a^4*b^9 + 6*a
^2*b^11 + b^13) + 3*(24*a^7*b*tan(d*x + c)^4 - 216*a^5*b^3*tan(d*x + c)^4
+ 216*a^3*b^5*tan(d*x + c)^4 - 24*a*b^7*tan(d*x + c)^4 - 5*a^8*tan(d*x + c
)^3 + 60*a^6*b^2*tan(d*x + c)^3 + 10*a^4*b^4*tan(d*x + c)^3 - 52*a^2*b^6*t
an(d*x + c)^3 + 3*b^8*tan(d*x + c)^3 + 16*a^7*b*tan(d*x + c)^2 - 384*a^5*b
^3*tan(d*x + c)^2 + 496*a^3*b^5*tan(d*x + c)^2 - 64*a*b^7*tan(d*x + c)^2 -
 3*a^8*tan(d*x + c) + 52*a^6*b^2*tan(d*x + c) - 10*a^4*b^4*tan(d*x + c) -
60*a^2*b^6*tan(d*x + c) + 5*b^8*tan(d*x + c) - 160*a^5*b^3 + 272*a^3*b^5 -
 48*a*b^7)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*
a^2*b^10 + b^12)*(tan(d*x + c)^2 + 1)^2) - 8*(22*a^7*b^4*tan(d*x + c)^3 -
198*a^5*b^6*tan(d*x + c)^3 + 198*a^3*b^8*tan(d*x + c)^3 - 22*a*b^10*tan(d*
x + c)^3 + 75*a^8*b^3*tan(d*x + c)^2 - 630*a^6*b^5*tan(d*x + c)^2 + 567*a^
4*b^7*tan(d*x + c)^2 - 48*a^2*b^9*tan(d*x + c)^2 + 87*a^9*b^2*tan(d*x + c)
 - 666*a^7*b^4*tan(d*x + c) + 531*a^5*b^6*tan(d*x + c) - 36*a^3*b^8*tan(d*
x + c) + 35*a^10*b - 231*a^8*b^3 + 165*a^6*b^5 - 9*a^4*b^7)/((a^12 + 6*...
```

3.73.9 Mupad [B] (verification not implemented)

Time = 7.61 (sec) , antiderivative size = 962, normalized size of antiderivative = 2.63

$$\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{\ln(a+b\tan(c+dx))\left(\frac{4ab}{(a^2+b^2)^3}-\frac{48ab^3}{(a^2+b^2)^4}+\frac{120ab^5}{(a^2+b^2)^5}-\frac{80ab^7}{(a^2+b^2)^6}\right)}{d}$$
$$-\frac{\frac{2(11a^8b-38a^6b^3+11a^4b^5)}{3(a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10})}-\frac{\tan(c+dx)^6(-29a^6b^3+185a^4b^5-103a^2b^7+3b^9)}{8(a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10})}+\frac{\tan(c+dx)^5(71a^7b^2-411a^5b^4+}{8(a^{10}+5a^8b^2+10a^6b^4+10a^4}}{d\left(\tan(c+dx)^2(2a^3+3ab^2)+\tan(c+dx)^5(\right.}$$
$$+\frac{\ln(\tan(c+dx)-\mathrm{i})(-3a^2+ab\,14\mathrm{i}+3b^2)}{16d(-a^6\,1\mathrm{i}+6a^5b+a^4b^2\,15\mathrm{i}-20a^3b^3-a^2b^4\,15\mathrm{i}+6ab^5+b^6\,1\mathrm{i})}$$
$$-\frac{\ln(\tan(c+dx)+1\mathrm{i})(3a^2+ab\,14\mathrm{i}-3b^2)}{16d(a^6\,1\mathrm{i}+6a^5b-a^4b^2\,15\mathrm{i}-20a^3b^3+a^2b^4\,15\mathrm{i}+6ab^5-b^6\,1\mathrm{i})}$$

input
```
int(sin(c + d*x)^4/(a + b*tan(c + d*x))^4,x)
```

output
```
(log(a + b*tan(c + d*x))*((4*a*b)/(a^2 + b^2)^3 - (48*a*b^3)/(a^2 + b^2)^4
 + (120*a*b^5)/(a^2 + b^2)^5 - (80*a*b^7)/(a^2 + b^2)^6))/d - ((2*(11*a^8*
b + 11*a^4*b^5 - 38*a^6*b^3))/(3*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 1
0*a^6*b^4 + 5*a^8*b^2)) - (tan(c + d*x)^6*(3*b^9 - 103*a^2*b^7 + 185*a^4*b
^5 - 29*a^6*b^3))/(8*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 +
5*a^8*b^2)) + (tan(c + d*x)^5*(7*a*b^8 + 165*a^3*b^6 - 411*a^5*b^4 + 71*a^
7*b^2))/(8*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)
) + (tan(c + d*x)^2*(331*a^8*b + 315*a^2*b^7 - 239*a^4*b^5 - 1183*a^6*b^3)
)/(24*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) + (
tan(c + d*x)^3*(9*a*b^8 + 5*a^9 + 320*a^3*b^6 - 822*a^5*b^4 + 152*a^7*b^2)
)/(8*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)) - (t
an(c + d*x)^4*(15*b^9 - 149*a^8*b - 600*a^2*b^7 + 1006*a^4*b^5 + 512*a^6*b
^3))/(24*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))
+ (a*tan(c + d*x)*(3*a^8 + 147*a^2*b^6 - 423*a^4*b^4 + 73*a^6*b^2))/(8*(a^
10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2)))/(d*(tan(c +
 d*x)^2*(3*a*b^2 + 2*a^3) + tan(c + d*x)^5*(3*a^2*b + 2*b^3) + a^3 + tan(c
 + d*x)^4*(6*a*b^2 + a^3) + tan(c + d*x)^3*(6*a^2*b + b^3) + b^3*tan(c + d
*x)^7 + 3*a*b^2*tan(c + d*x)^6 + 3*a^2*b*tan(c + d*x))) + (log(tan(c + d*x
) - 1i)*(a*b*14i - 3*a^2 + 3*b^2))/(16*d*(6*a*b^5 + 6*a^5*b - a^6*1i + b^6
*1i - a^2*b^4*15i - 20*a^3*b^3 + a^4*b^2*15i)) - (log(tan(c + d*x) + 1i...
```

3.73. $\int \frac{\sin^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

3.74 $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

3.74.1 Optimal result

Integrand size = 21, antiderivative size = 264

$$\begin{aligned}\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx &= \frac{(a^6-25a^4b^2+35a^2b^4-3b^6)\,x}{2\,(a^2+b^2)^5} \\ &+ \frac{4ab(a^4-5a^2b^2+2b^4)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^5\,d} \\ &- \frac{a^2b}{3\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^3} \\ &- \frac{ab(a^2-b^2)}{(a^2+b^2)^3\,d(a+b\tan(c+dx))^2} \\ &- \frac{b(3a^4-8a^2b^2+b^4)}{(a^2+b^2)^4\,d(a+b\tan(c+dx))} \\ &- \frac{\cos^2(c+dx)\,(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\tan(c+dx))}{2\,(a^2+b^2)^4\,d}\end{aligned}$$

output
```
1/2*(a^6-25*a^4*b^2+35*a^2*b^4-3*b^6)*x/(a^2+b^2)^5+4*a*b*(a^4-5*a^2*b^2+2
*b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^5/d-1/3*a^2*b/(a^2+b^2)^2/d/
(a+b*tan(d*x+c))^3-a*b*(a^2-b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))^2-b*(3*a^4
-8*a^2*b^2+b^4)/(a^2+b^2)^4/d/(a+b*tan(d*x+c))-1/2*cos(d*x+c)^2*(4*a*b*(a^
2-b^2)+(a^4-6*a^2*b^2+b^4)*tan(d*x+c))/(a^2+b^2)^4/d
```

3.74.2 Mathematica [A] (verified)

Time = 3.95 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.50

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx =$$
$$-\frac{b\left(\frac{3(a^2+b^2)(a^4-6a^2b^2+b^4)\arctan(\tan(c+dx))}{b}+12a(a-b)(a+b)\left(a^2+b^2\right)\cos^2(c+dx)+3\left(4a^5-20a^3b^2+8ab\right.}{}$$

input
```
Integrate[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

output
```
-1/6*(b*((3*(a^2 + b^2)*(a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[c + d*x]])/b +
12*a*(a - b)*(a + b)*(a^2 + b^2)*Cos[c + d*x]^2 + 3*(4*a^5 - 20*a^3*b^2 +
8*a*b^4 + (-a^6 + 15*a^4*b^2 - 15*a^2*b^4 + b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2
] - b*Tan[c + d*x]] - 24*a*(a^4 - 5*a^2*b^2 + 2*b^4)*Log[a + b*Tan[c + d*x
]] + 3*(4*a^5 - 20*a^3*b^2 + 8*a*b^4 + (a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^
6)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (3*(a^2 + b^2)*(a^4 - 6*
a^2*b^2 + b^4)*Sin[2*(c + d*x)])/(2*b) + (2*a^2*(a^2 + b^2)^3)/(a + b*Tan[
c + d*x])^3 + (6*a*(a - b)*(a + b)*(a^2 + b^2)^2)/(a + b*Tan[c + d*x])^2 +
 (6*(a^2 + b^2)*(3*a^4 - 8*a^2*b^2 + b^4))/(a + b*Tan[c + d*x]))/((a^2 +
b^2)^5*d)
```

3.74.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3999, 601, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(c+dx)^2}{(a+b\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{ 3999}$$

$$\frac{b\int \frac{b^2\tan^2(c+dx)}{(a+b\tan(c+dx))^4(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 601$$

$$\frac{b\left(-\frac{\int -\frac{-\frac{\left(a^4-6b^2a^2+b^4\right)\tan^4(c+dx)b^6}{\left(a^2+b^2\right)^4}-\frac{4a\left(a^4-4b^2a^2-b^4\right)\tan^3(c+dx)b^5}{\left(a^2+b^2\right)^4}-\frac{2\left(3a^4-6b^2a^2-b^4\right)\tan^2(c+dx)b^4}{\left(a^2+b^2\right)^3}-\frac{4a^3\left(a^4+4b^2a^2-b^4\right)\tan(c+dx)b^3}{\left(a^2+b^2\right)^4}+\frac{a^4\left(a^4 \cdots\right.}{\left(\cdots\right.}}{(a+b\tan(c+dx))^4\left(\tan^2(c+dx)b^2+b^2\right)}}{2b^2}\cdots\right.}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{\int \frac{-\frac{\left(a^4-6b^2a^2+b^4\right)\tan^4(c+dx)b^6}{\left(a^2+b^2\right)^4}-\frac{4a\left(a^4-4b^2a^2-b^4\right)\tan^3(c+dx)b^5}{\left(a^2+b^2\right)^4}-\frac{2\left(3a^4-6b^2a^2-b^4\right)\tan^2(c+dx)b^4}{\left(a^2+b^2\right)^3}-\frac{4a^3\left(a^4+4b^2a^2-b^4\right)\tan(c+dx)b^3}{\left(a^2+b^2\right)^4}+\frac{a^4\left(a^4-6b^2 \cdots\right.}{\left(a^2+\cdots\right.}}{(a+b\tan(c+dx))^4\left(\tan^2(c+dx)b^2+b^2\right)}}{2b^2}\cdots\right.}{d}$$

$$\Big\downarrow \; 2160$$

$$\frac{b\left(\frac{\int\left(\frac{8a\left(a^4-5b^2a^2+2b^4\right)b^2}{\left(a^2+b^2\right)^5(a+b\tan(c+dx))}+\frac{\left(a^6-25b^2a^4+35b^4a^2-8b\left(a^4-5b^2a^2+2b^4\right)\tan(c+dx)a-3b^6\right)b^2}{\left(a^2+b^2\right)^5\left(\tan^2(c+dx)b^2+b^2\right)}+\frac{4a\left(a^2-b^2\right)b^2}{\left(a^2+b^2\right)^3(a+b\tan(c+dx))^3}+\frac{2a^2b^2}{\left(a^2+b^2\right)^2(a+b\tan(c+dx)\cdots}\cdots\right.}{2b^2}\cdots\right.}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\left(\frac{-\frac{2ab^2\left(a^2-b^2\right)}{\left(a^2+b^2\right)^3(a+b\tan(c+dx))^2}-\frac{2a^2b^2}{3\left(a^2+b^2\right)^2(a+b\tan(c+dx))^3}-\frac{2b^2\left(3a^4-8a^2b^2+b^4\right)}{\left(a^2+b^2\right)^4(a+b\tan(c+dx))}-\frac{4ab^2\left(a^4-5a^2b^2+2b^4\right)\log\left(b^2\tan^2(c+dx)+b^2\right)}{\left(a^2+b^2\right)^5}+\frac{8ab^2\left(a^4-5a^2\cdots\right.}{\cdots}}{2b^2}\cdots\right.}{d}$$

input
```
Int[Sin[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

3.74. $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

output
```
(b*(-1/2*(4*a*b^2*(a^2 - b^2) + b*(a^4 - 6*a^2*b^2 + b^4)*Tan[c + d*x])/((
a^2 + b^2)^4*(b^2 + b^2*Tan[c + d*x]^2)) + ((b*(a^6 - 25*a^4*b^2 + 35*a^2*
b^4 - 3*b^6)*ArcTan[Tan[c + d*x]])/(a^2 + b^2)^5 + (8*a*b^2*(a^4 - 5*a^2*b
^2 + 2*b^4)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^5 - (4*a*b^2*(a^4 - 5*a^2
*b^2 + 2*b^4)*Log[b^2 + b^2*Tan[c + d*x]^2])/(a^2 + b^2)^5 - (2*a^2*b^2)/(
3*(a^2 + b^2)^2*(a + b*Tan[c + d*x])^3) - (2*a*b^2*(a^2 - b^2))/((a^2 + b^
2)^3*(a + b*Tan[c + d*x])^2) - (2*b^2*(3*a^4 - 8*a^2*b^2 + b^4))/((a^2 + b
^2)^4*(a + b*Tan[c + d*x]))))/(2*b^2)))/d
```

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.74. $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4} \, dx$

3.74.4 Maple [A] (verified)

Time = 17.84 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{\left(-\frac{1}{2}a^6+\frac{5}{2}a^4b^2+\frac{5}{2}a^2b^4-\frac{1}{2}b^6\right)\tan(dx+c)-2a^5b+2a\,b^5}{1+\tan^2(dx+c)}+\frac{\left(-8a^5b+40a^3b^3-16a\,b^5\right)\ln\left(1+\tan^2(dx+c)\right)}{4}+\frac{\left(a^6-25a^4b^2+35a^2b^4-3b^6\right.}{2}}{\left(a^2+b^2\right)^5}$
default	$\frac{\frac{\left(-\frac{1}{2}a^6+\frac{5}{2}a^4b^2+\frac{5}{2}a^2b^4-\frac{1}{2}b^6\right)\tan(dx+c)-2a^5b+2a\,b^5}{1+\tan^2(dx+c)}+\frac{\left(-8a^5b+40a^3b^3-16a\,b^5\right)\ln\left(1+\tan^2(dx+c)\right)}{4}+\frac{\left(a^6-25a^4b^2+35a^2b^4-3b^6\right.}{2}}{\left(a^2+b^2\right)^5}$
risch	$-\frac{3ixb}{2(5ia^4b-10ia^2b^3+ib^5-a^5+10a^3b^2-5a\,b^4)}-\frac{xa}{2(5ia^4b-10ia^2b^3+ib^5-a^5+10a^3b^2-5a\,b^4)}+\frac{i\mathrm{e}^{2i(dx+c)}}{8(-4ia^3b+4ia\,b^3+a^4-}$

input
```
int(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^5*(((-1/2*a^6+5/2*a^4*b^2+5/2*a^2*b^4-1/2*b^6)*tan(d*x+c)
-2*a^5*b+2*a*b^5)/(1+tan(d*x+c)^2)+1/4*(-8*a^5*b+40*a^3*b^3-16*a*b^5)*ln(1
+tan(d*x+c)^2)+1/2*(a^6-25*a^4*b^2+35*a^2*b^4-3*b^6)*arctan(tan(d*x+c)))-1
/3*a^2*b/(a^2+b^2)^2/(a+b*tan(d*x+c))^3-b*(3*a^4-8*a^2*b^2+b^4)/(a^2+b^2)^
4/(a+b*tan(d*x+c))-a*b*(a^2-b^2)/(a^2+b^2)^3/(a+b*tan(d*x+c))^2+4*a*b*(a^4
-5*a^2*b^2+2*b^4)/(a^2+b^2)^5*ln(a+b*tan(d*x+c)))
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(258) = 516$.

Time = 0.33 (sec) , antiderivative size = 802, normalized size of antiderivative = 3.04

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx =$$
$$-\frac{3\left(a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9\right)\cos\left(dx+c\right)^5+\left(3\,a^8b+111\,a^6b^3-231\,a^4b^5+65\,a^2b^7-12\,b^9-3\right.}{}$$

input
```
integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/6*(3*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^5 +
 (3*a^8*b + 111*a^6*b^3 - 231*a^4*b^5 + 65*a^2*b^7 - 12*b^9 - 3*(a^9 - 28*
a^7*b^2 + 110*a^5*b^4 - 108*a^3*b^6 + 9*a*b^8)*d*x)*cos(d*x + c)^3 - 3*(25
*a^6*b^3 - 51*a^4*b^5 + 25*a^2*b^7 - 3*b^9 + 3*(a^7*b^2 - 25*a^5*b^4 + 35*
a^3*b^6 - 3*a*b^8)*d*x)*cos(d*x + c) - 12*((a^8*b - 8*a^6*b^3 + 17*a^4*b^5
 - 6*a^2*b^7)*cos(d*x + c)^3 + 3*(a^6*b^3 - 5*a^4*b^5 + 2*a^2*b^7)*cos(d*x
 + c) + (a^5*b^4 - 5*a^3*b^6 + 2*a*b^8 + (3*a^7*b^2 - 16*a^5*b^4 + 11*a^3*
b^6 - 2*a*b^8)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*
x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (32*a^5*b^4 - 66*a^3*b^6 + 6*
a*b^8 - 3*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cos(d*x + c)^4
 + 3*(a^6*b^3 - 25*a^4*b^5 + 35*a^2*b^7 - 3*b^9)*d*x + (45*a^7*b^2 - 143*a
^5*b^4 + 219*a^3*b^6 - 9*a*b^8 + 3*(3*a^8*b - 76*a^6*b^3 + 130*a^4*b^5 - 4
4*a^2*b^7 + 3*b^9)*d*x)*cos(d*x + c)^2)*sin(d*x + c))/((a^13 + 2*a^11*b^2
- 5*a^9*b^4 - 20*a^7*b^6 - 25*a^5*b^8 - 14*a^3*b^10 - 3*a*b^12)*d*cos(d*x
+ c)^3 + 3*(a^11*b^2 + 5*a^9*b^4 + 10*a^7*b^6 + 10*a^5*b^8 + 5*a^3*b^10 +
a*b^12)*d*cos(d*x + c) + ((3*a^12*b + 14*a^10*b^3 + 25*a^8*b^5 + 20*a^6*b^
7 + 5*a^4*b^9 - 2*a^2*b^11 - b^13)*d*cos(d*x + c)^2 + (a^10*b^3 + 5*a^8*b^
5 + 10*a^6*b^7 + 10*a^4*b^9 + 5*a^2*b^11 + b^13)*d)*sin(d*x + c))
```

3.74.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(sin(d*x+c)**2/(a+b*tan(d*x+c))**4,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.74.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(258) = 516$.

Time = 0.34 (sec) , antiderivative size = 662, normalized size of antiderivative = 2.51

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

$$= \frac{\frac{3\,(a^6-25\,a^4b^2+35\,a^2b^4-3\,b^6)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} + \frac{24\,(a^5b-5\,a^3b^3+2\,ab^5)\log(b\tan(dx+c)+a)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - \frac{12\,(a^5b-5\,a^3b^3+2\,ab^5)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} -}{}$$

input

```
integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

output

```
1/6*(3*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*(d*x + c)/(a^10 + 5*a^8*b^2
 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 24*(a^5*b - 5*a^3*b^3 + 2
*a*b^5)*log(b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^
6 + 5*a^2*b^8 + b^10) - 12*(a^5*b - 5*a^3*b^3 + 2*a*b^5)*log(tan(d*x + c)^
2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - (
38*a^6*b - 56*a^4*b^3 + 2*a^2*b^5 + 3*(7*a^4*b^3 - 22*a^2*b^5 + 3*b^7)*tan
(d*x + c)^4 + 3*(17*a^5*b^2 - 46*a^3*b^4 + a*b^6)*tan(d*x + c)^3 + (35*a^6
*b - 44*a^4*b^3 - 73*a^2*b^5 + 6*b^7)*tan(d*x + c)^2 + 3*(a^7 + 20*a^5*b^2
 - 43*a^3*b^4 + 2*a*b^6)*tan(d*x + c))/(a^11 + 4*a^9*b^2 + 6*a^7*b^4 + 4*a
^5*b^6 + a^3*b^8 + (a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*ta
n(d*x + c)^5 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*ta
n(d*x + c)^4 + (3*a^10*b + 13*a^8*b^3 + 22*a^6*b^5 + 18*a^4*b^7 + 7*a^2*b^
9 + b^11)*tan(d*x + c)^3 + (a^11 + 7*a^9*b^2 + 18*a^7*b^4 + 22*a^5*b^6 + 1
3*a^3*b^8 + 3*a*b^10)*tan(d*x + c)^2 + 3*(a^10*b + 4*a^8*b^3 + 6*a^6*b^5 +
 4*a^4*b^7 + a^2*b^9)*tan(d*x + c)))/d
```

3.74.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(258) = 516$.

Time = 0.83 (sec) , antiderivative size = 642, normalized size of antiderivative = 2.43

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

$$= \frac{\frac{3\,(a^6-25\,a^4b^2+35\,a^2b^4-3\,b^6)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - \frac{12\,(a^5b-5\,a^3b^3+2\,ab^5)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} + \frac{24\,(a^5b^2-5\,a^3b^4+2\,ab^6)\log(|b\tan(dx+c)+a|)}{a^{10}b+5\,a^8b^3+10\,a^6b^5+10\,a^4b^7+5\,a^2b^9+b^{11}} +}{}$$

input
```
integrate(sin(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/6*(3*(a^6 - 25*a^4*b^2 + 35*a^2*b^4 - 3*b^6)*(d*x + c)/(a^10 + 5*a^8*b^2
 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 12*(a^5*b - 5*a^3*b^3 + 2
*a*b^5)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^
6 + 5*a^2*b^8 + b^10) + 24*(a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*log(abs(b*tan(d
*x + c) + a))/(a^10*b + 5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 +
b^11) + 3*(4*a^5*b*tan(d*x + c)^2 - 20*a^3*b^3*tan(d*x + c)^2 + 8*a*b^5*ta
n(d*x + c)^2 - a^6*tan(d*x + c) + 5*a^4*b^2*tan(d*x + c) + 5*a^2*b^4*tan(d
*x + c) - b^6*tan(d*x + c) - 20*a^3*b^3 + 12*a*b^5)/((a^10 + 5*a^8*b^2 + 1
0*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10)*(tan(d*x + c)^2 + 1)) - 2*(22*a
^5*b^4*tan(d*x + c)^3 - 110*a^3*b^6*tan(d*x + c)^3 + 44*a*b^8*tan(d*x + c)
^3 + 75*a^6*b^3*tan(d*x + c)^2 - 345*a^4*b^5*tan(d*x + c)^2 + 111*a^2*b^7*
tan(d*x + c)^2 + 3*b^9*tan(d*x + c)^2 + 87*a^7*b^2*tan(d*x + c) - 357*a^5*
b^4*tan(d*x + c) + 87*a^3*b^6*tan(d*x + c) + 3*a*b^8*tan(d*x + c) + 35*a^8
*b - 119*a^6*b^3 + 23*a^4*b^5 + a^2*b^7)/((a^10 + 5*a^8*b^2 + 10*a^6*b^4 +
 10*a^4*b^6 + 5*a^2*b^8 + b^10)*(b*tan(d*x + c) + a)^3))/d
```

3.74.9 Mupad [B] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.26

$$\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{4\,a\,b}{{\left(a^2+b^2\right)}^3}-\frac{28\,a\,b^3}{{\left(a^2+b^2\right)}^4}+\frac{32\,a\,b^5}{{\left(a^2+b^2\right)}^5}\right)}{d}$$
$$-\frac{\frac{{\tan(c+d\,x)}^2\,\left(35\,a^4\,b-79\,a^2\,b^3+6\,b^5\right)}{6\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{{\tan(c+d\,x)}^4\,\left(7\,a^4\,b^3-22\,a^2\,b^5+3\,b^7\right)}{2\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}+\frac{{\tan(c+d\,x)}^3\,\left(17\,a^5\,b^2-46\,a^3\,b^4+a\,b^6\right)}{2\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}+\frac{a^2\,\left(19\,a^4\,b-\right.}{3\,\left(a^2+b^2\right)\,\left(a^6+3\right.}}{d\,\left(a^3+{\tan\left(c+d\,x\right)}^2\,\left(a^3+3\,a\,b^2\right)+{\tan\left(c+d\,x\right)}^3\,\left(3\,a^2\,b+b^3\right)+b^3\,{\tan\left(c+d\,x\right)}^5+3\,a\,b^2\right.}$$
$$-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(3\,b+a\,1{}\mathrm{i}\right)}{4\,d\,\left(a^5+a^4\,b\,5{}\mathrm{i}-10\,a^3\,b^2-a^2\,b^3\,10{}\mathrm{i}+5\,a\,b^4+b^5\,1{}\mathrm{i}\right)}$$
$$+\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(-3\,b+a\,1{}\mathrm{i}\right)}{4\,d\,\left(a^5-a^4\,b\,5{}\mathrm{i}-10\,a^3\,b^2+a^2\,b^3\,10{}\mathrm{i}+5\,a\,b^4-b^5\,1{}\mathrm{i}\right)}$$

input
```
int(sin(c + d*x)^2/(a + b*tan(c + d*x))^4,x)
```

output
```
(log(a + b*tan(c + d*x))*((4*a*b)/(a^2 + b^2)^3 - (28*a*b^3)/(a^2 + b^2)^4
 + (32*a*b^5)/(a^2 + b^2)^5))/d - ((tan(c + d*x)^2*(35*a^4*b + 6*b^5 - 79*
a^2*b^3))/(6*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)^4*(3*b^7
 - 22*a^2*b^5 + 7*a^4*b^3))/(2*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*
b^2)) + (tan(c + d*x)^3*(a*b^6 - 46*a^3*b^4 + 17*a^5*b^2))/(2*(a^8 + b^8 +
 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (a^2*(19*a^4*b + b^5 - 28*a^2*b^3))
/(3*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*tan(c + d*x)*(a^
6 + 2*b^6 - 43*a^2*b^4 + 20*a^4*b^2))/(2*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^
4 + 3*a^4*b^2)))/(d*(a^3 + tan(c + d*x)^2*(3*a*b^2 + a^3) + tan(c + d*x)^3
*(3*a^2*b + b^3) + b^3*tan(c + d*x)^5 + 3*a*b^2*tan(c + d*x)^4 + 3*a^2*b*t
an(c + d*x))) - (log(tan(c + d*x) - 1i)*(a*1i + 3*b))/(4*d*(5*a*b^4 + a^4*
b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)) + (log(tan(c + d*x) + 1i)
*(a*1i - 3*b))/(4*d*(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i + a^2*b^3*10i - 10*
a^3*b^2))
```

3.74. $\int \frac{\sin^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

3.75 $\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

3.75.1 Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{\cot(c+dx)}{a^4d} - \frac{4b\log(\tan(c+dx))}{a^5d} + \frac{4b\log(a+b\tan(c+dx))}{a^5d} - \frac{b}{3a^2d(a+b\tan(c+dx))^3} - \frac{b}{a^3d(a+b\tan(c+dx))^2} - \frac{3b}{a^4d(a+b\tan(c+dx))}$$

output
```
-cot(d*x+c)/a^4/d-4*b*ln(tan(d*x+c))/a^5/d+4*b*ln(a+b*tan(d*x+c))/a^5/d-1/
3*b/a^2/d/(a+b*tan(d*x+c))^3-b/a^3/d/(a+b*tan(d*x+c))^2-3*b/a^4/d/(a+b*tan
(d*x+c))
```

3.75.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 259 vs. 2(116) = 232.

Time = 3.28 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.23

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

$$= \frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(-3a(b+a\cot(c+dx))^3\sin^2(c+dx)+\frac{a^2b^4\tan(c+dx)}{a^2+b^2}+\frac{b^2(18a^4}{}\right.}{}$$

input
```
Integrate[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*(-3*a*(b + a*Cot[c + d*x
])^3*Sin[c + d*x]^2 + (a^2*b^4*Tan[c + d*x])/(a^2 + b^2) + (b^2*(18*a^4 +
23*a^2*b^2 + 9*b^4)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2*Tan[c + d*x])/(a^2
 + b^2)^2 - (2*a^2*b^3*(3*a^2 + 2*b^2)*(a + b*Tan[c + d*x]))/(a^2 + b^2)^2
 - 12*b*Cos[c + d*x]^2*Log[Sin[c + d*x]]*(a + b*Tan[c + d*x])^3 + 12*b*Cos
[c + d*x]^2*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]*(a + b*Tan[c + d*x])^3))/
(3*a^5*d*(a + b*Tan[c + d*x])^4)
```

3.75.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3999, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^2(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3999}$$

$$\frac{b\int \frac{\cot^2(c+dx)}{b^2(a+b\tan(c+dx))^4}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$\frac{b\int \left(\frac{\cot^2(c+dx)}{a^4b^2}-\frac{4\cot(c+dx)}{a^5b}+\frac{4}{a^5(a+b\tan(c+dx))}+\frac{3}{a^4(a+b\tan(c+dx))^2}+\frac{2}{a^3(a+b\tan(c+dx))^3}+\frac{1}{a^2(a+b\tan(c+dx))^4}\right)d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{4\log(b\tan(c+dx))}{a^5}+\frac{4\log(a+b\tan(c+dx))}{a^5}-\frac{3}{a^4(a+b\tan(c+dx))}-\frac{\cot(c+dx)}{a^4b}-\frac{1}{a^3(a+b\tan(c+dx))^2}-\frac{1}{3a^2(a+b\tan(c+dx))^3}\right)}{d}$$

input
```
Int[Csc[c + d*x]^2/(a + b*Tan[c + d*x])^4,x]
```

output
```
(b*(-(Cot[c + d*x]/(a^4*b)) - (4*Log[b*Tan[c + d*x]])/a^5 + (4*Log[a + b*T
an[c + d*x]])/a^5 - 1/(3*a^2*(a + b*Tan[c + d*x])^3) - 1/(a^3*(a + b*Tan[c
 + d*x])^2) - 3/(a^4*(a + b*Tan[c + d*x]))))/d
```

3.75.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.75.4 Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{1}{a^4\tan(dx+c)}-\frac{4b\ln(\tan(dx+c))}{a^5}-\frac{b}{3a^2(a+b\tan(dx+c))^3}+\frac{4b\ln(a+b\tan(dx+c))}{a^5}-\frac{3b}{a^4(a+b\tan(dx+c))}-\frac{b}{a^3(a+b\tan(dx+c))^2}}{d}$
default	$\frac{-\frac{1}{a^4\tan(dx+c)}-\frac{4b\ln(\tan(dx+c))}{a^5}-\frac{b}{3a^2(a+b\tan(dx+c))^3}+\frac{4b\ln(a+b\tan(dx+c))}{a^5}-\frac{3b}{a^4(a+b\tan(dx+c))}-\frac{b}{a^3(a+b\tan(dx+c))^2}}{d}$
risch	$-\frac{2i\left(96ia^3b^3e^{2i(dx+c)}+36ia^5b\,e^{4i(dx+c)}-120ia\,b^5e^{4i(dx+c)}-3a^6e^{6i(dx+c)}+63a^4b^2e^{6i(dx+c)}-120a^2b^4e^{6i(dx+c)}+12b^6e\right.}{}$

input
```
int(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^4/tan(d*x+c)-4/a^5*b*ln(tan(d*x+c))-1/3/a^2*b/(a+b*tan(d*x+c))^3
+4/a^5*b*ln(a+b*tan(d*x+c))-3/a^4*b/(a+b*tan(d*x+c))-1/a^3*b/(a+b*tan(d*x+
c))^2)
```

3.75.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(114) = 228$.

Time = 0.30 (sec) , antiderivative size = 874, normalized size of antiderivative = 7.53

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{13\,a^6b^4 + 15\,a^4b^6 + 6\,a^2b^8 - (3\,a^{10} + 18\,a^8b^2 - 49\,a^6b^4 - 84\,a^4b^6 - 36\,a^2b^8)\cos{(dx+c)}^4 + (9\,a^8b^2 - 71}{}$$

input
```
integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/3*(13*a^6*b^4 + 15*a^4*b^6 + 6*a^2*b^8 - (3*a^10 + 18*a^8*b^2 - 49*a^6*
b^4 - 84*a^4*b^6 - 36*a^2*b^8)*cos(d*x + c)^4 + (9*a^8*b^2 - 71*a^6*b^4 -
102*a^4*b^6 - 42*a^2*b^8)*cos(d*x + c)^2 + 6*(a^6*b^4 + 3*a^4*b^6 + 3*a^2*
b^8 + b^10 - (3*a^8*b^2 + 8*a^6*b^4 + 6*a^4*b^6 - b^10)*cos(d*x + c)^4 + (
3*a^8*b^2 + 7*a^6*b^4 + 3*a^4*b^6 - 3*a^2*b^8 - 2*b^10)*cos(d*x + c)^2 + (
(a^9*b - 6*a^5*b^5 - 8*a^3*b^7 - 3*a*b^9)*cos(d*x + c)^3 + 3*(a^7*b^3 + 3*
a^5*b^5 + 3*a^3*b^7 + a*b^9)*cos(d*x + c))*sin(d*x + c))*log(2*a*b*cos(d*x
 + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*(a^6*b^4 + 3*a^
4*b^6 + 3*a^2*b^8 + b^10 - (3*a^8*b^2 + 8*a^6*b^4 + 6*a^4*b^6 - b^10)*cos(
d*x + c)^4 + (3*a^8*b^2 + 7*a^6*b^4 + 3*a^4*b^6 - 3*a^2*b^8 - 2*b^10)*cos(
d*x + c)^2 + ((a^9*b - 6*a^5*b^5 - 8*a^3*b^7 - 3*a*b^9)*cos(d*x + c)^3 + 3
*(a^7*b^3 + 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*cos(d*x + c))*sin(d*x + c))*log
(-1/4*cos(d*x + c)^2 + 1/4) - ((9*a^9*b + 78*a^7*b^3 + 69*a^5*b^5 + 4*a^3*
b^7 - 12*a*b^9)*cos(d*x + c)^3 - 3*(9*a^7*b^3 + 3*a^5*b^5 - 6*a^3*b^7 - 4*
a*b^9)*cos(d*x + c))*sin(d*x + c))/((3*a^13*b + 8*a^11*b^3 + 6*a^9*b^5 - a
^5*b^9)*d*cos(d*x + c)^4 - (3*a^13*b + 7*a^11*b^3 + 3*a^9*b^5 - 3*a^7*b^7
- 2*a^5*b^9)*d*cos(d*x + c)^2 - (a^11*b^3 + 3*a^9*b^5 + 3*a^7*b^7 + a^5*b^
9)*d - ((a^14 - 6*a^10*b^4 - 8*a^8*b^6 - 3*a^6*b^8)*d*cos(d*x + c)^3 + 3*(
a^12*b^2 + 3*a^10*b^4 + 3*a^8*b^6 + a^6*b^8)*d*cos(d*x + c))*sin(d*x + c))
```

3.75.6 Sympy [F]

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

input
```
integrate(csc(d*x+c)**2/(a+b*tan(d*x+c))**4,x)
```

output
```
Integral(csc(c + d*x)**2/(a + b*tan(c + d*x))**4, x)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.21

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{\frac{12\,b^3\tan(dx+c)^3+30\,ab^2\tan(dx+c)^2+22\,a^2b\tan(dx+c)+3\,a^3}{a^4b^3\tan(dx+c)^4+3\,a^5b^2\tan(dx+c)^3+3\,a^6b\tan(dx+c)^2+a^7\tan(dx+c)} - \frac{12\,b\log(b\tan(dx+c)+a)}{a^5} + \frac{12\,b\log(\tan(dx+c))}{a^5}}{3\,d}$$

input
```
integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/3*((12*b^3*tan(d*x + c)^3 + 30*a*b^2*tan(d*x + c)^2 + 22*a^2*b*tan(d*x
+ c) + 3*a^3)/(a^4*b^3*tan(d*x + c)^4 + 3*a^5*b^2*tan(d*x + c)^3 + 3*a^6*b
*tan(d*x + c)^2 + a^7*tan(d*x + c)) - 12*b*log(b*tan(d*x + c) + a)/a^5 + 1
2*b*log(tan(d*x + c))/a^5)/d
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.11

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{\frac{12\,b\log(|b\tan(dx+c)+a|)}{a^5} - \frac{12\,b\log(|\tan(dx+c)|)}{a^5} + \frac{3\,(4\,b\tan(dx+c)-a)}{a^5\tan(dx+c)} - \frac{22\,b^4\tan(dx+c)^3+75\,ab^3\tan(dx+c)^2+87\,a^2b^2\tan(dx+c)+35\,a^3}{(b\tan(dx+c)+a)^3a^5}}{3\,d}$$

input
```
integrate(csc(d*x+c)^2/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(12*b*log(abs(b*tan(d*x + c) + a))/a^5 - 12*b*log(abs(tan(d*x + c)))/a
^5 + 3*(4*b*tan(d*x + c) - a)/(a^5*tan(d*x + c)) - (22*b^4*tan(d*x + c)^3
+ 75*a*b^3*tan(d*x + c)^2 + 87*a^2*b^2*tan(d*x + c) + 35*a^3*b)/((b*tan(d*
x + c) + a)^3*a^5))/d
```

3.75.9 Mupad [B] (verification not implemented)

Time = 5.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.13

$$\int \frac{\csc^2(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{8\,b\,\mathrm{atanh}\left(\frac{2\,b\tan(c+d\,x)}{a}+1\right)}{a^5\,d} - \frac{\frac{1}{a}+\frac{10\,b^2\tan(c+d\,x)^2}{a^3}+\frac{4\,b^3\tan(c+d\,x)^3}{a^4}+\frac{22\,b\tan(c+d\,x)}{3\,a^2}}{d\,\left(a^3\tan\left(c+d\,x\right)+3\,a^2\,b\tan\left(c+d\,x\right)^2+3\,a\,b^2\tan\left(c+d\,x\right)^3+b^3\tan\left(c+d\,x\right)^4\right)}$$

input
```
int(1/(sin(c + d*x)^2*(a + b*tan(c + d*x))^4),x)
```

output
```
(8*b*atanh((2*b*tan(c + d*x))/a + 1))/(a^5*d) - (1/a + (10*b^2*tan(c + d*x
)^2)/a^3 + (4*b^3*tan(c + d*x)^3)/a^4 + (22*b*tan(c + d*x))/(3*a^2))/(d*(a
^3*tan(c + d*x) + b^3*tan(c + d*x)^4 + 3*a^2*b*tan(c + d*x)^2 + 3*a*b^2*ta
n(c + d*x)^3))
```

3.76 $\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

3.76.1 Optimal result

Integrand size = 21, antiderivative size = 205

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{(a^2+10b^2)\cot(c+dx)}{a^6d}+\frac{2b\cot^2(c+dx)}{a^5d}-\frac{\cot^3(c+dx)}{3a^4d}-\frac{4b(a^2+5b^2)\log(\tan(c+dx))}{a^7d}+\frac{4b(a^2+5b^2)\log(a+b\tan(c+dx))}{a^7d}-\frac{b(a^2+b^2)}{3a^4d(a+b\tan(c+dx))^3}-\frac{b(a^2+2b^2)}{a^5d(a+b\tan(c+dx))^2}-\frac{b(3a^2+10b^2)}{a^6d(a+b\tan(c+dx))}$$

output

```
-(a^2+10*b^2)*cot(d*x+c)/a^6/d+2*b*cot(d*x+c)^2/a^5/d-1/3*cot(d*x+c)^3/a^4
/d-4*b*(a^2+5*b^2)*ln(tan(d*x+c))/a^7/d+4*b*(a^2+5*b^2)*ln(a+b*tan(d*x+c))
/a^7/d-1/3*b*(a^2+b^2)/a^4/d/(a+b*tan(d*x+c))^3-b*(a^2+2*b^2)/a^5/d/(a+b*t
an(d*x+c))^2-b*(3*a^2+10*b^2)/a^6/d/(a+b*tan(d*x+c))
```

3.76.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 528 vs. 2(205) = 410.

Time = 2.88 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.58

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{\sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(-192b(a^2+5b^2)\log(\sin(c+dx))(a\cos(c+dx)+b\sin(c+d\right.}{}$$

input
```
Integrate[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])*(-192*b*(a^2 + 5*b^2)*Lo
g[Sin[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 + 192*b*(a^2 + 5*b^2)*
Log[a*Cos[c + d*x] + b*Sin[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 -
 (Csc[c + d*x]^3*(8*a^8 - 4*a^6*b^2 - 50*a^4*b^4 - 190*a^2*b^6 - 150*b^8 +
 3*(3*a^8 + 10*a^6*b^2 + 45*a^4*b^4 + 115*a^2*b^6 + 75*b^8)*Cos[2*(c + d*x
)] + 6*(2*a^6*b^2 - 17*a^4*b^4 - 35*a^2*b^6 - 15*b^8)*Cos[4*(c + d*x)] - a
^8*Cos[6*(c + d*x)] - 22*a^6*b^2*Cos[6*(c + d*x)] + 17*a^4*b^4*Cos[6*(c +
d*x)] + 55*a^2*b^6*Cos[6*(c + d*x)] + 15*b^8*Cos[6*(c + d*x)] - 3*a^7*b*Si
n[2*(c + d*x)] + 3*a^5*b^3*Sin[2*(c + d*x)] - 75*a^3*b^5*Sin[2*(c + d*x)]
- 75*a*b^7*Sin[2*(c + d*x)] - 6*a^7*b*Sin[4*(c + d*x)] + 84*a^5*b^3*Sin[4*
(c + d*x)] + 156*a^3*b^5*Sin[4*(c + d*x)] + 60*a*b^7*Sin[4*(c + d*x)] - 3*
a^7*b*Sin[6*(c + d*x)] - 65*a^5*b^3*Sin[6*(c + d*x)] - 79*a^3*b^5*Sin[6*(c
 + d*x)] - 15*a*b^7*Sin[6*(c + d*x)]))/(a^2 + b^2)))/(48*a^7*d*(a + b*Tan[
c + d*x])^4)
```

3.76.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3999, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(c+dx)^4(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3999}$$

$$\frac{b\int \frac{\cot^4(c+dx)\left(\tan^2(c+dx)b^2+b^2\right)}{b^4(a+b\tan(c+dx))^4}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{b\int \left(\frac{\cot^4(c+dx)}{a^4b^2}-\frac{4\cot^3(c+dx)}{a^5b}+\frac{(a^2+10b^2)\cot^2(c+dx)}{a^6b^2}-\frac{4(a^2+5b^2)\cot(c+dx)}{a^7b}+\frac{4(a^2+5b^2)}{a^7(a+b\tan(c+dx))}+\frac{3a^2+10b^2}{a^6(a+b\tan(c+dx))^2}+\frac{}{a^5(a}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{2\cot^2(c+dx)}{a^5}-\frac{\cot^3(c+dx)}{3a^4b}-\frac{4(a^2+5b^2)\log(b\tan(c+dx))}{a^7}+\frac{4(a^2+5b^2)\log(a+b\tan(c+dx))}{a^7}-\frac{3a^2+10b^2}{a^6(a+b\tan(c+dx))}-\frac{(a^2+10b^2)\cot(c+}{a^6b}\right.}{d}$$

input
```
Int[Csc[c + d*x]^4/(a + b*Tan[c + d*x])^4,x]
```

output
```
(b*(-(((a^2 + 10*b^2)*Cot[c + d*x])/(a^6*b)) + (2*Cot[c + d*x]^2)/a^5 - Co
t[c + d*x]^3/(3*a^4*b) - (4*(a^2 + 5*b^2)*Log[b*Tan[c + d*x]])/a^7 + (4*(a
^2 + 5*b^2)*Log[a + b*Tan[c + d*x]])/a^7 - (a^2 + b^2)/(3*a^4*(a + b*Tan[c
 + d*x])^3) - (a^2 + 2*b^2)/(a^5*(a + b*Tan[c + d*x])^2) - (3*a^2 + 10*b^2
)/(a^6*(a + b*Tan[c + d*x]))))/d
```

3.76.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.76.4 Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{b\left(3a^2+10b^2\right)}{a^6(a+b\tan(dx+c))}-\frac{\left(a^2+b^2\right)b}{3a^4(a+b\tan(dx+c))^3}-\frac{b\left(a^2+2b^2\right)}{a^5(a+b\tan(dx+c))^2}+\frac{4b\left(a^2+5b^2\right)\ln(a+b\tan(dx+c))}{a^7}-\frac{1}{3a^4\tan(dx+c)^3}-\frac{a^2+10}{a^6\tan(d}}{d}$
default	$\frac{-\frac{b\left(3a^2+10b^2\right)}{a^6(a+b\tan(dx+c))}-\frac{\left(a^2+b^2\right)b}{3a^4(a+b\tan(dx+c))^3}-\frac{b\left(a^2+2b^2\right)}{a^5(a+b\tan(dx+c))^2}+\frac{4b\left(a^2+5b^2\right)\ln(a+b\tan(dx+c))}{a^7}-\frac{1}{3a^4\tan(dx+c)^3}-\frac{a^2+10}{a^6\tan(d}}{d}$
risch	$-\frac{4i\left(ia^7-30b^7-26a^4b^3-56b^5a^2-a^6b+32ia^5b^2e^{6i(dx+c)}+280ia^3b^4e^{6i(dx+c)}-300ia\,b^6e^{4i(dx+c)}-300ia^3b^4e^{4i(dx+c)}-4\right.}{}$

input
```
int(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b*(3*a^2+10*b^2)/a^6/(a+b*tan(d*x+c))-1/3*(a^2+b^2)*b/a^4/(a+b*tan(d
*x+c))^3-b*(a^2+2*b^2)/a^5/(a+b*tan(d*x+c))^2+4*b*(a^2+5*b^2)/a^7*ln(a+b*t
an(d*x+c))-1/3/a^4/tan(d*x+c)^3-(a^2+10*b^2)/a^6/tan(d*x+c)+2/a^5*b/tan(d*
x+c)^2-4*b*(a^2+5*b^2)/a^7*ln(tan(d*x+c)))
```

3.76.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1235 vs. $2(201) = 402$.

Time = 0.32 (sec) , antiderivative size = 1235, normalized size of antiderivative = 6.02

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/3*(19*a^6*b^4 + 51*a^4*b^6 + 30*a^2*b^8 + 2*(a^10 + 23*a^8*b^2 - 22*a^6*
b^4 - 138*a^4*b^6 - 90*a^2*b^8)*cos(d*x + c)^6 - 3*(a^10 + 25*a^8*b^2 - 46
*a^6*b^4 - 206*a^4*b^6 - 130*a^2*b^8)*cos(d*x + c)^4 + 3*(9*a^8*b^2 - 38*a
^6*b^4 - 131*a^4*b^6 - 80*a^2*b^8)*cos(d*x + c)^2 + 6*(a^6*b^4 + 7*a^4*b^6
 + 11*a^2*b^8 + 5*b^10 + (3*a^8*b^2 + 20*a^6*b^4 + 26*a^4*b^6 + 4*a^2*b^8
- 5*b^10)*cos(d*x + c)^6 - 3*(2*a^8*b^2 + 13*a^6*b^4 + 15*a^4*b^6 - a^2*b^
8 - 5*b^10)*cos(d*x + c)^4 + 3*(a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 - 6*a^2*b^
8 - 5*b^10)*cos(d*x + c)^2 - ((a^9*b + 4*a^7*b^3 - 10*a^5*b^5 - 28*a^3*b^7
 - 15*a*b^9)*cos(d*x + c)^5 - (a^9*b + a^7*b^3 - 31*a^5*b^5 - 61*a^3*b^7 -
 30*a*b^9)*cos(d*x + c)^3 - 3*(a^7*b^3 + 7*a^5*b^5 + 11*a^3*b^7 + 5*a*b^9)
*cos(d*x + c))*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
b^2)*cos(d*x + c)^2 + b^2) - 6*(a^6*b^4 + 7*a^4*b^6 + 11*a^2*b^8 + 5*b^10
+ (3*a^8*b^2 + 20*a^6*b^4 + 26*a^4*b^6 + 4*a^2*b^8 - 5*b^10)*cos(d*x + c)^
6 - 3*(2*a^8*b^2 + 13*a^6*b^4 + 15*a^4*b^6 - a^2*b^8 - 5*b^10)*cos(d*x + c
)^4 + 3*(a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 - 6*a^2*b^8 - 5*b^10)*cos(d*x + c
)^2 - ((a^9*b + 4*a^7*b^3 - 10*a^5*b^5 - 28*a^3*b^7 - 15*a*b^9)*cos(d*x +
c)^5 - (a^9*b + a^7*b^3 - 31*a^5*b^5 - 61*a^3*b^7 - 30*a*b^9)*cos(d*x + c)
^3 - 3*(a^7*b^3 + 7*a^5*b^5 + 11*a^3*b^7 + 5*a*b^9)*cos(d*x + c))*sin(d*x
+ c))*log(-1/4*cos(d*x + c)^2 + 1/4) + (2*(3*a^9*b + 77*a^7*b^3 + 142*a^5*
b^5 + 34*a^3*b^7 - 30*a*b^9)*cos(d*x + c)^5 - (3*a^9*b + 193*a^7*b^3 + ...
```

3.76.6 Sympy [F]

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

input
```
integrate(csc(d*x+c)**4/(a+b*tan(d*x+c))**4,x)
```

output
```
Integral(csc(c + d*x)**4/(a + b*tan(c + d*x))**4, x)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.11

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{3\,a^4b\tan(dx+c)-12\,(a^2b^3+5\,b^5)\tan(dx+c)^5-a^5-30\,(a^3b^2+5\,ab^4)\tan(dx+c)^4-22\,(a^4b+5\,a^2b^3)\tan(dx+c)^3-3\,(a^5+5\,a^3b^2)\tan(dx+c)^2}{a^6b^3\tan(dx+c)^6+3\,a^7b^2\tan(dx+c)^5+3\,a^8b\tan(dx+c)^4+a^9\tan(dx+c)^3}+12}{3\,d}$$

input
```
integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/3*((3*a^4*b*tan(d*x + c) - 12*(a^2*b^3 + 5*b^5)*tan(d*x + c)^5 - a^5 - 3
0*(a^3*b^2 + 5*a*b^4)*tan(d*x + c)^4 - 22*(a^4*b + 5*a^2*b^3)*tan(d*x + c)
^3 - 3*(a^5 + 5*a^3*b^2)*tan(d*x + c)^2)/(a^6*b^3*tan(d*x + c)^6 + 3*a^7*b
^2*tan(d*x + c)^5 + 3*a^8*b*tan(d*x + c)^4 + a^9*tan(d*x + c)^3) + 12*(a^2
*b + 5*b^3)*log(b*tan(d*x + c) + a)/a^7 - 12*(a^2*b + 5*b^3)*log(tan(d*x +
 c))/a^7)/d
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.08

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx =$$
$$-\frac{\frac{12\,(a^2b+5\,b^3)\log(|\tan(dx+c)|)}{a^7}-\frac{12\,(a^2b^2+5\,b^4)\log(|b\tan(dx+c)+a|)}{a^7b}+\frac{12\,a^2b^3\tan(dx+c)^5+60\,b^5\tan(dx+c)^5+30\,a^3b^2\tan(dx+c)^4+15}{}}{3\,d}$$

input
```
integrate(csc(d*x+c)^4/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(12*(a^2*b + 5*b^3)*log(abs(tan(d*x + c)))/a^7 - 12*(a^2*b^2 + 5*b^4)
*log(abs(b*tan(d*x + c) + a))/(a^7*b) + (12*a^2*b^3*tan(d*x + c)^5 + 60*b^
5*tan(d*x + c)^5 + 30*a^3*b^2*tan(d*x + c)^4 + 150*a*b^4*tan(d*x + c)^4 +
22*a^4*b*tan(d*x + c)^3 + 110*a^2*b^3*tan(d*x + c)^3 + 3*a^5*tan(d*x + c)^
2 + 15*a^3*b^2*tan(d*x + c)^2 - 3*a^4*b*tan(d*x + c) + a^5)/((b*tan(d*x +
c)^2 + a*tan(d*x + c))^3*a^6))/d
```

3.76.9 Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.13

$$\int \frac{\csc^4(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{8\,b\,\mathrm{atanh}\left(\frac{4\,b\,(a^2+5\,b^2)\,(a+2\,b\tan(c+d\,x))}{a\,(4\,a^2\,b+20\,b^3)}\right)\,(a^2+5\,b^2)}{a^7\,d} - \frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,(a^2+5\,b^2)}{a^3}-\frac{b\tan(c+d\,x)}{a^2}+\frac{22\,b\tan(c+d\,x)^3\,(a^2+5\,b^2)}{3\,a^4}+\frac{10\,b^2\tan(c+d\,x)^4\,(a^2+5\,b^2)}{a^5}+\frac{4\,b^3\tan(c+d\,x)^5\,(a^2+5\,b^2)}{a^6}}{d\,\left(a^3\tan\left(c+d\,x\right)^3+3\,a^2\,b\tan\left(c+d\,x\right)^4+3\,a\,b^2\tan\left(c+d\,x\right)^5+b^3\tan\left(c+d\,x\right)^6\right)}$$

input
```
int(1/(sin(c + d*x)^4*(a + b*tan(c + d*x))^4),x)
```

output
```
(8*b*atanh((4*b*(a^2 + 5*b^2)*(a + 2*b*tan(c + d*x)))/(a*(4*a^2*b + 20*b^3
)))*(a^2 + 5*b^2))/(a^7*d) - (1/(3*a) + (tan(c + d*x)^2*(a^2 + 5*b^2))/a^3
 - (b*tan(c + d*x))/a^2 + (22*b*tan(c + d*x)^3*(a^2 + 5*b^2))/(3*a^4) + (1
0*b^2*tan(c + d*x)^4*(a^2 + 5*b^2))/a^5 + (4*b^3*tan(c + d*x)^5*(a^2 + 5*b
^2))/a^6)/(d*(a^3*tan(c + d*x)^3 + b^3*tan(c + d*x)^6 + 3*a^2*b*tan(c + d*
x)^4 + 3*a*b^2*tan(c + d*x)^5))
```

3.77 $\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

3.77.1 Optimal result

Integrand size = 21, antiderivative size = 300

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{(a^4+20a^2b^2+35b^4)\cot(c+dx)}{a^8d}+\frac{2b(2a^2+5b^2)\cot^2(c+dx)}{a^7d}-\frac{2(a^2+5b^2)\cot^3(c+dx)}{3a^6d}+\frac{b\cot^4(c+dx)}{a^5d}-\frac{\cot^5(c+dx)}{5a^4d}-\frac{4b(a^4+10a^2b^2+14b^4)\log(\tan(c+dx))}{a^9d}+\frac{4b(a^4+10a^2b^2+14b^4)\log(a+b\tan(c+dx))}{a^9d}-\frac{b(a^2+b^2)^2}{3a^6d(a+b\tan(c+dx))^3}-\frac{b(a^2+b^2)(a^2+3b^2)}{a^7d(a+b\tan(c+dx))^2}-\frac{b(3a^4+20a^2b^2+21b^4)}{a^8d(a+b\tan(c+dx))}$$

output
```
-(a^4+20*a^2*b^2+35*b^4)*cot(d*x+c)/a^8/d+2*b*(2*a^2+5*b^2)*cot(d*x+c)^2/a
^7/d-2/3*(a^2+5*b^2)*cot(d*x+c)^3/a^6/d+b*cot(d*x+c)^4/a^5/d-1/5*cot(d*x+c
)^5/a^4/d-4*b*(a^4+10*a^2*b^2+14*b^4)*ln(tan(d*x+c))/a^9/d+4*b*(a^4+10*a^2
*b^2+14*b^4)*ln(a+b*tan(d*x+c))/a^9/d-1/3*b*(a^2+b^2)^2/a^6/d/(a+b*tan(d*x
+c))^3-b*(a^2+b^2)*(a^2+3*b^2)/a^7/d/(a+b*tan(d*x+c))^2-b*(3*a^4+20*a^2*b^
2+21*b^4)/a^8/d/(a+b*tan(d*x+c))
```

3.77.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 673 vs. 2(300) = 600.

Time = 3.10 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.24

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{\sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(-7680b(a^4+10a^2b^2+14b^4)\log(\sin(c+dx))(a\cos(c+dx)+\right.}{}$$

input
```
Integrate[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])*(-7680*b*(a^4 + 10*a^2*b
^2 + 14*b^4)*Log[Sin[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 + 7680*
b*(a^4 + 10*a^2*b^2 + 14*b^4)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]*(a*Cos[
c + d*x] + b*Sin[c + d*x])^3 + Csc[c + d*x]^5*(-200*a^8 + 380*a^6*b^2 + 30
70*a^4*b^4 + 11375*a^2*b^6 + 11025*b^8 - 4*(52*a^8 + 194*a^6*b^2 + 1510*a^
4*b^4 + 5705*a^2*b^6 + 4410*b^8)*Cos[2*(c + d*x)] + 4*(4*a^8 - 16*a^6*b^2
+ 1010*a^4*b^4 + 4585*a^2*b^6 + 2205*b^8)*Cos[4*(c + d*x)] + 16*a^8*Cos[6*
(c + d*x)] + 776*a^6*b^2*Cos[6*(c + d*x)] - 1000*a^4*b^4*Cos[6*(c + d*x)]
- 8540*a^2*b^6*Cos[6*(c + d*x)] - 2520*b^8*Cos[6*(c + d*x)] - 8*a^8*Cos[8*
(c + d*x)] - 316*a^6*b^2*Cos[8*(c + d*x)] - 70*a^4*b^4*Cos[8*(c + d*x)] +
1645*a^2*b^6*Cos[8*(c + d*x)] + 315*b^8*Cos[8*(c + d*x)] + 264*a^7*b*Sin[2
*(c + d*x)] + 372*a^5*b^3*Sin[2*(c + d*x)] + 4830*a^3*b^5*Sin[2*(c + d*x)]
 + 1470*a*b^7*Sin[2*(c + d*x)] + 144*a^7*b*Sin[4*(c + d*x)] - 2476*a^5*b^3
*Sin[4*(c + d*x)] - 9730*a^3*b^5*Sin[4*(c + d*x)] - 1470*a*b^7*Sin[4*(c +
d*x)] - 24*a^7*b*Sin[6*(c + d*x)] + 2756*a^5*b^3*Sin[6*(c + d*x)] + 7670*a
^3*b^5*Sin[6*(c + d*x)] + 630*a*b^7*Sin[6*(c + d*x)] - 24*a^7*b*Sin[8*(c +
 d*x)] - 922*a^5*b^3*Sin[8*(c + d*x)] - 2095*a^3*b^5*Sin[8*(c + d*x)] - 10
5*a*b^7*Sin[8*(c + d*x)])))/(1920*a^9*d*(a + b*Tan[c + d*x])^4)
```

3.77.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3999, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\sin(c+dx)^6(a+b\tan(c+dx))^4}dx$$

$\Big\downarrow$ 3999

$$\frac{b\int \frac{\cot^6(c+dx)\left(\tan^2(c+dx)b^2+b^2\right)^2}{b^6(a+b\tan(c+dx))^4}d(b\tan(c+dx))}{d}$$

$\Big\downarrow$ 522

$$\frac{b\int \left(\frac{\cot^6(c+dx)}{a^4b^2}-\frac{4\cot^5(c+dx)}{a^5b}+\frac{2(a^2+5b^2)\cot^4(c+dx)}{a^6b^2}-\frac{4(5b^4+2a^2b^2)\cot^3(c+dx)}{a^7b^3}+\frac{(a^4+20b^2a^2+35b^4)\cot^2(c+dx)}{a^8b^2}-\frac{4(a^4+10b^2a}{}\right.}{d}$$

$\Big\downarrow$ 2009

$$\frac{b\left(\frac{\cot^4(c+dx)}{a^5}-\frac{\cot^5(c+dx)}{5a^4b}-\frac{(a^2+b^2)(a^2+3b^2)}{a^7(a+b\tan(c+dx))^2}+\frac{2(2a^2+5b^2)\cot^2(c+dx)}{a^7}-\frac{(a^2+b^2)^2}{3a^6(a+b\tan(c+dx))^3}-\frac{2(a^2+5b^2)\cot^3(c+dx)}{3a^6b}-\frac{4(a^4}{}\right.}{d}$$

input
```
Int[Csc[c + d*x]^6/(a + b*Tan[c + d*x])^4,x]
```

output
```
(b*(-(((a^4 + 20*a^2*b^2 + 35*b^4)*Cot[c + d*x])/(a^8*b)) + (2*(2*a^2 + 5*
b^2)*Cot[c + d*x]^2)/a^7 - (2*(a^2 + 5*b^2)*Cot[c + d*x]^3)/(3*a^6*b) + Co
t[c + d*x]^4/a^5 - Cot[c + d*x]^5/(5*a^4*b) - (4*(a^4 + 10*a^2*b^2 + 14*b^
4)*Log[b*Tan[c + d*x]])/a^9 + (4*(a^4 + 10*a^2*b^2 + 14*b^4)*Log[a + b*Tan
[c + d*x]])/a^9 - (a^2 + b^2)^2/(3*a^6*(a + b*Tan[c + d*x])^3) - ((a^2 + b
^2)*(a^2 + 3*b^2))/(a^7*(a + b*Tan[c + d*x])^2) - (3*a^4 + 20*a^2*b^2 + 21
*b^4)/(a^8*(a + b*Tan[c + d*x]))))/d
```

3.77.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.77.4 Maple [A] (verified)

Time = 7.96 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{b\left(3a^4+20a^2b^2+21b^4\right)}{a^8(a+b\tan(dx+c))}-\frac{\left(a^4+2a^2b^2+b^4\right)b}{3a^6(a+b\tan(dx+c))^3}-\frac{b\left(a^4+4a^2b^2+3b^4\right)}{a^7(a+b\tan(dx+c))^2}+\frac{4b\left(a^4+10a^2b^2+14b^4\right)\ln(a+b\tan(dx+c))}{a^9}-\frac{1}{5a^4\tan(dx+c}}{d}$
default	$\frac{-\frac{b\left(3a^4+20a^2b^2+21b^4\right)}{a^8(a+b\tan(dx+c))}-\frac{\left(a^4+2a^2b^2+b^4\right)b}{3a^6(a+b\tan(dx+c))^3}-\frac{b\left(a^4+4a^2b^2+3b^4\right)}{a^7(a+b\tan(dx+c))^2}+\frac{4b\left(a^4+10a^2b^2+14b^4\right)\ln(a+b\tan(dx+c))}{a^9}-\frac{1}{5a^4\tan(dx+c}}{d}$
risch	Expression too large to display

input
```
int(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b*(3*a^4+20*a^2*b^2+21*b^4)/a^8/(a+b*tan(d*x+c))-1/3*(a^4+2*a^2*b^2+
b^4)*b/a^6/(a+b*tan(d*x+c))^3-b*(a^4+4*a^2*b^2+3*b^4)/a^7/(a+b*tan(d*x+c))
^2+4*b*(a^4+10*a^2*b^2+14*b^4)/a^9*ln(a+b*tan(d*x+c))-1/5/a^4/tan(d*x+c)^5
-1/3*(2*a^2+10*b^2)/a^6/tan(d*x+c)^3-(a^4+20*a^2*b^2+35*b^4)/a^8/tan(d*x+c
)+1/a^5*b/tan(d*x+c)^4+2*b*(2*a^2+5*b^2)/a^7/tan(d*x+c)^2-4*b*(a^4+10*a^2*
b^2+14*b^4)/a^9*ln(tan(d*x+c)))
```

3.77.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1536 vs. 2(294) = 588.

Time = 0.37 (sec) , antiderivative size = 1536, normalized size of antiderivative = 5.12

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/15*(110*a^6*b^4 + 510*a^4*b^6 + 420*a^2*b^8 - 4*(2*a^10 + 81*a^8*b^2 +
29*a^6*b^4 - 660*a^4*b^6 - 630*a^2*b^8)*cos(d*x + c)^8 + 2*(10*a^10 + 423*
a^8*b^2 - 47*a^6*b^4 - 4320*a^4*b^6 - 3990*a^2*b^8)*cos(d*x + c)^6 - 15*(a
^10 + 47*a^8*b^2 - 44*a^6*b^4 - 658*a^4*b^6 - 588*a^2*b^8)*cos(d*x + c)^4
+ 20*(9*a^8*b^2 - 28*a^6*b^4 - 219*a^4*b^6 - 189*a^2*b^8)*cos(d*x + c)^2 +
 30*(a^6*b^4 + 11*a^4*b^6 + 24*a^2*b^8 + 14*b^10 - (3*a^8*b^2 + 32*a^6*b^4
 + 61*a^4*b^6 + 18*a^2*b^8 - 14*b^10)*cos(d*x + c)^8 + (9*a^8*b^2 + 95*a^6
*b^4 + 172*a^4*b^6 + 30*a^2*b^8 - 56*b^10)*cos(d*x + c)^6 - 3*(3*a^8*b^2 +
 31*a^6*b^4 + 50*a^4*b^6 - 6*a^2*b^8 - 28*b^10)*cos(d*x + c)^4 + (3*a^8*b^
2 + 29*a^6*b^4 + 28*a^4*b^6 - 54*a^2*b^8 - 56*b^10)*cos(d*x + c)^2 + ((a^9
*b + 8*a^7*b^3 - 9*a^5*b^5 - 58*a^3*b^7 - 42*a*b^9)*cos(d*x + c)^7 - (2*a^
9*b + 13*a^7*b^3 - 51*a^5*b^5 - 188*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^5 +
(a^9*b + 2*a^7*b^3 - 75*a^5*b^5 - 202*a^3*b^7 - 126*a*b^9)*cos(d*x + c)^3
+ 3*(a^7*b^3 + 11*a^5*b^5 + 24*a^3*b^7 + 14*a*b^9)*cos(d*x + c))*sin(d*x +
 c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^
2) - 30*(a^6*b^4 + 11*a^4*b^6 + 24*a^2*b^8 + 14*b^10 - (3*a^8*b^2 + 32*a^6
*b^4 + 61*a^4*b^6 + 18*a^2*b^8 - 14*b^10)*cos(d*x + c)^8 + (9*a^8*b^2 + 95
*a^6*b^4 + 172*a^4*b^6 + 30*a^2*b^8 - 56*b^10)*cos(d*x + c)^6 - 3*(3*a^8*b
^2 + 31*a^6*b^4 + 50*a^4*b^6 - 6*a^2*b^8 - 28*b^10)*cos(d*x + c)^4 + (3*a^
8*b^2 + 29*a^6*b^4 + 28*a^4*b^6 - 54*a^2*b^8 - 56*b^10)*cos(d*x + c)^2 ...
```

3.77.6 Sympy [F]

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \int \frac{\csc^6{(c+dx)}}{(a+b\tan{(c+dx)})^4}\,dx$$

input
```
integrate(csc(d*x+c)**6/(a+b*tan(d*x+c))**4,x)
```

output
```
Integral(csc(c + d*x)**6/(a + b*tan(c + d*x))**4, x)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.08

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{6\,a^6 b\tan(dx+c)-60\,(a^4b^3+10\,a^2b^5+14\,b^7)\tan(dx+c)^7-3\,a^7-150\,(a^5b^2+10\,a^3b^4+14\,ab^6)\tan(dx+c)^6-110\,(a^6b+10\,a^4b^3+14\,a^2b^5)\tan(dx+c)^5}{a^8b^3\tan(dx+c)^8+3\,a^9b^2\tan(dx+c)^7+3\,a^{10}b\tan(dx+c)^6+a}$$

input
```
integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/15*((6*a^6*b*tan(d*x + c) - 60*(a^4*b^3 + 10*a^2*b^5 + 14*b^7)*tan(d*x +
 c)^7 - 3*a^7 - 150*(a^5*b^2 + 10*a^3*b^4 + 14*a*b^6)*tan(d*x + c)^6 - 110
*(a^6*b + 10*a^4*b^3 + 14*a^2*b^5)*tan(d*x + c)^5 - 15*(a^7 + 10*a^5*b^2 +
 14*a^3*b^4)*tan(d*x + c)^4 + 6*(5*a^6*b + 7*a^4*b^3)*tan(d*x + c)^3 - 2*(
5*a^7 + 7*a^5*b^2)*tan(d*x + c)^2)/(a^8*b^3*tan(d*x + c)^8 + 3*a^9*b^2*tan
(d*x + c)^7 + 3*a^10*b*tan(d*x + c)^6 + a^11*tan(d*x + c)^5) + 60*(a^4*b +
 10*a^2*b^3 + 14*b^5)*log(b*tan(d*x + c) + a)/a^9 - 60*(a^4*b + 10*a^2*b^3
 + 14*b^5)*log(tan(d*x + c))/a^9)/d
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.43

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{\frac{60\,(a^4b+10\,a^2b^3+14\,b^5)\log(|\tan(dx+c)|)}{a^9}-\frac{60\,(a^4b^2+10\,a^2b^4+14\,b^6)\log(|b\tan(dx+c)+a|)}{a^9b}+\frac{5\left(22\,a^4b^4\tan(dx+c)^3+220\,a^2b^6\tan(dx+c)\right.}{}}{}$$

input
```
integrate(csc(d*x+c)^6/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/15*(60*(a^4*b + 10*a^2*b^3 + 14*b^5)*log(abs(tan(d*x + c)))/a^9 - 60*(a
^4*b^2 + 10*a^2*b^4 + 14*b^6)*log(abs(b*tan(d*x + c) + a))/(a^9*b) + 5*(22
*a^4*b^4*tan(d*x + c)^3 + 220*a^2*b^6*tan(d*x + c)^3 + 308*b^8*tan(d*x + c
)^3 + 75*a^5*b^3*tan(d*x + c)^2 + 720*a^3*b^5*tan(d*x + c)^2 + 987*a*b^7*t
an(d*x + c)^2 + 87*a^6*b^2*tan(d*x + c) + 792*a^4*b^4*tan(d*x + c) + 1059*
a^2*b^6*tan(d*x + c) + 35*a^7*b + 294*a^5*b^3 + 381*a^3*b^5)/((b*tan(d*x +
 c) + a)^3*a^9) - (137*a^4*b*tan(d*x + c)^5 + 1370*a^2*b^3*tan(d*x + c)^5
+ 1918*b^5*tan(d*x + c)^5 - 15*a^5*tan(d*x + c)^4 - 300*a^3*b^2*tan(d*x +
c)^4 - 525*a*b^4*tan(d*x + c)^4 + 60*a^4*b*tan(d*x + c)^3 + 150*a^2*b^3*ta
n(d*x + c)^3 - 10*a^5*tan(d*x + c)^2 - 50*a^3*b^2*tan(d*x + c)^2 + 15*a^4*
b*tan(d*x + c) - 3*a^5)/(a^9*tan(d*x + c)^5))/d
```

3.77.9 Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.12

$$\int \frac{\csc^6(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

$$= \frac{8\,b\,\mathrm{atanh}\left(\frac{4\,b\,(a+2\,b\tan(c+d\,x))\,(a^4+10\,a^2\,b^2+14\,b^4)}{a\,(4\,a^4\,b+40\,a^2\,b^3+56\,b^5)}\right)\,\left(a^4+10\,a^2\,b^2+14\,b^4\right)}{a^9\,d} - \frac{\frac{1}{5\,a}+\frac{\tan(c+d\,x)^4\,(a^4+10\,a^2\,b^2+14\,b^4)}{a^5}+\frac{2\tan(c+d\,x)^2\,(5\,a^2+7\,b^2)}{15\,a^3}-\frac{2\,b\tan(c+d\,x)}{5\,a^2}+\frac{22\,b\tan(c+d\,x)^5\,(a^4+10\,a^2\,b^2+14\,b^4)}{3\,a^6}+\frac{10\,b^2 \cdots}{}}{d\,\left(a^3\tan\left(c+d\,x\right)^5+3\,a^2\,b\tan\left(c+d\,x\right)^6+3\,a\,b^2\tan \cdots\right)}$$

input
```
int(1/(sin(c + d*x)^6*(a + b*tan(c + d*x))^4),x)
```

output
```
(8*b*atanh((4*b*(a + 2*b*tan(c + d*x))*(a^4 + 14*b^4 + 10*a^2*b^2))/(a*(4*
a^4*b + 56*b^5 + 40*a^2*b^3)))*(a^4 + 14*b^4 + 10*a^2*b^2))/(a^9*d) - (1/(
5*a) + (tan(c + d*x)^4*(a^4 + 14*b^4 + 10*a^2*b^2))/a^5 + (2*tan(c + d*x)^
2*(5*a^2 + 7*b^2))/(15*a^3) - (2*b*tan(c + d*x))/(5*a^2) + (22*b*tan(c + d
*x)^5*(a^4 + 14*b^4 + 10*a^2*b^2))/(3*a^6) + (10*b^2*tan(c + d*x)^6*(a^4 +
 14*b^4 + 10*a^2*b^2))/a^7 + (4*b^3*tan(c + d*x)^7*(a^4 + 14*b^4 + 10*a^2*
b^2))/a^8 - (2*b*tan(c + d*x)^3*(5*a^2 + 7*b^2))/(5*a^4))/(d*(a^3*tan(c +
d*x)^5 + b^3*tan(c + d*x)^8 + 3*a^2*b*tan(c + d*x)^6 + 3*a*b^2*tan(c + d*x
)^7))
```

3.78 $\int \frac{\csc(x)}{1+\tan(x)}\,dx$

3.78.1 Optimal result

Integrand size = 9, antiderivative size = 26

$$\int \frac{\csc(x)}{1+\tan(x)}\,dx = -\operatorname{arctanh}(\cos(x)) + \frac{\operatorname{arctanh}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
-arctanh(cos(x))+1/2*arctanh(1/2*(cos(x)-sin(x))*2^(1/2))*2^(1/2)
```

3.78.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{\csc(x)}{1+\tan(x)}\,dx = (1+i)(-1)^{3/4}\operatorname{arctanh}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Csc[x]/(1 + Tan[x]),x]
```

output
```
(1 + I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] - Log[Cos[x/2]] + Log[
Sin[x/2]]
```

3.78.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 4001, 3042, 3589, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{\tan(x)+1}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(x)(\tan(x)+1)}dx$$
$$\Big\downarrow \text{4001}$$
$$\int \frac{\cot(x)}{\sin(x)+\cos(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(x)}{\sin(x)(\sin(x)+\cos(x))}dx$$
$$\Big\downarrow \text{3589}$$
$$\int \left(\csc(x)+\frac{1}{-\sin(x)-\cos(x)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\text{arctanh}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\text{arctanh}(\cos(x))$$

input
```
Int[Csc[x]/(1 + Tan[x]),x]
```

output
```
-ArcTanh[Cos[x]] + ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2]
```

3.78.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3589
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

rule 4001
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

3.78.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
default	$\ln\left(\tan\left(\frac{x}{2}\right)\right)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tan\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)$	26
risch	$\ln\left(\mathrm{e}^{ix}-1\right)-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{ix}-\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{ix}+\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)}{2}-\ln\left(\mathrm{e}^{ix}+1\right)$	66

input
```
int(csc(x)/(1+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(tan(1/2*x))-2^(1/2)*arctanh(1/4*(2*tan(1/2*x)-2)*2^(1/2))
```

3.78.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(24) = 48$.

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.12

$$\int \frac{\csc(x)}{1+\tan(x)}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}+\cos(x)\right)\sin(x)-2\sqrt{2}\cos(x)-3}{2\cos(x)\sin(x)+1}\right) - \frac{1}{2}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

input
```
integrate(csc(x)/(1+tan(x)),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((2*(sqrt(2) + cos(x))*sin(x) - 2*sqrt(2)*cos(x) - 3)/(2*co
s(x)*sin(x) + 1)) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

3.78.6 Sympy [F]

$$\int \frac{\csc(x)}{1+\tan(x)}\,dx = \int \frac{\csc(x)}{\tan(x)+1}\,dx$$

input
```
integrate(csc(x)/(1+tan(x)),x)
```

output
```
Integral(csc(x)/(tan(x) + 1), x)
```

3.78.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(24) = 48$.

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{\csc(x)}{1+\tan(x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}+1}{\sqrt{2}+\frac{\sin(x)}{\cos(x)+1}-1}\right)+\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(csc(x)/(1+tan(x)),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) + 1)/(sqrt(2) + sin(x)/(co
s(x) + 1) - 1)) + log(sin(x)/(cos(x) + 1))
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \frac{\csc(x)}{1+\tan(x)}\,dx = \frac{1}{2}\,\sqrt{2}\log\left(\frac{\left|-2\,\sqrt{2}+2\,\tan\left(\frac{1}{2}\,x\right)-2\right|}{\left|2\,\sqrt{2}+2\,\tan\left(\frac{1}{2}\,x\right)-2\right|}\right)+\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

input
```
integrate(csc(x)/(1+tan(x)),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) - 2)/abs(2*sqrt(2) + 2*tan(1
/2*x) - 2)) + log(abs(tan(1/2*x)))
```

3.78.9 Mupad [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{\csc(x)}{1+\tan(x)}\,dx = \ln\left(\tan\left(\frac{x}{2}\right)\right)-\sqrt{2}\,\mathrm{atanh}\left(\frac{5\,\sqrt{2}\tan\left(\frac{x}{2}\right)+2\,\sqrt{2}}{7\tan\left(\frac{x}{2}\right)+3}\right)$$

input
```
int(1/(sin(x)*(tan(x) + 1)),x)
```

output
```
log(tan(x/2)) - 2^(1/2)*atanh((5*2^(1/2)*tan(x/2) + 2*2^(1/2))/(7*tan(x/2)
 + 3))
```

3.79 $\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx$

3.79.1 Optimal result

Integrand size = 21, antiderivative size = 229

$$\begin{aligned}&\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx\\&=\frac{a^3\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}\\&\quad+\frac{3a^2b\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},\sin^2(c+dx)\right)\sin^{2+m}(c+dx)}{d(2+m)}\\&\quad+\frac{3ab^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5+m}{2},\sin^2(c+dx)\right)\sec(c+dx)\sin^{3+m}(c+dx)}{d(3+m)}\\&\quad+\frac{b^3\,\text{Hypergeometric2F1}\left(2,\frac{4+m}{2},\frac{6+m}{2},\sin^2(c+dx)\right)\sin^{4+m}(c+dx)}{d(4+m)}\end{aligned}$$

output
```
3*a^2*b*hypergeom([1, 1+1/2*m],[2+1/2*m],sin(d*x+c)^2)*sin(d*x+c)^(2+m)/d/
(2+m)+b^3*hypergeom([2, 2+1/2*m],[3+1/2*m],sin(d*x+c)^2)*sin(d*x+c)^(4+m)/
d/(4+m)+a^3*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2
)*sin(d*x+c)^(1+m)/d/(1+m)/(cos(d*x+c)^2)^(1/2)+3*a*b^2*hypergeom([3/2, 3/
2+1/2*m],[5/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*sin(d*x+c)^(3+m)*(cos(d*x+c)
^2)^(1/2)/d/(3+m)
```

3.79.2 Mathematica [A] (verified)

Time = 2.76 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.90

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{\sin^{1+m}(c+dx)\left(\frac{a^3\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)}{1+m}+b\sin(c+dx)\left(\frac{3a^2\,\text{Hypergeometric2I}\right.\right.}{}$$

input
```
Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sin[c + d*x]^(1 + m)*((a^3*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1
 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x])/(1 + m) + b*Sin[c + d*x]
*((3*a^2*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2])/(2 +
m) + b*((b*Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[c + d*x]^2]*Sin[
c + d*x]^2)/(4 + m) + (3*a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3
+ m)/2, (5 + m)/2, Sin[c + d*x]^2]*Tan[c + d*x])/(3 + m)))))/d
```

3.79.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(c+dx)^m(a+b\tan(c+dx))^3dx$$
$$\Big\downarrow \text{4901}$$
$$\int \left(a^3\sin^m(c+dx)+3a^2b\sec(c+dx)\sin^{m+1}(c+dx)+3ab^2\sec^2(c+dx)\sin^{m+2}(c+dx)+b^3\sec^3(c+dx)\sin^{m+}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{a^3\cos(c+dx)\sin^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{3a^2b\sin^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},\sin^2(c+dx)\right)}{d(m+2)}+\frac{3ab^2\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{m+3}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+3}{2},\frac{m+5}{2},\sin^2(c+dx)\right)}{d(m+3)}+\frac{b^3\sin^{m+4}(c+dx)\,\text{Hypergeometric2F1}\left(2,\frac{m+4}{2},\frac{m+6}{2},\sin^2(c+dx)\right)}{d(m+4)}$$

input
```
Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^3,x]
```

output
```
(a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^2*b*Hyp
ergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 +
m))/(d*(2 + m)) + (3*a*b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3
+ m)/2, (5 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(3 + m))/(d*(
3 + m)) + (b^3*Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[c + d*x]^2]*
Sin[c + d*x]^(4 + m))/(d*(4 + m))
```

3.79.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.79.4 Maple [F]

$$\int \left(\sin^{m}\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^{3}dx$$

input
```
int(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x)
```

output
```
int(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x)
```

3.79.5 Fricas [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(b\tan\left(dx+c\right)+a\right)^{3}\sin\left(dx+c\right)^{m}\,dx$$

input
```
integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral((b^3*tan(d*x + c)^3 + 3*a*b^2*tan(d*x + c)^2 + 3*a^2*b*tan(d*x +
c) + a^3)*sin(d*x + c)^m, x)
```

3.79.6 Sympy [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^{3}\sin^{m}\left(c+dx\right)dx$$

input
```
integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*sin(c + d*x)**m, x)
```

3.79.7 Maxima [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx = \int (b\tan(dx+c)+a)^3\sin(dx+c)^m\,dx$$

input
```
integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^3*sin(d*x + c)^m, x)
```

3.79.8 Giac [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx = \int (b\tan(dx+c)+a)^3\sin(dx+c)^m\,dx$$

input
```
integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^3*sin(d*x + c)^m, x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^3\,dx = \int \sin(c+d\,x)^m\,(a+b\tan(c+d\,x))^3\,dx$$

input
```
int(sin(c + d*x)^m*(a + b*tan(c + d*x))^3,x)
```

output
```
int(sin(c + d*x)^m*(a + b*tan(c + d*x))^3, x)
```

3.80 $\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx$

3.80.1 Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{a^2\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}} + \frac{2ab\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},\sin^2(c+dx)\right)\sin^{2+m}(c+dx)}{d(2+m)} + \frac{b^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5+m}{2},\sin^2(c+dx)\right)\sec(c+dx)\sin^{3+m}(c+dx)}{d(3+m)}$$

output
```
2*a*b*hypergeom([1, 1+1/2*m],[2+1/2*m],sin(d*x+c)^2)*sin(d*x+c)^(2+m)/d/(2
+m)+a^2*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*si
n(d*x+c)^(1+m)/d/(1+m)/(cos(d*x+c)^2)^(1/2)+b^2*hypergeom([3/2, 3/2+1/2*m]
,[5/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*sin(d*x+c)^(3+m)*(cos(d*x+c)^2)^(1/2
)/d/(3+m)
```

3.80.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.93

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{\sin^{1+m}(c+dx)\left(\frac{a^2\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)}{1+m}+\frac{b\sin(c+dx)\left(2a(3+m)\,\text{Hypergeometric2F}\right.}{}\right.}{d}$$

input
```
Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sin[c + d*x]^(1 + m)*((a^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1
 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x])/(1 + m) + (b*Sin[c + d*x
]*(2*a*(3 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]
+ b*(2 + m)*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)
/2, Sin[c + d*x]^2]*Tan[c + d*x]))/((2 + m)*(3 + m))))/d
```

3.80.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(c+dx)^m(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \; 4901$$

$$\int \left(a^2\sin^m(c+dx)+2ab\sec(c+dx)\sin^{m+1}(c+dx)+b^2\sec^2(c+dx)\sin^{m+2}(c+dx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2 \cos(c+dx) \sin^{m+1}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}} + \frac{2ab \sin^{m+2}(c+dx) \operatorname{Hypergeometric2F1}\left(1, \frac{m+2}{2}, \frac{m+4}{2}, \sin^2(c+dx)\right)}{d(m+2)} + \frac{b^2 \sqrt{\cos^2(c+dx)} \sec(c+dx) \sin^{m+3}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{3}{2}, \frac{m+3}{2}, \frac{m+5}{2}, \sin^2(c+dx)\right)}{d(m+3)}$$

input
```
Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^2,x]
```

output
```
(a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*a*b*Hyper
geometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m)
)/(d*(2 + m)) + (b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2
, (5 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(3 + m))/(d*(3 + m)
)
```

3.80.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.80.4 Maple [F]

$$\int \left(\sin^m (dx+c)\right) \left(a+b\tan (dx+c)\right)^2 dx$$

input
```
int(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x)
```

output
```
int(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x)
```

3.80.5 Fricas [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx = \int (b\tan(dx+c)+a)^2\sin(dx+c)^m\,dx$$

input
```
integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)*sin(d*x + c)^m, x
)
```

3.80.6 Sympy [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\sin^m(c+dx)\,dx$$

input
```
integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sin(c + d*x)**m, x)
```

3.80.7 Maxima [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx = \int (b\tan(dx+c)+a)^2\sin(dx+c)^m\,dx$$

input
```
integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^2*sin(d*x + c)^m, x)
```

3.80.8 Giac [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx=\int (b\tan(dx+c)+a)^2\sin(dx+c)^m\,dx$$

input
```
integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^2*sin(d*x + c)^m, x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^2\,dx=\int \sin(c+d\,x)^m\,(a+b\tan(c+d\,x))^2\,dx$$

input
```
int(sin(c + d*x)^m*(a + b*tan(c + d*x))^2,x)
```

output
```
int(sin(c + d*x)^m*(a + b*tan(c + d*x))^2, x)
```

3.81 $\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx$

3.81.1 Optimal result

Integrand size = 19, antiderivative size = 109

$$\begin{aligned}\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx &= \frac{a\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sin^{1+m}(c+dx)}{d(1+m)\sqrt{\cos^2(c+dx)}}\\ &\quad+\frac{b\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},\sin^2(c+dx)\right)\sin^{2+m}(c+dx)}{d(2+m)}\end{aligned}$$

output
```
b*hypergeom([1, 1+1/2*m],[2+1/2*m],sin(d*x+c)^2)*sin(d*x+c)^(2+m)/d/(2+m)+
a*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sin(d*x+
c)^(1+m)/d/(1+m)/(cos(d*x+c)^2)^(1/2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx &= \frac{a\sqrt{\cos^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(c+dx)\right)\sec(c+dx)\sin^{1+m}(c+dx)}{d(1+m)}\\ &\quad+\frac{b\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},\sin^2(c+dx)\right)\sin^{2+m}(c+dx)}{d(2+m)}\end{aligned}$$

input
```
Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x]),x]
```

output
```
(a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c
 + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + m))/(d*(1 + m)) + (b*Hypergeomet
ric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(
2 + m))
```

3.81.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)^m(a+b\tan(c+dx))dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(a\sin^m(c+dx)+b\sec(c+dx)\sin^{m+1}(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\cos(c+dx)\sin^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{b\sin^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},\sin^2(c+dx)\right)}{d(m+2)}$$

input
```
Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x]),x]
```

output
```
(a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^
2]*Sin[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (b*Hypergeomet
ric2F1[1, (2 + m)/2, (4 + m)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + m))/(d*(
2 + m))
```

3.81.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.81.4 Maple [F]

$$\int (\sin^m (dx + c)) (a + b \tan (dx + c)) \, dx$$

input
```
int(sin(d*x+c)^m*(a+b*tan(d*x+c)),x)
```

output
```
int(sin(d*x+c)^m*(a+b*tan(d*x+c)),x)
```

3.81.5 Fricas [F]

$$\int \sin^m(c + dx)(a + b\tan(c + dx)) \, dx = \int (b \tan (dx + c) + a) \sin (dx + c)^m \, dx$$

input
```
integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)
```

3.81.6 Sympy [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\sin^m(c+dx)\,dx$$

input
```
integrate(sin(d*x+c)**m*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*sin(c + d*x)**m, x)
```

3.81.7 Maxima [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx = \int (b\tan(dx+c)+a)\sin(dx+c)^m\,dx$$

input
```
integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)
```

3.81.8 Giac [F]

$$\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx = \int (b\tan(dx+c)+a)\sin(dx+c)^m\,dx$$

input
```
integrate(sin(d*x+c)^m*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)*sin(d*x + c)^m, x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \sin^m(c+dx)(a+b\tan(c+dx))\,dx = \int \sin\left(c+d\,x\right)^m\,\left(a+b\tan(c+d\,x)\right)\,dx$$

input
```
int(sin(c + d*x)^m*(a + b*tan(c + d*x)),x)
```

output
```
int(sin(c + d*x)^m*(a + b*tan(c + d*x)), x)
```

3.82 $\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx$

3.82.1 Optimal result

Integrand size = 21, antiderivative size = 765

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$= \frac{2^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},1+m,\frac{3+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^m (1+ \cdots}{ad(1+m)}$$

$$+\frac{2^{1+m}b\,\text{AppellF1}\left(\frac{2+m}{2},1+m,1,\frac{4+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\left(b-\sqrt{a^2+b^2}\right)^2}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+ \cdots}{1+\tan^2\left(\frac{1}{2}(c \cdots}\right. \cdots}{\sqrt{a^2+b^2}\left(b-\sqrt{a^2+b^2}\right)d(2+m)}$$

$$-\frac{2^{1+m}b\,\text{AppellF1}\left(\frac{2+m}{2},1+m,1,\frac{4+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\left(b+\sqrt{a^2+b^2}\right)^2}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+ \cdots}{1+\tan^2\left(\frac{1}{2}(c \cdots}\right. \cdots}{\sqrt{a^2+b^2}\left(b+\sqrt{a^2+b^2}\right)d(2+m)}$$

$$+\frac{2^{1+m}ab\,\text{AppellF1}\left(\frac{3+m}{2},1+m,1,\frac{5+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\left(b-\sqrt{a^2+b^2}\right)^2}\right)\tan^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c \cdots}{1+\tan^2\left(\frac{1}{2} \cdots}\right. \cdots}{\sqrt{a^2+b^2}\left(b-\sqrt{a^2+b^2}\right)^2 d(3+m)}$$

$$-\frac{2^{1+m}ab\,\text{AppellF1}\left(\frac{3+m}{2},1+m,1,\frac{5+m}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{a^2\tan^2\left(\frac{1}{2}(c+dx)\right)}{\left(b+\sqrt{a^2+b^2}\right)^2}\right)\tan^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c \cdots}{1+\tan^2\left(\frac{1}{2} \cdots}\right. \cdots}{\sqrt{a^2+b^2}\left(b+\sqrt{a^2+b^2}\right)^2 d(3+m)}$$

output
```
2^(1+m)*hypergeom([1+m, 1/2+1/2*m],[3/2+1/2*m],-tan(1/2*d*x+1/2*c)^2)*tan(
1/2*d*x+1/2*c)*(tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2))^m*(1+tan(1/2*
d*x+1/2*c)^2)^m/a/d/(1+m)+2^(1+m)*b*AppellF1(1+1/2*m,1,1+m,2+1/2*m,a^2*tan
(1/2*d*x+1/2*c)^2/(b-(a^2+b^2)^(1/2))^2,-tan(1/2*d*x+1/2*c)^2)*tan(1/2*d*x
+1/2*c)^2*(tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2))^m*(1+tan(1/2*d*x+1
/2*c)^2)^m/d/(2+m)/(b-(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)-2^(1+m)*b*AppellF1(
1+1/2*m,1,1+m,2+1/2*m,a^2*tan(1/2*d*x+1/2*c)^2/(b+(a^2+b^2)^(1/2))^2,-tan(
1/2*d*x+1/2*c)^2)*tan(1/2*d*x+1/2*c)^2*(tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+
1/2*c)^2))^m*(1+tan(1/2*d*x+1/2*c)^2)^m/d/(2+m)/(a^2+b^2)^(1/2)/(b+(a^2+b^
2)^(1/2))+2^(1+m)*a*b*AppellF1(3/2+1/2*m,1,1+m,5/2+1/2*m,a^2*tan(1/2*d*x+1
/2*c)^2/(b-(a^2+b^2)^(1/2))^2,-tan(1/2*d*x+1/2*c)^2)*tan(1/2*d*x+1/2*c)^3*
(tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2))^m*(1+tan(1/2*d*x+1/2*c)^2)^m
/d/(3+m)/(b-(a^2+b^2)^(1/2))^2/(a^2+b^2)^(1/2)-2^(1+m)*a*b*AppellF1(3/2+1/
2*m,1,1+m,5/2+1/2*m,a^2*tan(1/2*d*x+1/2*c)^2/(b+(a^2+b^2)^(1/2))^2,-tan(1/
2*d*x+1/2*c)^2)*tan(1/2*d*x+1/2*c)^3*(tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/
2*c)^2))^m*(1+tan(1/2*d*x+1/2*c)^2)^m/d/(3+m)/(a^2+b^2)^(1/2)/(b+(a^2+b^2)
^(1/2))^2
```

3.82.2 Mathematica [F]

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx=\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx$$

input
```
Integrate[Sin[c + d*x]^m/(a + b*Tan[c + d*x]),x]
```

output
```
Integrate[Sin[c + d*x]^m/(a + b*Tan[c + d*x]), x]
```

3.82.3 Rubi [A] (verified)

Time = 2.42 (sec) , antiderivative size = 580, normalized size of antiderivative = 0.76, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4902, 27, 7270, 7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(c+dx)^m}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \; 4902$$

$$\frac{2\int \frac{2^m \cot\left(\frac{1}{2}(c+dx)\right)\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^{m+1}}{-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{2^{m+1}\int \frac{\cot\left(\frac{1}{2}(c+dx)\right)\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^{m+1}}{-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \; 7270$$

$$\frac{2^{m+1}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m\int \frac{\tan^m\left(\frac{1}{2}(c+dx)\right)\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\tan^2\left(\frac{1}{2}(c+dx)\right)\right.}{-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}}{d}$$

$$\Big\downarrow \; 7279$$

$$\frac{2^{m+1}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m\int\left(\frac{\tan^m\left(\frac{1}{2}(c+dx)\right)\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^{-m-1}}{a}-\frac{2b\,t}{a}\right.}{d}$$

$$\Big\downarrow \; 2009$$

$$2^{m+1}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m\left(\frac{b\tan^{m+2}\left(\frac{1}{2}(c+dx)\right)\text{AppellF1}\left(\frac{m+2}{2},m+1,1,\frac{m+4}{2},-\right.}{(m+2)\sqrt{a^2+b^2}\left(b-\sqrt{a^2}\right.}\right.$$

input
```
Int[Sin[c + d*x]^m/(a + b*Tan[c + d*x]),x]
```

3.82. $\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx$

output
```
(2^(1 + m)*(Tan[(c + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^m*(1 + Tan[(c + d*x
)/2]^2)^m*((Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/
2]^2]*Tan[(c + d*x)/2]^(1 + m))/(a*(1 + m)) + (b*AppellF1[(2 + m)/2, 1 + m
, 1, (4 + m)/2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b - Sqrt[a^
2 + b^2])^2]*Tan[(c + d*x)/2]^(2 + m))/(Sqrt[a^2 + b^2]*(b - Sqrt[a^2 + b^
2])*(2 + m)) - (b*AppellF1[(2 + m)/2, 1 + m, 1, (4 + m)/2, -Tan[(c + d*x)/
2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b + Sqrt[a^2 + b^2])^2]*Tan[(c + d*x)/2]^(
2 + m))/(Sqrt[a^2 + b^2]*(b + Sqrt[a^2 + b^2])*(2 + m)) + (a*b*AppellF1[(3
 + m)/2, 1 + m, 1, (5 + m)/2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^2
)/(b - Sqrt[a^2 + b^2])^2]*Tan[(c + d*x)/2]^(3 + m))/(Sqrt[a^2 + b^2]*(b -
 Sqrt[a^2 + b^2])^2*(3 + m)) - (a*b*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m)/
2, -Tan[(c + d*x)/2]^2, (a^2*Tan[(c + d*x)/2]^2)/(b + Sqrt[a^2 + b^2])^2]*
Tan[(c + d*x)/2]^(3 + m))/(Sqrt[a^2 + b^2]*(b + Sqrt[a^2 + b^2])^2*(3 + m)
)))/(d*Tan[(c + d*x)/2]^m)
```

3.82.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.82.4 Maple [F]

$$\int \frac{\sin^{m}\left(dx+c\right)}{a+b\tan\left(dx+c\right)}dx$$

input
```
int(sin(d*x+c)^m/(a+b*tan(d*x+c)),x)
```

output
```
int(sin(d*x+c)^m/(a+b*tan(d*x+c)),x)
```

3.82.5 Fricas [F]

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sin\left(dx+c\right)^{m}}{b\tan\left(dx+c\right)+a}\,dx$$

input
```
integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)
```

3.82.6 Sympy [F]

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sin^{m}\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

input
```
integrate(sin(d*x+c)**m/(a+b*tan(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)**m/(a + b*tan(c + d*x)), x)
```

3.82.7 Maxima [F]

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sin\left(dx+c\right)^m}{b\tan\left(dx+c\right)+a}\,dx$$

input
```
integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)
```

3.82.8 Giac [F]

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sin\left(dx+c\right)^m}{b\tan\left(dx+c\right)+a}\,dx$$

input
```
integrate(sin(d*x+c)^m/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sin(d*x + c)^m/(b*tan(d*x + c) + a), x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^m(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sin\left(c+d\,x\right)^m}{a+b\tan\left(c+d\,x\right)}\,dx$$

input
```
int(sin(c + d*x)^m/(a + b*tan(c + d*x)),x)
```

output
```
int(sin(c + d*x)^m/(a + b*tan(c + d*x)), x)
```

3.83 $\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$

3.83.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx = \text{Int}(\sin^m(c+dx)(a+b\tan(c+dx))^n, x)$$

output
```
CannotIntegrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x)
```

3.83.2 Mathematica [N/A]

Not integrable

Time = 6.55 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$$

input
```
Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n,x]
```

output
```
Integrate[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n, x]
```

3.83.3 Rubi [N/A]

Not integrable

Time = 2.85 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4902, 27, 7270, 7274, 7292, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)^m(a+b\tan(c+dx))^n dx$$

$$\Big\downarrow \text{4902}$$

$$\frac{2\int 2^m\cot\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^{m+1}\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2^{m+1}\int \cot\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^{m+1}\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{7270}$$

$$\frac{2^{m+1}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m\int \tan^m\left(\frac{1}{2}(c+dx)\right)\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^-}{d}$$

$$\Big\downarrow \text{7274}$$

$$2^{m+1}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx\right)}\right.$$

$$\Big\downarrow \text{7292}$$

$$2^{m+1}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx\right)}\right.$$

$$\Big\downarrow \text{7299}$$

$$\frac{2^{m+1}\tan^{-m}\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}\right)^m\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^m\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right.}{}$$

input
```
Int[Sin[c + d*x]^m*(a + b*Tan[c + d*x])^n,x]
```

output
```
$Aborted
```

3.83.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

rule 7274
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Simp[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p])   I
nt[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Integ
erQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.83.4 Maple [N/A] (verified)

Not integrable

Time = 1.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (\sin^m (dx + c)) (a + b \tan (dx + c))^n \, dx$$

input
```
int(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x)
```

output
```
int(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x)
```

3.83.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n \, dx = \int (b \tan (dx + c) + a)^n \sin (dx + c)^m \, dx$$

input
```
integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

3.83.6 Sympy [N/A]

Not integrable

Time = 54.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx = \int (a+b\tan{(c+dx)})^n \sin^m{(c+dx)}\,dx$$

input
```
integrate(sin(d*x+c)**m*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*sin(c + d*x)**m, x)
```

3.83.7 Maxima [N/A]

Not integrable

Time = 2.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan{(dx+c)}+a)^n \sin{(dx+c)}^m\,dx$$

input
```
integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

3.83.8 Giac [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan{(dx+c)}+a)^n \sin{(dx+c)}^m\,dx$$

input
```
integrate(sin(d*x+c)^m*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^m, x)
```

3.83.9 Mupad [N/A]

Not integrable

Time = 8.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^m(c+dx)(a+b\tan(c+dx))^n\,dx=\int \sin\left(c+d\,x\right)^m\,\left(a+b\tan(c+d\,x)\right)^n\,dx$$

input
```
int(sin(c + d*x)^m*(a + b*tan(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)^m*(a + b*tan(c + d*x))^n, x)
```

3.84 $\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx$

3.84.1 Optimal result

Integrand size = 21, antiderivative size = 435

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx =$$
$$-\frac{\left(ab^2n(5a^2+b^2(3+2n))+\sqrt{-b^2}(3a^4+a^2b^2(6+6n-n^2)+b^4(3+4n+n^2))\right)\text{Hypergeometric2F1}\left(1\right.}{16b\left(a^2+b^2\right)^2\left(a-\sqrt{-b^2}\right)d(1+n)}$$
$$-\frac{\left(ab^2n(5a^2+b^2(3+2n))-\sqrt{-b^2}(3a^4+a^2b^2(6+6n-n^2)+b^4(3+4n+n^2))\right)\text{Hypergeometric2F1}\left(1\right.}{16b\left(a^2+b^2\right)^2\left(a+\sqrt{-b^2}\right)d(1+n)}$$
$$+\frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d}$$
$$-\frac{\cos^2(c+dx)(a+b\tan(c+dx))^{1+n}\left(b(a^2(7-n)+b^2(5+n))+a(5a^2+b^2(3+2n))\tan(c+dx)\right)}{8\left(a^2+b^2\right)^2d}$$

output
```
-1/16*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))*(a*b^2*n
*(5*a^2+b^2*(3+2*n))-(3*a^4+a^2*b^2*(-n^2+6*n+6)+b^4*(n^2+4*n+3))*(-b^2)^(
1/2))*(a+b*tan(d*x+c))^(1+n)/b/(a^2+b^2)^2/d/(1+n)/(a+(-b^2)^(1/2))-1/16*h
ypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)))*(a*b^2*n*(5*a^2
+b^2*(3+2*n))+(3*a^4+a^2*b^2*(-n^2+6*n+6)+b^4*(n^2+4*n+3))*(-b^2)^(1/2))*(
a+b*tan(d*x+c))^(1+n)/b/(a^2+b^2)^2/d/(1+n)/(a-(-b^2)^(1/2))+1/4*cos(d*x+c
)^4*(b+a*tan(d*x+c))*(a+b*tan(d*x+c))^(1+n)/(a^2+b^2)/d-1/8*cos(d*x+c)^2*(
a+b*tan(d*x+c))^(1+n)*(b*(a^2*(7-n)+b^2*(5+n))+a*(5*a^2+b^2*(3+2*n))*tan(d
*x+c))/(a^2+b^2)^2/d
```

3.84.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 910 vs. $2(435) = 870$.

Time = 6.81 (sec) , antiderivative size = 910, normalized size of antiderivative = 2.09

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$= \frac{b\left(\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+n}}{2\sqrt{-b^2}\left(a-\sqrt{-b^2}\right)(1+n)} - \frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)(a+b\tan(c+dx)}{2\sqrt{-b^2}\left(a+\sqrt{-b^2}\right)(1+n)}\right.}{}$$

input
```
Integrate[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

output
```
(b*((Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^
2])]*(a + b*Tan[c + d*x])^(1 + n))/(2*Sqrt[-b^2]*(a - Sqrt[-b^2])*(1 + n))
 - (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2
])]*(a + b*Tan[c + d*x])^(1 + n))/(2*Sqrt[-b^2]*(a + Sqrt[-b^2])*(1 + n))
- (Cos[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b*Tan[c + d*x]))/(
b^2*(a^2 + b^2)) + (Cos[c + d*x]^4*(a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b
*Tan[c + d*x]))/(4*b^2*(a^2 + b^2)) + (((Sqrt[-b^2]*(a^2 + b^2*(1 - n)) -
a*b^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt
[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(b^2*(a - Sqrt[-b^2])*(1 + n)) - ((
a^2*Sqrt[-b^2] - (-b^2)^(3/2)*(1 - n) + a*b^2*n)*Hypergeometric2F1[1, 1 +
n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 +
 n))/(b^2*(a + Sqrt[-b^2])*(1 + n)))/(2*(a^2 + b^2)) - (b^2*((Cos[c + d*x]
^2*(a + b*Tan[c + d*x])^(1 + n)*(b^2*(-3*a^2 - b^2*(3 - n)) + a^2*b^2*(2 -
 n) + b*(a*(-3*a^2 - b^2*(3 - n)) - a*b^2*(2 - n))*Tan[c + d*x]))/(2*b^4*(
a^2 + b^2)) - (((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n - Sqrt[-b^2]*(3*a^4 + a^2
*b^2*(6 - 2*n - n^2) + b^4*(3 - 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2
 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))
/(2*b^2*(a - Sqrt[-b^2])*(1 + n)) + ((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sq
rt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2*n - n^2) + b^4*(3 - 4*n + n^2)))*Hypergeo
metric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + ...
```

3.84.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3999, 602, 2180, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(c+dx)^4(a+b\tan(c+dx))^ndx$$

$$\Big\downarrow \text{ 3999}$$

$$\frac{b\int \frac{b^4\tan^4(c+dx)(a+b\tan(c+dx))^n}{\left(\tan^2(c+dx)b^2+b^2\right)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 602}$$

$$b\Bigg(\frac{b^2\left(ab\tan(c+dx)+b^2\right)(a+b\tan(c+dx))^{n+1}}{4(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}-\frac{\int \frac{(a+b\tan(c+dx))^n\left(-a(2-n)\tan(c+dx)b^5-4\left(a^2+b^2\right)\tan^2(c+dx)b^4+\left(a^2+b^2(n+1)\right)b^4\right)}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx}{4b^2(a^2+b^2)}$$

$$d$$

$$\Big\downarrow \text{ 2180}$$

$$b\Bigg(\frac{b^2\left(ab\tan(c+dx)+b^2\right)(a+b\tan(c+dx))^{n+1}}{4(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}-\frac{\frac{b^2(a+b\tan(c+dx))^{n+1}\left(ab\left(5a^2+b^2(2n+3)\right)\tan(c+dx)+b^2\left(a^2(7-n)+b^2(n+5)\right)\right)}{2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{\int\frac{b^4(a+b\tan(c+dx))}{}}{}}{4b^2(a^2+b^2}$$

$$d$$

$$\Big\downarrow \text{ 27}$$

$$b\Bigg(\frac{b^2\left(ab\tan(c+dx)+b^2\right)(a+b\tan(c+dx))^{n+1}}{4(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}-\frac{\frac{b^2(a+b\tan(c+dx))^{n+1}\left(ab\left(5a^2+b^2(2n+3)\right)\tan(c+dx)+b^2\left(a^2(7-n)+b^2(n+5)\right)\right)}{2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{b^2\int\frac{(a+b\tan(c+dx))}{}}{}}{4b^2(a^2+b^2}$$

$$d$$

$$\Big\downarrow \text{ 657}$$

$$b\left(\frac{b^2(ab\tan(c+dx)+b^2)(a+b\tan(c+dx))^{n+1}}{4(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}-\frac{\frac{b^2(a+b\tan(c+dx))^{n+1}\left(ab\left(5a^2+b^2(2n+3)\right)\tan(c+dx)+b^2\left(a^2(7-n)+b^2(n+5)\right)\right)}{2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{b^2\int\left(\frac{\left(\sqrt{-b^2}\left(3a^4+b\right.\right.}{}\right.}{}}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$b\left(\frac{b^2(ab\tan(c+dx)+b^2)(a+b\tan(c+dx))^{n+1}}{4(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}-\frac{\frac{b^2(a+b\tan(c+dx))^{n+1}\left(ab\left(5a^2+b^2(2n+3)\right)\tan(c+dx)+b^2\left(a^2(7-n)+b^2(n+5)\right)\right)}{2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{b^2\left(-\frac{\left(ab^2n\left(5a^2+b^2\right.\right.}{}\right.}{}}{}\right.$$

input
```
Int[Sin[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

output
```
(b*((b^2*(a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b*Tan[c + d*x]))/(4*(a^2 +
b^2)*(b^2 + b^2*Tan[c + d*x]^2)^2) - ((b^2*(a + b*Tan[c + d*x])^(1 + n)*(b
^2*(a^2*(7 - n) + b^2*(5 + n)) + a*b*(5*a^2 + b^2*(3 + 2*n))*Tan[c + d*x])
)/(2*(a^2 + b^2)*(b^2 + b^2*Tan[c + d*x]^2)) - (b^2*(-1/2*((a*b^2*n*(5*a^2
 + b^2*(3 + 2*n)) + Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 + 6*n - n^2) + b^4*(3 +
 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a -
 Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(b^2*(a - Sqrt[-b^2])*(1 + n))
 - ((a*b^2*n*(5*a^2 + b^2*(3 + 2*n)) - Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 + 6*
n - n^2) + b^4*(3 + 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b
*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(2*b^2*(a +
 Sqrt[-b^2])*(1 + n))))/(2*(a^2 + b^2)))/(4*b^2*(a^2 + b^2))))/d
```

3.84. $\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx$

3.84.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 602
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomia
lRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
+ b*x^2, x], x, 1]}, Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^(p + 1)*((a*(d*e
 - c*f) + (b*c*e + a*d*f)*x)/(2*a*(p + 1)*(b*c^2 + a*d^2))), x] + Simp[1/(2
*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*x^2)^(p + 1)*ExpandToS
um[2*a*(p + 1)*(b*c^2 + a*d^2)*Qx + e*(b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3
)) - a*c*d*f*n + d*(b*c*e + a*d*f)*(n + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, d, n}, x] && IGtQ[m, 1] && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.84.4 Maple [F]

$$\int \left(\sin^4\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^n dx$$

input
```
int(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

output
```
int(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

3.84.5 Fricas [F]

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int \left(b\tan\left(dx+c\right)+a\right)^n\sin\left(dx+c\right)^4\,dx$$

input
```
integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(b*tan(d*x + c) + a)^n, x
)
```

3.84.6 Sympy [F]

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^n\sin^4\left(c+dx\right)dx$$

input
```
integrate(sin(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*sin(c + d*x)**4, x)
```

3.84.7 Maxima [F]

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n \sin(dx+c)^4\,dx$$

input
```
integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

3.84.8 Giac [F]

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n \sin(dx+c)^4\,dx$$

input
```
integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \sin^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin(c+d\,x)^4\,(a+b\tan(c+d\,x))^n\,dx$$

input
```
int(sin(c + d*x)^4*(a + b*tan(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)^4*(a + b*tan(c + d*x))^n, x)
```

3.85 $\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx$

3.85.1 Optimal result

Integrand size = 21, antiderivative size = 276

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx = -\frac{\left(ab^2n+\sqrt{-b^2}(a^2+b^2(1+n))\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1}}{4b\left(a^2+b^2\right)\left(a-\sqrt{-b^2}\right)d(1+n)} - \frac{\left(ab^2n-\sqrt{-b^2}(a^2+b^2(1+n))\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1}}{4b\left(a^2+b^2\right)\left(a+\sqrt{-b^2}\right)d(1+n)} - \frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2\left(a^2+b^2\right)d}$$

output
```
-1/4*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))*(a*b^2*n-
(a^2+b^2*(1+n))*(-b^2)^(1/2))*(a+b*tan(d*x+c))^(1+n)/b/(a^2+b^2)/d/(1+n)/(
a+(-b^2)^(1/2))-1/4*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1
/2)))*(a*b^2*n+(a^2+b^2*(1+n))*(-b^2)^(1/2))*(a+b*tan(d*x+c))^(1+n)/b/(a^2
+b^2)/d/(1+n)/(a-(-b^2)^(1/2))-1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))*(a+b*tan(
d*x+c))^(1+n)/(a^2+b^2)/d
```

3.85.2 Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.98

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$=\frac{\left(\left(a^3\sqrt{-b^2}+a^2b^2(-1+n)-b^4(1+n)-a(-b^2)^{3/2}(1+2n)\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan}{a-}\right.\right.}{}$$

input
```
Integrate[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

output
```
(((a^3*Sqrt[-b^2] + a^2*b^2*(-1 + n) - b^4*(1 + n) - a*(-b^2)^(3/2)*(1 + 2
*n))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^
2])] - (a^3*Sqrt[-b^2] - a^2*b^2*(-1 + n) + b^4*(1 + n) - a*(-b^2)^(3/2)*(
1 + 2*n))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqr
t[-b^2])] + 2*b*(a^2 + b^2)*(1 + n)*Cos[c + d*x]*(b*Cos[c + d*x] + a*Sin[c
 + d*x]))*(a + b*Tan[c + d*x])^(1 + n))/(4*b*(a^2 + b^2)*(-a + Sqrt[-b^2])
*(a + Sqrt[-b^2])*d*(1 + n))
```

3.85.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3999, 602, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(c+dx)^2(a+b\tan(c+dx))^ndx$$
$$\Big\downarrow \text{3999}$$
$$\frac{b\int \frac{b^2\tan^2(c+dx)(a+b\tan(c+dx))^n}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{d}$$
$$\Big\downarrow \text{602}$$

$$\frac{b\left(-\frac{\int -\frac{b^2(a+b\tan(c+dx))^n\left(a^2+bn\tan(c+dx)a+b^2(n+1)\right)}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2(a^2+b^2)}-\frac{(ab\tan(c+dx)+b^2)(a+b\tan(c+dx))^{n+1}}{2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{\int \frac{b^2(a+b\tan(c+dx))^n\left(a^2+bn\tan(c+dx)a+b^2(n+1)\right)}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2(a^2+b^2)}-\frac{(ab\tan(c+dx)+b^2)(a+b\tan(c+dx))^{n+1}}{2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$\Big\downarrow$ 27

$$\frac{b\left(\frac{\int \frac{(a+b\tan(c+dx))^n\left(a^2+bn\tan(c+dx)a+b^2(n+1)\right)}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2(a^2+b^2)}-\frac{(ab\tan(c+dx)+b^2)(a+b\tan(c+dx))^{n+1}}{2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$\Big\downarrow$ 657

$$\frac{b\left(\frac{\int \left(\frac{\left(\sqrt{-b^2}\left(a^2+b^2(n+1)\right)-ab^2n\right)(a+b\tan(c+dx))^n}{2b^2\left(\sqrt{-b^2}-b\tan(c+dx)\right)}+\frac{\left(anb^2+\sqrt{-b^2}\left(a^2+b^2(n+1)\right)\right)(a+b\tan(c+dx))^n}{2b^2\left(b\tan(c+dx)+\sqrt{-b^2}\right)}\right)d(b\tan(c+dx))}{2(a^2+b^2)}-\frac{(ab\tan(c+dx)+b^2)(a+b\tan(c+dx))^{n+1}}{2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$\Big\downarrow$ 2009

$$\frac{b\left(\frac{-\frac{\left(\sqrt{-b^2}\left(a^2+b^2(n+1)\right)+ab^2n\right)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{2b^2(n+1)\left(a-\sqrt{-b^2}\right)}-\frac{\left(ab^2n-\sqrt{-b^2}\left(a^2+b^2(n+1)\right)\right)(a+b\tan(c+dx))^{n+1}}{2b^2(n+1)(}}{2(a^2+b^2)}\right)}{d}$$

input
```
Int[Sin[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

output
```
(b*(-1/2*((a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b*Tan[c + d*x]))/((a^2 + b
^2)*(b^2 + b^2*Tan[c + d*x]^2)) + (-1/2*((a*b^2*n + Sqrt[-b^2]*(a^2 + b^2*
(1 + n)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqr
t[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(b^2*(a - Sqrt[-b^2])*(1 + n)) - (
(a*b^2*n - Sqrt[-b^2]*(a^2 + b^2*(1 + n)))*Hypergeometric2F1[1, 1 + n, 2 +
 n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(
2*b^2*(a + Sqrt[-b^2])*(1 + n)))/(2*(a^2 + b^2))))/d
```

3.85. $\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx$

3.85.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 602
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomia
lRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
+ b*x^2, x], x, 1]}, Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^(p + 1)*((a*(d*e
 - c*f) + (b*c*e + a*d*f)*x)/(2*a*(p + 1)*(b*c^2 + a*d^2))), x] + Simp[1/(2
*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*x^2)^(p + 1)*ExpandToS
um[2*a*(p + 1)*(b*c^2 + a*d^2)*Qx + e*(b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3
)) - a*c*d*f*n + d*(b*c*e + a*d*f)*(n + 2*p + 4)*x, x], x], x]] /; FreeQ[{a
, b, c, d, n}, x] && IGtQ[m, 1] && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.85.4 Maple [F]

$$\int \left(\sin^2(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

input
```
int(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

output
```
int(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

3.85.5 Fricas [F]

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\sin(dx+c)^2\,dx$$

input
```
integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(-(cos(d*x + c)^2 - 1)*(b*tan(d*x + c) + a)^n, x)
```

3.85.6 Sympy [F]

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (a+b\tan(c+dx))^n\sin^2(c+dx)\,dx$$

input
```
integrate(sin(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*sin(c + d*x)**2, x)
```

3.85.7 Maxima [F]

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\sin(dx+c)^2\,dx$$

input
```
integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

3.85.8 Giac [F]

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\sin(dx+c)^2\,dx$$

input
```
integrate(sin(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \sin^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin(c+d\,x)^2\,(a+b\tan(c+d\,x))^n\,dx$$

input
```
int(sin(c + d*x)^2*(a + b*tan(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)^2*(a + b*tan(c + d*x))^n, x)
```

3.86 $\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx$

3.86.1 Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^{1+n}}{a^2d(1+n)}$$

output
```
b*hypergeom([2, 1+n],[2+n],1+b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^(1+n)/a^2/d/
(1+n)
```

3.86.2 Mathematica [A] (verified)

Time = 6.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^{1+n}}{a^2d(1+n)}$$

input
```
Integrate[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

output
```
(b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c
 + d*x])^(1 + n))/(a^2*d*(1 + n))
```

3.86.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3999, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^n}{\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3999}$$

$$\frac{b\int \frac{\cot^2(c+dx)(a+b\tan(c+dx))^n}{b^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{75}$$

$$\frac{b(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{b\tan(c+dx)}{a}+1\right)}{a^2d(n+1)}$$

input
```
Int[Csc[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

output
```
(b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c
 + d*x])^(1 + n))/(a^2*d*(1 + n))
```

3.86.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.86.4 Maple [F]

$$\int \left(\csc^2\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^n dx$$

input
```
int(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

output
```
int(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

3.86.5 Fricas [F]

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int \left(b\tan\left(dx+c\right)+a\right)^n \csc\left(dx+c\right)^2\,dx$$

input
```
integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

3.86.6 Sympy [F]

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^n \csc^2\left(c+dx\right)dx$$

input
```
integrate(csc(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*csc(c + d*x)**2, x)
```

3.86.7 Maxima [F]

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\csc(dx+c)^2\,dx$$

input
```
integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

3.86.8 Giac [F]

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\csc(dx+c)^2\,dx$$

input
```
integrate(csc(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \csc^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int \frac{(a+b\tan(c+d\,x))^n}{\sin(c+d\,x)^2}\,dx$$

input
```
int((a + b*tan(c + d*x))^n/sin(c + d*x)^2,x)
```

output
```
int((a + b*tan(c + d*x))^n/sin(c + d*x)^2, x)
```

3.87 $\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx$

3.87.1 Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b(2-n)\cot^2(c+dx)(a+b\tan(c+dx))^{1+n}}{6a^2d} - \frac{\cot^3(c+dx)(a+b\tan(c+dx))^{1+n}}{3ad} + \frac{b(6a^2+b^2(2-3n+n^2))\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^{1+n}}{6a^4d(1+n)}$$

output
```
1/6*b*(2-n)*cot(d*x+c)^2*(a+b*tan(d*x+c))^(1+n)/a^2/d-1/3*cot(d*x+c)^3*(a+
b*tan(d*x+c))^(1+n)/a/d+1/6*b*(6*a^2+b^2*(n^2-3*n+2))*hypergeom([2, 1+n],[
2+n],1+b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^(1+n)/a^4/d/(1+n)
```

3.87.2 Mathematica [A] (verified)

Time = 7.97 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.56

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b\left(a^2\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{b\tan(c+dx)}{a}\right)+b^2\,\text{Hypergeometric2F1}\left(4,1+n,2+n,1+\frac{b}{}\right.\right.}{a^4d(1+n)}$$

input
```
Integrate[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

output
```
(b*(a^2*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a] + b^2*H
ypergeometric2F1[4, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a])*(a + b*Tan[c +
d*x])^(1 + n))/(a^4*d*(1 + n))
```

3.87.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3999, 520, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^n}{\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{3999}$$

$$\frac{b\int \frac{\cot^4(c+dx)(a+b\tan(c+dx))^n\left(\tan^2(c+dx)b^2+b^2\right)}{b^4}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{520}$$

$$\frac{b\left(-\frac{\int \frac{\cot^3(c+dx)(a+b\tan(c+dx))^n\left(b^2(2-n)-3ab\tan(c+dx)\right)}{b^3}d(b\tan(c+dx))}{3a}-\frac{\cot^3(c+dx)(a+b\tan(c+dx))^{n+1}}{3ab}\right)}{d}$$

$$\Big\downarrow \text{87}$$

$$\frac{b\left(-\frac{-\frac{\left(6a^2+b^2(1-n)(2-n)\right)\int \frac{\cot^2(c+dx)(a+b\tan(c+dx))^n}{b^2}d(b\tan(c+dx))}{2a}-\frac{(2-n)\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2a}}{3a}-\frac{\cot^3(c+dx)(a+b\tan(c+dx))^{n+1}}{3ab}\right)}{d}$$

$$\Big\downarrow \text{75}$$

$$\frac{b\left(-\frac{-\frac{\left(6a^2+b^2(1-n)(2-n)\right)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{b\tan(c+dx)}{a}+1\right)}{2a^3(n+1)}-\frac{(2-n)\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2a}}{3a}-\cot^3(c+dx)\right.}{d}$$

input
```
Int[Csc[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

output
```
(b*(-1/3*(Cot[c + d*x]^3*(a + b*Tan[c + d*x])^(1 + n))/(a*b) - (-1/2*((2 -
 n)*Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n))/a - ((6*a^2 + b^2*(1 - n)
*(2 - n))*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a +
b*Tan[c + d*x])^(1 + n))/(2*a^3*(1 + n)))/(3*a)))/d
```

3.87.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 520
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2)^p, e*x, x], R = Pol
ynomialRemainder[(a + b*x^2)^p, e*x, x]}, Simp[R*(e*x)^(m + 1)*((c + d*x)^(
n + 1)/((m + 1)*(e*c))), x] + Simp[1/((m + 1)*(e*c))   Int[(e*x)^(m + 1)*(c
 + d*x)^n*ExpandToSum[(m + 1)*(e*c)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[m, -1] &&  !IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3999
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[b/f   Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

3.87.4 Maple [F]

$$\int \left(\csc^4 (dx + c)\right) (a + b \tan (dx + c))^n \, dx$$

input
```
int(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

output
```
int(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

3.87.5 Fricas [F]

$$\int \csc^4(c + dx)(a + b\tan(c + dx))^n \, dx = \int (b \tan (dx + c) + a)^n \csc (dx + c)^4 \, dx$$

input
```
integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

3.87.6 Sympy [F]

$$\int \csc^4(c + dx)(a + b\tan(c + dx))^n \, dx = \int (a + b \tan (c + dx))^n \csc^4 (c + dx) \, dx$$

input
```
integrate(csc(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*csc(c + d*x)**4, x)
```

3.87.7 Maxima [F]

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan(dx+c)+a)^n\csc(dx+c)^4\,dx$$

input
```
integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

3.87.8 Giac [F]

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan(dx+c)+a)^n\csc(dx+c)^4\,dx$$

input
```
integrate(csc(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \csc^4(c+dx)(a+b\tan(c+dx))^n\,dx=\int \frac{(a+b\tan(c+d\,x))^n}{\sin(c+d\,x)^4}\,dx$$

input
```
int((a + b*tan(c + d*x))^n/sin(c + d*x)^4,x)
```

output
```
int((a + b*tan(c + d*x))^n/sin(c + d*x)^4, x)
```

3.88 $\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$

3.88.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx=\text{Int}\left(\sin^3(c+dx)(a+b\tan(c+dx))^n,x\right)$$

output
```
CannotIntegrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

3.88.2 Mathematica [N/A]

Not integrable

Time = 4.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx=\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

input
```
Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

output
```
Integrate[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

3.88.3 Rubi [N/A]

Not integrable

Time = 2.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4902, 27, 7274, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(c+dx)^3(a+b\tan(c+dx))^n dx$$

$$\Big\downarrow \ 4902$$

$$\frac{2\int \frac{8\tan^3\left(\frac{1}{2}(c+dx)\right)\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n}{\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^4}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{16\int \frac{\tan^3\left(\frac{1}{2}(c+dx)\right)\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n}{\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^4}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \ 7274$$

$$\frac{16\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n\left(a\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^{-n}\int \frac{\tan^3\left(\frac{1}{2}(c+dx\right.}{}}{d}$$

$$\Big\downarrow \ 7292$$

$$\frac{16\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n\left(a\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^{-n}\int \frac{\tan^3\left(\frac{1}{2}(c+dx\right.}{}}{d}$$

$$\Big\downarrow \ 7293$$

$$16\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n\left(a\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^{-n}\int \left(\frac{\tan\left(\frac{1}{2}(c+d\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$16\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n\left(a\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^{-n}\left(\int\frac{\tan\left(\frac{1}{2}(c+d\right.}{}\right.$$

input
```
Int[Sin[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

output
```
$Aborted
```

3.88.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

rule 7274
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Simp[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p])   I
nt[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Integ
erQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.88.4 Maple [N/A] (verified)

Not integrable

Time = 3.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(\sin^3\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^n dx$$

input
```
int(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

output
```
int(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

3.88.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.52

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx=\int \left(b\tan\left(dx+c\right)+a\right)^n\sin\left(dx+c\right)^3\,dx$$

input
```
integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(-(cos(d*x + c)^2 - 1)*(b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

3.88.6 Sympy [F(-1)]

Timed out.

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx=\text{Timed out}$$

input
```
integrate(sin(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
```

output
```
Timed out
```

3.88.7 Maxima [N/A]

Not integrable

Time = 4.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n \sin(dx+c)^3\,dx$$

input
```
integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

3.88.8 Giac [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n \sin(dx+c)^3\,dx$$

input
```
integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

3.88.9 Mupad [N/A]

Not integrable

Time = 7.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin(c+d\,x)^3\,(a+b\tan(c+d\,x))^n\,dx$$

input
```
int(sin(c + d*x)^3*(a + b*tan(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)^3*(a + b*tan(c + d*x))^n, x)
```

3.89 $\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$

3.89.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx = \text{Int}(\sin(c+dx)(a+b\tan(c+dx))^n,x)$$

output
```
CannotIntegrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

3.89.2 Mathematica [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$$

input
```
Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

output
```
Integrate[Sin[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

3.89.3 Rubi [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4902, 27, 7274, 7292, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)(a+b\tan(c+dx))^n dx$$

$$\Big\downarrow \text{4902}$$

$$\frac{2\int \frac{2\tan\left(\frac{1}{2}(c+dx)\right)\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n}{\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^2}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n}{\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^2}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{7274}$$

$$\frac{4\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n\left(a\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^{-n}\int \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{}}{d}$$

$$\Big\downarrow \text{7292}$$

$$\frac{4\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n\left(a\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^{-n}\int \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{}}{d}$$

$$\Big\downarrow \text{7299}$$

$$\frac{4\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n\left(a\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^{-n}\int \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{}}{d}$$

input
```
Int[Sin[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

output
```
$Aborted
```

3.89.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

rule 7274
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Simp[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p])   I
nt[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Integ
erQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.89.4 Maple [N/A] (verified)

Not integrable

Time = 0.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sin (dx+c) (a+b\tan (dx+c))^n dx$$

input
```
int(sin(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

output
```
int(sin(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

3.89.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sin(c+dx)(a+b\tan(c+dx))^n dx = \int (b\tan (dx+c)+a)^n \sin (dx+c)\, dx$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

3.89.6 Sympy [N/A]

Not integrable

Time = 4.72 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sin(c+dx)(a+b\tan(c+dx))^n dx = \int (a+b\tan (c+dx))^n \sin (c+dx)\, dx$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*sin(c + d*x), x)
```

3.89.7 Maxima [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\sin(dx+c)\,dx$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

3.89.8 Giac [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\sin(dx+c)\,dx$$

input
```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sin(d*x + c), x)
```

3.89.9 Mupad [N/A]

Not integrable

Time = 5.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sin(c+dx)(a+b\tan(c+dx))^n\,dx = \int \sin(c+d\,x)\,(a+b\tan(c+d\,x))^n\,dx$$

input
```
int(sin(c + d*x)*(a + b*tan(c + d*x))^n,x)
```

output
```
int(sin(c + d*x)*(a + b*tan(c + d*x))^n, x)
```

3.90 $\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$

3.90.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx = \mathrm{Int}(\csc(c+dx)(a+b\tan(c+dx))^n,x)$$

output
```
CannotIntegrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

3.90.2 Mathematica [N/A]

Not integrable

Time = 4.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx = \int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$$

input
```
Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

output
```
Integrate[Csc[c + d*x]*(a + b*Tan[c + d*x])^n, x]
```

3.90.3 Rubi [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4902, 27, 7274, 2084, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^n}{\sin(c+dx)}dx$$

$$\Big\downarrow \text{4902}$$

$$\frac{2\int \frac{1}{2}\cot\left(\frac{1}{2}(c+dx)\right)\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \cot\left(\frac{1}{2}(c+dx)\right)\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{7274}$$

$$\frac{\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n\left(a\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^{-n}\int \cot\left(\frac{1}{2}(c+dx\right.}{d}$$

$$\Big\downarrow \text{2084}$$

$$\frac{\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n\left(a\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^{-n}\int \cot\left(\frac{1}{2}(c+dx\right.}{d}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n\left(a\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^{-n}\int \cot\left(\frac{1}{2}(c+dx\right.}{d}$$

input
```
Int[Csc[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

output
```
$Aborted
```

3.90.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2084
```
Int[(u_)^(p_.)*(v_)^(q_.)*(z_)^(m_.), x_Symbol] :> Int[ExpandToSum[z, x]^m*
ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{m, p, q}, x] && Linea
rQ[z, x] && QuadraticQ[{u, v}, x] &&  !(LinearMatchQ[z, x] && QuadraticMatc
hQ[{u, v}, x]) &&  !MatchQ[z^m*u^p*v^q, ((d_.) + (e_.)*x)^m*((f_.) + (g_.)*
x)^2*((a_.) + (b_.)*x + (c_.)*x^2)^(t_.) /; FreeQ[{a, b, c, d, e, f, g, t},
 x]] &&  !MatchQ[z^m*u^p*v^q, ((d_.) + (e_.)*x)^m*((f_.) + (g_.)*x)^2*((a_.
) + (c_.)*x^2)^(t_.) /; FreeQ[{a, c, d, e, f, g, t}, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

rule 7274
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Simp[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p])   I
nt[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Integ
erQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.90.4 Maple [N/A] (verified)

Not integrable

Time = 1.53 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \csc\left(dx+c\right)\left(a+b\tan\left(dx+c\right)\right)^{n} dx$$

input
```
int(csc(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

output
```
int(csc(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

3.90.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx = \int \left(b\tan\left(dx+c\right)+a\right)^{n}\csc\left(dx+c\right)\,dx$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*csc(d*x + c), x)
```

3.90.6 Sympy [N/A]

Not integrable

Time = 3.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^{n}\csc\left(c+dx\right)dx$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*csc(c + d*x), x)
```

3.90.7 Maxima [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan(dx+c)+a)^n\csc(dx+c)\,dx$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*csc(d*x + c), x)
```

3.90.8 Giac [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan(dx+c)+a)^n\csc(dx+c)\,dx$$

input
```
integrate(csc(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*csc(d*x + c), x)
```

3.90.9 Mupad [N/A]

Not integrable

Time = 4.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \csc(c+dx)(a+b\tan(c+dx))^n\,dx=\int \frac{(a+b\tan(c+d\,x))^n}{\sin(c+d\,x)}\,dx$$

input
```
int((a + b*tan(c + d*x))^n/sin(c + d*x),x)
```

output
```
int((a + b*tan(c + d*x))^n/sin(c + d*x), x)
```

3.91 $\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$

3.91.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx=\mathrm{Int}\left(\csc^3(c+dx)(a+b\tan(c+dx))^n,x\right)$$

output
```
CannotIntegrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

3.91.2 Mathematica [N/A]

Not integrable

Time = 23.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx=\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

input
```
Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

output
```
Integrate[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n, x]
```

3.91.3 Rubi [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4902, 27, 7274, 7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^n}{\sin(c+dx)^3}dx$$

$$\Big\downarrow \text{4902}$$

$$\frac{2\int \frac{1}{8}\cot^3\left(\frac{1}{2}(c+dx)\right)\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^2\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \cot^3\left(\frac{1}{2}(c+dx)\right)\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^2\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n d\tan\left(\frac{1}{2}(c+dx)\right)}{4d}$$

$$\Big\downarrow \text{7274}$$

$$\left(1-\tan^2\left(\tfrac{1}{2}(c+dx)\right)\right)^n\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n\left(a\left(1-\tan^2\left(\tfrac{1}{2}(c+dx)\right)\right)+2b\tan\left(\tfrac{1}{2}(c+dx)\right)\right)^{-n}\int \cot^3\left(\tfrac{1}{2}(c+d\right.$$

$$\Big\downarrow \text{7292}$$

$$\left(1-\tan^2\left(\tfrac{1}{2}(c+dx)\right)\right)^n\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n\left(a\left(1-\tan^2\left(\tfrac{1}{2}(c+dx)\right)\right)+2b\tan\left(\tfrac{1}{2}(c+dx)\right)\right)^{-n}\int \cot^3\left(\tfrac{1}{2}(c+d\right.$$

$$\Big\downarrow \text{7293}$$

$$\left(1-\tan^2\left(\tfrac{1}{2}(c+dx)\right)\right)^n\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n\left(a\left(1-\tan^2\left(\tfrac{1}{2}(c+dx)\right)\right)+2b\tan\left(\tfrac{1}{2}(c+dx)\right)\right)^{-n}\int \left(\cot^3\left(\tfrac{1}{2}(c+\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(a+\frac{2b\tan\left(\frac{1}{2}(c+dx)\right)}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n\left(a\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^{-n}\left(\int\tan\left(\frac{1}{2}(c+\right.\right.$$

input
```
Int[Csc[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

output
```
$Aborted
```

3.91.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

rule 7274
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Simp[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p])   I
nt[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Integ
erQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.91.4 Maple [N/A] (verified)

Not integrable

Time = 1.57 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(\csc^3\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^n dx$$

input
```
int(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

output
```
int(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

3.91.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx=\int \left(b\tan\left(dx+c\right)+a\right)^n\csc\left(dx+c\right)^3\,dx$$

input
```
integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

3.91.6 Sympy [N/A]

Not integrable

Time = 34.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx=\int \left(a+b\tan\left(c+dx\right)\right)^n\csc^3\left(c+dx\right)dx$$

input
```
integrate(csc(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*csc(c + d*x)**3, x)
```

3.91.7 Maxima [N/A]

Not integrable

Time = 2.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\csc(dx+c)^3\,dx$$

input
```
integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

3.91.8 Giac [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\csc(dx+c)^3\,dx$$

input
```
integrate(csc(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

3.91.9 Mupad [N/A]

Not integrable

Time = 6.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \csc^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int \frac{(a+b\tan(c+d\,x))^n}{\sin(c+d\,x)^3}\,dx$$

input
```
int((a + b*tan(c + d*x))^n/sin(c + d*x)^3,x)
```

output
```
int((a + b*tan(c + d*x))^n/sin(c + d*x)^3, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```